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MULTIVARIABLE COMPUTER CONTROL OF AN EVAPORATOR

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled MULTIVARIABLE COMPUTER CONTROL OF AN EVAPORATOR submitted by ROBERT B. NEWELL, B.Sc.App., B.E., in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

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ABSTRACT

This thesis presents the design and development of optimal multivariable regulatory control systems suitable for implementation on computer controlled industrial processes. Experimental data from a pilot plant evaporation process are included to demonstrate the practicality of the methods developed and the significant improvement over conventional feedback control systems.

The optimal control problem is formulated with a quadratic performance index and solved using discrete dynamic programming to yield the basic optimal feedback controller reported previously in the literature. Extensions of the basic formulation are developed which generate multivariable proportional-plus-integral, feedforward, and setpoint control.

Simulated results show that small control intervals and heavy weighting of the critical controlled variables produce the best control. However experimental data show that larger control intervals and less weighting are more practical, demanding less computer time and reacting less to process noise.

Proportional feedback control by itself gives very satisfactory control because the multivariable gains are an order of magnitude greater than those for conventional control resulting in negligible offsets from constant loads.

A formulation for multivariable proportional-plus-integral control was developed which although it increases the order of the

problem removes offsets caused by unmeasured loads. The concept of the degrees of freedom of a control system proved to be important in multivariable systems.

A generalized formulation for multivariable feedforward control is presented which serves as a unifying basis for new and existing design methods which eliminate or minimize steady-state offsets or, in conjunction with feedback, minimize a summed quadratic index.

A model following approach for step changes in setpoints is compared experimentally with an approach which introduces setpoints directly into the basic multivariable formulation. Model following gives control over the form of the transient and minimizes interactions during setpoint changes.

Related aspects of this work include modelling, multiloop and noninteracting design, and implementation.

A generalized modelling procedure is developed which is applicable to the modelling of multi-effect industrial evaporators regardless of configuration. This procedure when applied to a pilot plant double effect evaporator at the University of Alberta produces a tenth-order nonlinear model. This is simplified to a fifth-order state space model which still represents the process well and is convenient for design.

The optimal multivariable control is compared to a multiloop configuration designed for the evaporator and a noninteracting control scheme designed for a distillation column model.

A FORTRAN program which implements multivariable control using a time-shared IBM 1800 control computer is described with practical experiences. Experimental data show that the optimal control system performs better and requires only slightly more execution time than Direct Digital Control. It is recommended as suitable for industry particularly for interacting processes and where control is critical.

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CHAPTER ONE

INTRODUCTION

Typical industrial control systems have been implemented by pneumatic or electronic analogue controllers arranged in a multiloop feedback configuration. In a small number of areas special purpose analogue "computers" have been used for feedforward control, usually with great improvement in control quality. However, there has been very little implementation of modern control techniques despite advances in the theory.

The flexibility and computational abilities of the digital control computer are now being used to centralize information and control. The large numbers of analogue instruments are being "simulated" by Direct Digital Control (DDC) systems and the use of control modes such as feedforward and ratio control is becoming more common in industry. These industrial computer installations now make the implementation of modern control techniques practical, and there is a need for standard proven design methods and implementation techniques.

Multivariable control is still mainly in the hands of the theorists although the number of simulation studies is increasing rapidly. Practical implementations of multivariable control, state driving or regulatory, are still few and far between.

Control studies in the Department of Chemical and Petroleum Engineering at the University of Alberta have been based upon a pilot

plant double effect evaporator and a nine inch eight tray distillation column. Evaporator control studies began with modelling [1] and the use of special purpose feedforward controllers [2]. The acquisition of an IBM1800 digital control computer in 1967 permitted rapid expansion of the scope of research to include multiloop DDC control [3,4] with extensions to feedforward and inferential control. Present research includes the development and implementation of optimal multivariable control. Optimal regulatory multivariable control has been studied in this work and parallel research has examined state driving control techniques [5].

1. AIMS OF THE STUDY

The general objective of this thesis is to develop and extend existing theory for the application of multivariable control in a process plant environment. The more specific objectives are summarized as follows:

(i) to examine the structure of the field of multivariable regulator design and to make a broad survey of the literature in this field.

(ii) to develop a model of existing pilot plant scaled process equipment that would be suitable for the design of multivariable control systems.

(iii) to develop a design basis for multivariable regulatory control. Multiloop, noninteracting, and optimal multivariable control approaches are to be considered with the emphasis on the latter.

(iv) to use a digital control computer to implement

multivariable control on process equipment and compare the results to those of conventional multiloop techniques.

2. THE STRUCTURE OF THE THESIS

The introduction and chapter containing the survey of control system design literature are followed by seven sections which form the main body of the thesis. These sections are presented in the format used by most technological journals. It is hoped that this approach will result in a concise presentation that will be convenient for the reader particularly if he wishes to refer to only part of the work. Chapters detailing additional supporting simulations and the conclusions and suggestions for further research conclude the thesis. A users manual for the control program and data books of experimental and simulated runs are compiled under separate cover.

The following paragraphs outline the main contributions of the thesis and indicate the connection and natural progression from one topic to the next. The material in each section is summarized in an ABSTRACT on the first page of each chapter.

A broad literature survey was undertaken with particular emphasis on multivariable control system design. In order to put the work in perspective, the literature was structured and is presented as the next chapter.

The pilot plant double effect evaporator in the department was selected as the primary application for this work. The development of a linear state space model required by the design

techniques examined is detailed in the third chapter. A generalized model building approach was taken to the development. The reduced and linearized model is evaluated using experimental data.

The multiloop control schemes used in previous work on the evaporator were based primarily on experience and frequency domain design techniques. An investigation was made into the use of a state space model as a basis for the design of a multiloop system of conventional feedback controllers. Both the design of a configuration and the choice of controller constants are examined.

Chapters five through eight deal with the design and evaluation of an optimal multivariable control scheme. The basic control problem formulation, the development of a calculational algorithm for its solution, and a study of the design parameters are presented in the first of these chapters. Chapter Six presents a generalized formulation of and evaluates some design methods for multivariable feedforward control from measurable disturbances. Multivariable integral control is formulated and evaluated in the next section as a solution to offsets caused by disturbances which cannot be measured. A setpoint following control mode which does not interfere with the regulatory action is then formulated to complete the control system.

Experience gained during implementation of the multivariable control by digital control computer on the pilot plant evaporator emphasized several practical criteria that must be considered. These are discussed in Chapter Nine. In addition, this section and the

program documentation manual are used to present some of the details of implementation that should be of interest to others who implement similar control techniques.

A number of simulation studies are made on a distillation column model taken from the literature in order to show that some of the conclusions are not specific to the evaporator model. The results of these studies and a comparison of multiloop, noninteracting, and optimal multivariable control of the column are presented in Chapter Ten.

Each section of the main body of the thesis contains conclusions pertaining to its specific subject. However the thesis concludes with an overall discussion of the multivariable control scheme and conclusions that can be drawn from the complete study. Suggested areas for further development are also presented in this final chapter.

CHAPTER TWO

MULTIVARIABLE CONTROL SYSTEM DESIGN

AN INTRODUCTION TO THE FIELD AND LITERATURE SURVEY

ABSTRACT

A structure of the field of multivariable control system design and a broad literature survey is presented. The structuring of the field provides an orientation for newcomers to multivariable control system design and gives a framework into which the different aspects of control system design can be placed in a logical manner. The literature surveyed is not definitive of the complete field but provides a starting point for more detailed examinations of particular aspects. The four major design approaches, classical, optimal, stochastic, and adaptive, are discussed together with process modelling and control systems implementation.

1. INTRODUCTION

Multivariable control has been defined as "an advanced level of control achieved when explicit control actions are calculated on an integrated basis after taking into account the multiplicity of variables and the complexities of their interrelations". [[1] page 24].

A multiloop control system, as found in most process plants today, can be considered a special case of multivariable design. The "interrelations" are not explicitly dealt with in the design of these multiloop systems although the interactions are often considered qualitatively and may be allowed for in the field tuning of the controllers. However, multivariable control systems as usually understood do take these interrelations or interactions into account explicitly, and for this reason should perform better than a multiloop system.

There are major hurdles in designing and implementing multivariable control schemes, that is, those schemes which consider the system as a whole rather than as a large number of small subsystems. The relevant mathematical relationships between variables are required and any subsequent control system can only be as good as this model. Implementation of these systems requires either a number of specialized "black boxes" or a single more flexible digital, hybrid, or analogue computer. Computing hardware is more in evidence in process plants today [2,3] but due to lack of applied technological knowledge only limited use is made of its flexibility for multivariable control.

The survey of the literature is not intended to be definitive

for the complete field, but rather to present a newcomer with an organization of the field and an introduction to the literature. The principle topic of this work developed into optimal regulatory control by dynamic programming and the later chapters include references to the directly applicable literature. However no attempt was made to present a complete, detailed review, or a critical comparison of alternative design or calculational techniques.

Figure 1 presents a block diagram summary of the design of a multivariable control system. The literature dealing with all four major design paths, classical, optimal, stochastic, and adaptive, will be discussed generally in this section but more detailed attention will be paid to the classical and optimal paths. The one technique common to all design paths is that of modelling and for this reason it will be discussed separately.

2. MODELLING MULTIVARIABLE SYSTEMS

Model classifications are generally very broad in nature [[1] page 39; [4] page 15] but can be simplified when the interest is in control system design. The major divisions in the classification of models are static and dynamic systems. Within these divisions such factors as dimension, derivation, and linearity are important.

2.1. Static Models

Static or steady state models are systems of linear or nonlinear algebraic equations and are usually derived empirically or

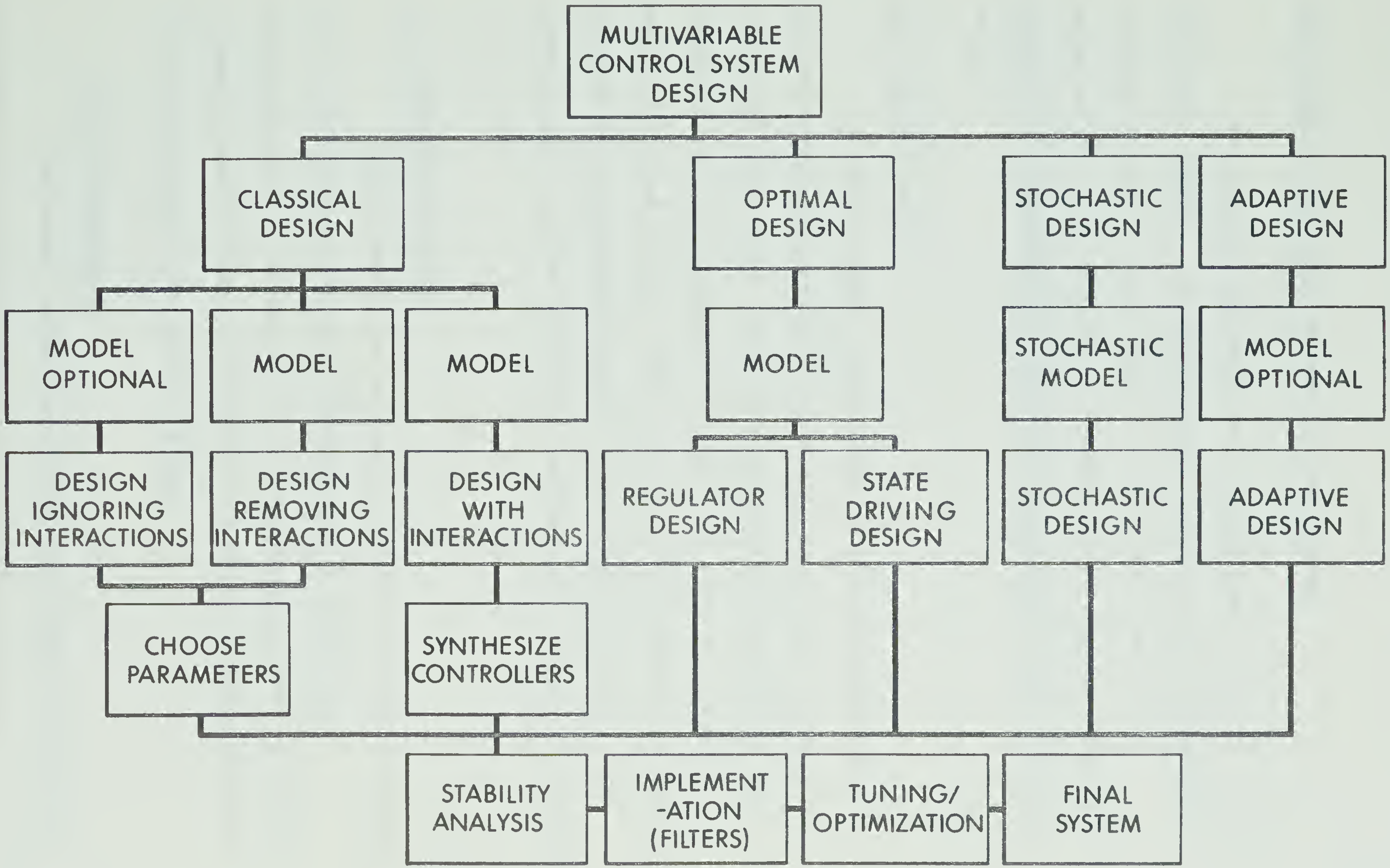


FIGURE 1. FLOWSHEET OF DESIGN PATHS

from material and energy balances on the process.

Such models have wide use in scheduling, inventory control, and process control. In multivariable control static mathematical relations find wide use in both on-line and off-line process optimization, and in material or energy balance control. They also find use in feedforward and nonlinear compensation modes of process control.

2.2. Dynamic Models

A dynamic model, particularly if derived analytically, requires a much greater knowledge of the quantitative behavior of individual plant units. They find wide application ranging from simulation studies of processes and control schemes to dynamic optimization and control. Empirical models or theoretical models with experimentally determined parameters are also widely used. The identification problem involved in evaluating parameters is a field of its own [5].

2.3. Modelling Domains

Dynamic models are basically systems of nonlinear differential or difference equations with time-varying coefficients. Transformation of linear models to the frequency domain is popular, although recent advances in computers and numerical mathematics has led to more analyses in the time domain.

2.3.1. Time Domain

Time domain representation takes the form of differential equations for continuous systems and difference equations for discrete

systems. The relations are generally nonlinear and time-varying although it is common practice to linearize and assume time invariant coefficients. These relations are often normalized about their steady state to remove their dependence on units of measurement and to remove numerical scaling problems, often encountered in simulation work. Repeated relinearization is often used where steady state operating conditions or time-varying coefficients change significantly. Nonlinear gain corrections [6] are applied where the linearization seriously affects the model gains.

2.3.2. Frequency Domain

Representation in the frequency domain is generally restricted to systems of linear differential or difference equations. The Laplace transform introduces the "s-domain" in which much of the classical control theory [7] has been developed and which is still in wide use for the single variable system, usually a subsystem with a single output and a single control variable. While the "s-domain" represents continuous systems, sampled or discrete systems are analysed by the more recently developed "z-transform". The representation and analysis techniques are analogous in most cases.

Signal flowgraph analysis [8,9] is also used for linear analysis, and recent extensions [10] enable sampled-data and nonlinear analysis to be included.

2.3.3. State Space

Linear models can always be reduced to a set of first order differential or difference equations and represented in state space in

matrix form [11].

$$\underline{\dot{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} + \underline{\underline{D}}\,\underline{d} \qquad (1)$$

$$\underline{y} = \underline{\underline{C}}\,\underline{x}\ (+\ \underline{\underline{E}}\,\underline{u} + \underline{\underline{F}}\,\underline{d} + \underline{\underline{G}}\,\underline{v}) \qquad (2)$$

State space is receiving the most attention in recent literature and several papers deal with transformations within this domain into canonical forms. Canonical forms are particularly convenient for the solution of Lyapunov stability relations [12], for improving a models performance [13], and for mathematically simplifying the design problem.

This simplification is achieved by transforming the system so that the redefined variables are decoupled either individually or into subsystems. Such simplification can be of the model itself [14,15] or of the control system design problem incorporating the model [16,17,18,19]. The procedure is likely to be particularly useful in designing control systems for very large processes [20].

Processes are often more amenable to modelling in one domain and analysis in another. Papers have outlined conversion procedures for transformations such as between flow-graphs and transfer functions [21], between the frequency domain and state space [22,23,24], and from continuous to discrete state space [25,11]. The commonly used transformations back and forth between transfer functions or differential equations and the state space form can be found in many texts [[11] Chapter Four].

2.4. Model Reduction

High order models often present problems in the formulation, the computational solution, and the implementation of multivariable control. The order of a model can be reduced by either fitting a simpler model by matching responses or simplifying the existing relations. In either case integral criteria are commonly used as a measure of "goodness of approximation" [26,27]. When a simpler model is empirically chosen its parameters are estimated from response data generated by the original model. There is a great variety of identification procedures which would be suitable and these are well reviewed by Nieman, Fisher, and Seborg [5].

The first step in simplification of the existing model involves the partitioning into independent subsystems [22] which can be solved as separate problems. Davison [28] introduced model reduction which has received attention in a number of papers [29,26, 30,31] and can be applied to dependent systems of equations. Aggregating states [32,33] and subspace projection methods [34] are also available. Many papers deal with mathematical transformations [13,15,35,36] but these often mask the physical significance of the original variables.

The optimization problem rather than the model can be reduced in order. A number of special cases [37,38,39,40] are treated in the literature but only decomposition of linear programming problems [41] can be considered general.

2.5. State Space Models of Process Equipment

This work used models of available equipment which includes a double-effect evaporator [42] and a distillation column [43]. Huckaba, et al. [44] and a number of other papers [45,46] also present distillation models.

Models with more than ten state variables include boilers [47,48,31] and the small process of Williams and Otto [49]. The six plate absorber of Lapidus and Luus [[50] page 49] appears in several optimal state driving papers. Many of the models presented in the literature do not include enough data to be able to put them to immediate use.

3. CLASSICAL MULTIVARIABLE DESIGN

The classical multivariable design approaches rely heavily on the established and accepted single variable design procedures. There have been three basic attacks on the problem, one resulting in a multiloop system while the others result in a more complex arrangement of compensators.

3.1. Multiloop Design

A control loop is based on a single output variable being controlled by a single manipulated variable. Classical multiloop systems are made up of a number of these control loops, but the interactions between variables in one loop and those in the other loops are not dealt with explicitly by the design techniques. These design techniques are often based on "experience" [51] although a sensitivity analysis [52,53,54,55] may be used for pairing the process variables.

Additional compensation such as ratio control is sometimes added on the basis of "experience" [[56] page 153] or loops are tuned in the field to handle severe interactions.

Once the control loops are chosen the form and parameters of the controllers must be specified. In classical single loop design controllers are usually proportional with the addition of integral and derivative action where required. Nonlinear controllers are used to advantage in some instances [[56] page 124]. Controller constants depend in the first instance on the broad aim of the system, servo or regulatory control [[56] part 1]. Constants can be estimated by empirical relations based on parameters determined by on-line testing such as those of Ziegler and Nichols or by design methods based upon time or frequency domain criteria [7]. Design techniques such as Bode and Nyquist analyses are basically in the frequency domain and are based on stability criteria [[7] page 476]. Feedforward control [[56] page 204] can be added to regulatory control systems with considerable advantage.

3.2. Noninteracting Design

A compensating scheme is first designed which will result in a noninteracting system [[1] page 153] and then the noninteracting pairs of manipulated and output variables are combined into separate loops for control purposes.

Several methods exist for the design of noninteracting systems. Diagonalization of the transfer function matrix can be achieved by feedback compensation [57,58,59,60,61,62,63,64,65,66], feedforward compensation [67], or a combination configuration

[68,69,70]. Signal flowgraphs [69,70,61] and z-transforms [59,71] also find use in the analysis. Difficulties often arise in realizing the required often complex compensators [72,[7] page 480] and some authors have considered a minimization of interactions [73,74] to escape this problem.

Falb and Wolovich [75] present necessary and sufficient conditions for achieving noninteraction by feedback in state space. Mufti [76] presents an alternate proof. Gilbert [17] and others [77,78,79,80] present design techniques based on canonical forms and transformations. Gilbert and Pivnichny [81] have developed a computer algorithm for this design approach. Partial decoupling has also received some attention [82].

Once a system is noninteracting the usual multiloop design can be applied for control although some authors [77,83,84,80,71] include this in their noninteracting design.

3.3. Design for a Specified Response

This more complex classical approach to designing multivariable control systems is dependent upon specifying the desired behaviour for the closed-loop system. With the desired closed-loop model and the open-loop model of the process, a system of compensators can be found by simple matrix algebra. The major problem in this type of design is often the physical synthesis of the required controllers. The technique often requires iterative modifications to the desired closed-loop model in order to realize the compensators.

In the frequency domain the desired model can be specified

as a matrix of transfer functions as Laplace transforms [85,86,87,88, 89,90] or z-transforms [91,92,71]. Stability criteria are also used to specify the desired behavior [93,94] requiring a trial-and-error approach.

In state space the response is specified either in terms of conventional criteria [95,96,97], of a coefficient matrix [98], or of pole and zero locations. Wonham [99] and Davison [100] present necessary and sufficient conditions for pole placement by state feedback control. A number of algorithms [16,80,101] are available to aid in pole placement but Rosenbrocks modal control [102] is perhaps the most versatile. A complete theory of modal analysis is developed by Simon and Mitter [36]. The technique is used [103, 104] particularly for distillation examples [45,102].

4. OPTIMAL MULTIVARIABLE DESIGN

Optimal control is the backbone of "modern"control theory. While some optimal techniques will analyse non-linear time-varying process models, the most practical methods from the viewpoints of complexity, computation, and implementation are restricted to linear models with time invariant parameters. For this reason the "state space" approach which arranges linear differential equations into a single vector relationship is popular [[11] page 1; [7] page 570].

An optimization problem is determining some action, subject to a set of constraints, that extremizes a criterion. The control system design basically involves the formulation of the optimization problem while the solution is more of a mathematical technique.

Although these aspects are to be discussed separately their interdependence should not be underestimated. A formulated problem must be solved before it can be used.

4.1. Problem Formulation

Control problems represented in the frequency domain can be optimized but suffer from the complexity of the algebraic manipulations required in their solution and from practical limitations in dimension. Usually the controller form is chosen and procedures such as calculus of variations, solved by the usual techniques [105,106,107,108] or Wiener's theory [109,110,111], are used to evaluate optimal parameters. A few algorithms are available for specific criteria such as settling time [112] and pole placement [101,113].

Generally speaking the state space formulation proves to be more flexible and presents more general techniques for the solution of the optimization problem. Kalman [114,115] presents much of the basic theory on controllability and observability and techniques for the formulation of the process control problem. Formulation of the more complex distributed parameter case is reviewed by Wang and Tung [116] but will not be considered in this review. General discussions on the effects of control on such system characteristics as eigenvalues [103,117] is also found in the literature.

The first factor to be considered in formulating an optimization problem is the criterion. The criteria basically separate into servo or regulatory control.

Servo or state driving control arises from the desire to

change the process operating conditions. Open-loop driving involves precalculation, usually off-line, of the manipulated variables as functions of time. Closed-loop servo control requires tight tuning of feedback control systems. Both the manipulated variables and the corresponding setpoints of a closed-loop can then correct for disturbances and also modelling errors. Minimum time is a common servo criterion. Optimal regulatory control formulated with an integral error criterion often results in a closed loop control law of the following form.

$$\underline{u} = \underline{\underline{K}}_{FB}\underline{x} + \underline{\underline{K}}_{FF}\underline{d} \tag{3}$$

The usual regulatory criterion is an integral quadratic function of errors and control actions.

$$J = \underline{x}^T(T)\underline{\underline{S}}\underline{x}(T) + \int_o^T e^{\beta t}(\underline{x}^T(t)\underline{\underline{Q}}(t)\underline{x}(t) + \underline{u}^T(t)\underline{\underline{R}}(t)\underline{u}(t))dt \tag{4}$$

A time weighting factor [118,119] and a parameter sensitivity function [120] can be added. This choice of criterion is made on the grounds of producing "good" control responses and, equally as important, of being mathematically tractable. The weighting matrix associated with the error vector is normally diagonal and positive definite although there are special exceptions [121, 122]. The choice of the elements of the matrix is still very much left to the judgement of the designer. Iterative methods exist [121] and for a few special cases the elements are related to other control criteria [123,124,125,126]. The weighting matrix on the control action may be chosen iteratively to prevent control saturation [127].

The other important factor is the constraint set. The model is the basic constraint and in some cases a specified control law is included. Methods differ greatly in their ability to handle other constraints. Many of the optimization methods, particularly the analytic techniques for continuous systems, work only in unconstrained or "softly" constrained domains. Other techniques permit "hard" constraints on the control variables [128] or control energy [68] and a few permit time-varying "hard" constraints on the state and control variables [129] but usually at considerable increase in complexity or calculational time.

4.2. Problem Solution

There are three techniques most commonly used to solve the process control optimization problem.

(a) Classical calculus of variations [[130] page 221; [131] page 171]

(b) Pontryagin's maximum principle [[130] page 284; [131] page 182; [50] page 87]

(c) Dynamic programming [[131] page 270; [50] page 69]

Other techniques include linear and nonlinear programming and the usual direct search methods.

Calculus of variations is perhaps the oldest and most general of the techniques. It is well introduced by Douglas and Denn [132] and detailed in such texts as Sage [[133] Chapters 3 and 6]. The general nature of the technique allows the incorporation of both time-delays and constraints [134, 135, 136, 137]. Formulation of a problem leads to vector two-point boundary value problems which can be solved

analytically in only a few special cases. Numerical solutions very often lead to prohibitive amounts of computation.

Pontryagins maximum principle [[133] Chapters 4 and 6] is a more recent formulation of the classical calculus of variations which makes it easier to introduce bounds on the state [129] and control vectors. It presents necessary but not sufficient conditions of optimality and there still remains a boundary value problem to be solved. A number of simulation techniques [138,139] are available for the solution. The maximum principle can be used for problems involving time-delays [140,141,142]. In its pure form an open-loop solution [143] of the control problem results, but if the problem is formulated with a quadratic [87] or time weighted quadratic [118] index, a transformation of the adjoint variables produces a closed-loop solution in the Ricatti equation [[50] page 165; [144]]. The Ricatti equation is solved by specialized algorithms [144,145] as well as by conventional techniques [146]. The Ricatti transformation is used to formulate proportional-plus-integral [147] and feedforward [148] control problems. Brosilow and Handley [149] present a solution for a fifteen tray distillation column and obtain good regulatory control of a pilot plant four-inch column.

The functional equations of dynamic programming are equivalent to the calculus of variations formulation with unconstrained control and this approach is treated in detail in several texts [150,151]. Advantages include easy handling of constraints in many formulations and the fact that it finds a global extremum. Computations are simplified to stage-by-stage rather than simultaneous

calculations but the technique generally suffers badly from "the curse of dimensionality". Formulations for the optimum linear regulator are plentiful [31,48,152,153,154,155,156,157] with necessary and sufficient conditions for a minimum derived by Moore and Colebatch [158]. Time delays can be included in the formulation [159,160,161,162]. Single-step optimal control by dynamic programming finds frequent application [31,46,163,164]. Noton and Choquette [165,166] report on one of the few attempts at implementation.

Design by Liapunov's second method, variations on minimizing the derivative of the Liapunov function, is presented by many authors [167,168,169,170, [50] page 334; 171,172]. An advantage of this technique is the guarantee of stability. Other procedures for the solution of the optimal control problem include second variation methods [173], gradient techniques [39,174,175,176,177,178], quadratic [179,180] and linear [181] programming, and a number of approximation techniques [182,183,184,185,186,187].

Optimal feedforward control receives only limited attention in the literature. Heideman and Esterson [154] combined dynamic programming and a static model and Anderson [48,152,153] formulated "error coordinates" for, effectively, "integral" feedforward. Other attempts exist which make use of calculus of variations [188] and the maximum principle [148].

5. STOCHASTIC MULTIVARIABLE DESIGN

Techniques for the design of optimal control schemes for stochastic systems constitute a rapidly expanding field. There are

two approaches to the problem, either incorporation of the stochastic properties into the design procedures [189,190] or invoking the concept of the Separation Theorem [191] which is restricted to linear systems.

The Separation Theorem [[133] page 312] essentially states that deterministic design techniques and control laws can be utilized provided that the control law acts upon an optimal estimate of the state. This estimate may be obtained by the use of an optimal filter. Stochastic estimation of variables began with the work of Wiener which is described in the text by Sage [[133] Chapter 8]. This is generally applicable to single variables and the extension to multivariable systems is presented by Kalman [192,193]. Kalman developed recursive relations for general systems which simplify for time invariant stationary systems. Park [194] presents a suboptimal filter for the time-varying process.

Most attempts to integrate statistics into the design procedure deal with dynamic programming. Most problems can be precisely stated and a number of particular cases solved explicitly, but the techniques are not yet of technological importance. Isolated attempts to incorporate statistical properties into classical design techniques produce complex formulations and the advantages are doubtful. There is some question whether the extra effort involved in the design and implementation of a stochastic control system is worthwhile. Wonham [195] believes that "in the case of feedback controls the general conclusion is that only marginal improvements can be obtained unless the disturbance level is very high". However, for open-loop systems, noise causes considerably more trouble and a number of techniques for

periodic correction have been examined [195].

6. ADAPTIVE MULTIVARIABLE DESIGN

Eveleigh [[196] page 135] defines an adaptive system "as one which measures its performance relative to a given performance index and modifies its parameters to approach an optimal set of values". Most industrial systems are controlled by a fixed parameter control system chosen to perform suitably over all likely conditions. In other cases time-varying preprogrammed control is used if the process is well enough defined. If conditions rule out these possibilities then adaptive control comes into its own.

Generalized design techniques for adaptive control are still very much in their infancy. Design is usually approached with the three major functions of adaptive control in mind. These functions, not always distinct, are identification or evaluation, decision, and modification [196].

(a) Identification in this sense refers to the evaluation of an index of performance. This varies from functions of single or multiple plant variables or parameters, which may require estimation or identification in the usual sense, to more sophisticated model reference systems.

(b) The decision process varies from custom logic based on "experience" to more general optimizing techniques. A common procedure is the gradient technique using steps or periodic perturbations in the "control parameters" to evaluate gradients. There are common peak-holding methods which also find application. A sophistication

of the usual adaptive decision making process is found in learning systems which work on the basis of an accumulation of knowledge. This is another rapidly expanding field.

(c) Modification is often simple implementation of "control parameter" changes. In dual- or multi-mode systems it is a simple switching of mode. Signal synthesis systems require more sophisticated modification schemes.

7. STABILITY OF MULTIVARIABLE SYSTEMS

The stability of multivariable systems centres mainly around the work of Lyapunov [[11] page 450]. A number of authors [197,198] have introduced methods mainly concerned with finding the elusive Lyapunov function, but widely applicable generalized procedures have yet to be developed.

Other stability procedures are based to a greater or lesser extent on the familiar single variable techniques. The Routh and Hurwitz criteria [[11] page 442] being most used.

8. IMPLEMENTATION OF MULTIVARIABLE SYSTEMS

In order to apply sophisticated measurement techniques and to simplify the application of multivariable control laws it becomes almost mandatory to install a digital control computer. Such hardware varies greatly in complexity and cost depending on a multitude of factors including the size of the application and the flexibility of the installation. Currently there is a growing number of such installations in industry basically involved in Direct Digital Control and

data logging [2,3,199]. Extensions of their use to multivariable control awaits the availability of persons with the necessary background and experience.

Another very real problem in the application of a multivariable control system to a process plant involves the measurement of variables.

In many cases measurements required by the control system are either physically inaccessible or their measurement is technically or economically unattractive. Inferential measurement can often be accomplished by the use of a process model and either used directly or "filtered" in with available measured data [200]. Kalman filters [[133] Chapter 9 to 11] and Luenberger predictors have received much attention in connection with this problem. Kalman filters can produce optimal estimates of measurements in the presence of noise and stochastic disturbances or give deterministic estimates [[133] page 306]. Luenberger [201,202] developed a predictor of order less than that of the state variable which gives a deterministic estimate of the state [203,204,205]. Porter and Woodhead [206] looked at the effect of such low order predictors on the optimal control.

Another solution to the measurement problem which has not received wide attention at this time is partial state control. Ferguson and Rekasius [207] and Rekasius [208] examine partial state feedback. Their transformation approaches become complex for systems above about second order. Davison and Goldberg [209] examine incomplete state feedback from the viewpoint of selecting the measurements that are most desirable. Levine and Athans [210] present an algorithm for

output control but do not guarantee its convergence. Kosut [211] considered suboptimal control from measureable states as a solution to the problem.

On-line tuning and optimization is often carried out as a second stage of implementation or on a continuing basis.

CHAPTER THREE

MODEL DEVELOPMENT, REDUCTION, AND EVALUATION FOR A DOUBLE EFFECT EVAPORATOR

ABSTRACT

A generalized approach to the modelling of evaporators is developed and used to obtain a ten equation, nonlinear model for a double effect evaporator at the University of Alberta. The same approach is applied to a six effect evaporator described in the literature.

Some of the methods and hazards of model simplification and linearization are illustrated by reduction of the tenth-order model to a fifth-order nonlinear and a fifth-order state space model. Second- and third-order linear models are derived from the fifth-order linear model.

Experimental and simulated responses are used to compare and evaluate the models. The performance of models in the experimental implementation of conventional, inferential, feed-forward, and multivariable optimal regulatory and state driving control systems is also reported.

1. INTRODUCTION

The aim of this work was to develop a model of a pilot plant, double effect evaporator located in the Department of Chemical and Petroleum Engineering at the University of Alberta. The model has already been used as the basis for experimental studies in multiloop, inferential, and multivariable control, including optimal regulatory and state driving control.

General theoretical relations are derived for a single effect and a "cookbook" procedure presented which allows the easy development of a model for any configuration of any number of effects. A six effect example is included.

The same general approach to modelling is also used to develop a tenth-order nonlinear model of the double effect evaporator which is then simulated and numerically linearized. The order of the model is reduced to five and is analytically linearized to give a linear state space model as a basis for control studies.

Lower order models are developed by simplification of the higher order models and direct derivation and are discussed relative to the assumptions required in their development.

The evaporator models are evaluated with the use of both simulated and experimental open-loop responses. Experimental studies of control schemes which use the evaporator model for implementation are also discussed relative to model evaluation.

2. LITERATURE SURVEY

Several models have been derived for evaporation equipment. They have been approached from two directions, empirically fitted models and theoretical derivations.

Johnson [1] presented a variety of empirical models of differing complexity and fitted parameters with experimental data from his falling film evaporator. Nisenfeld and Hoyle [2] considered simple empirical models for feed-forward control and used two first order lags and a time delay to dynamically compensate a six effect evaporation process.

Theoretical derivations have also been presented in the literature. Anderson, Glasson, and Lees [3] derived a six equation model for a single effect and simplified it to three differential equations. This was done by essentially neglecting vapour space and heat transfer dynamics. A frequency response comparison between the model and the equipment proved inconclusive. Manczak [4] presented an analytical procedure to determine the dynamic properties of single and multiple effects. His relations were extensively linearized which resulted in a small range of applicability about the operating point. Zavorka, Sutek, Aguado, and Delgado [5] developed a general model for a single effect of a commercial sugar evaporator. This model was extended to a triple effect system after simplification and included nonlinear relations for the heat transfer coefficients in terms of solution concentrations and liquid levels. However, their analysis neglected a general heat balance on the solution assuming vapourization to be proportional to heat transferred to the liquid. Andre and

Ritter [6] presented a direct derivation of a five equation model for the same double effect evaporator used for experimental work in this study.

Model reduction is a large field in its own right and will be only briefly examined. The first step in any model reduction problem is to partition the system into as many completely independent or sequential subsystems as possible. It then becomes necessary to make some approximations to further reduce the subsystems. With the danger of oversimplifying the field three approaches could be mentioned.

Procedures for reducing system order while maintaining the significant dynamic modes have been presented by Marshall [7], Davison [8,9], and Chen and Shieh [10]. This approach can be intuitively approximated with state space models by setting the derivatives of the first-order equations with small time constants equal to zero.

Transformation procedures such as that of Chidambara [11] have the disadvantage that the physical significance of the variables is often lost or obscured.

Fitting methods depend upon choosing a reduced model form and fitting it to data generated from the more complex model. This enters the field of parameter identification which has been surveyed by Nieman, Fisher, and Seborg [12] and Åström and Eykhoff [13]. Two identification methods aimed specifically at model reduction are presented by Chidambara [11] and Gibilaro and Lees [14].

3. DERIVATION OF GENERAL MODEL

Industrial evaporation processes are generally quite complex with a variety of types of evaporators arranged in parallel and series and with cocurrent and countercurrent feed. They also frequently include other units, such as settling tanks, between the effects. Because of this diversification a general model of evaporation processes is not possible and a "building block" approach is necessary.

A model of a complex evaporator system can be built up by combining sets of equations which describe a single effect with algebraic "configuration" equations plus the equations which describe special units.

General dynamic models for a single effect and a condenser are derived in the following sections.

3.1. The Single Effect

Figure 1 defines a general evaporator effect made up of four subsystems: the steam chest, the tube walls, the solution, and the vapour space.

3.1.1. Steam Chest

If the density and temperature gradients are assumed negligible the following lumped-parameter balances can be written.

$$V_s^i \frac{d\rho_s^i}{dt} = S^i - S_c^i \qquad (1)$$

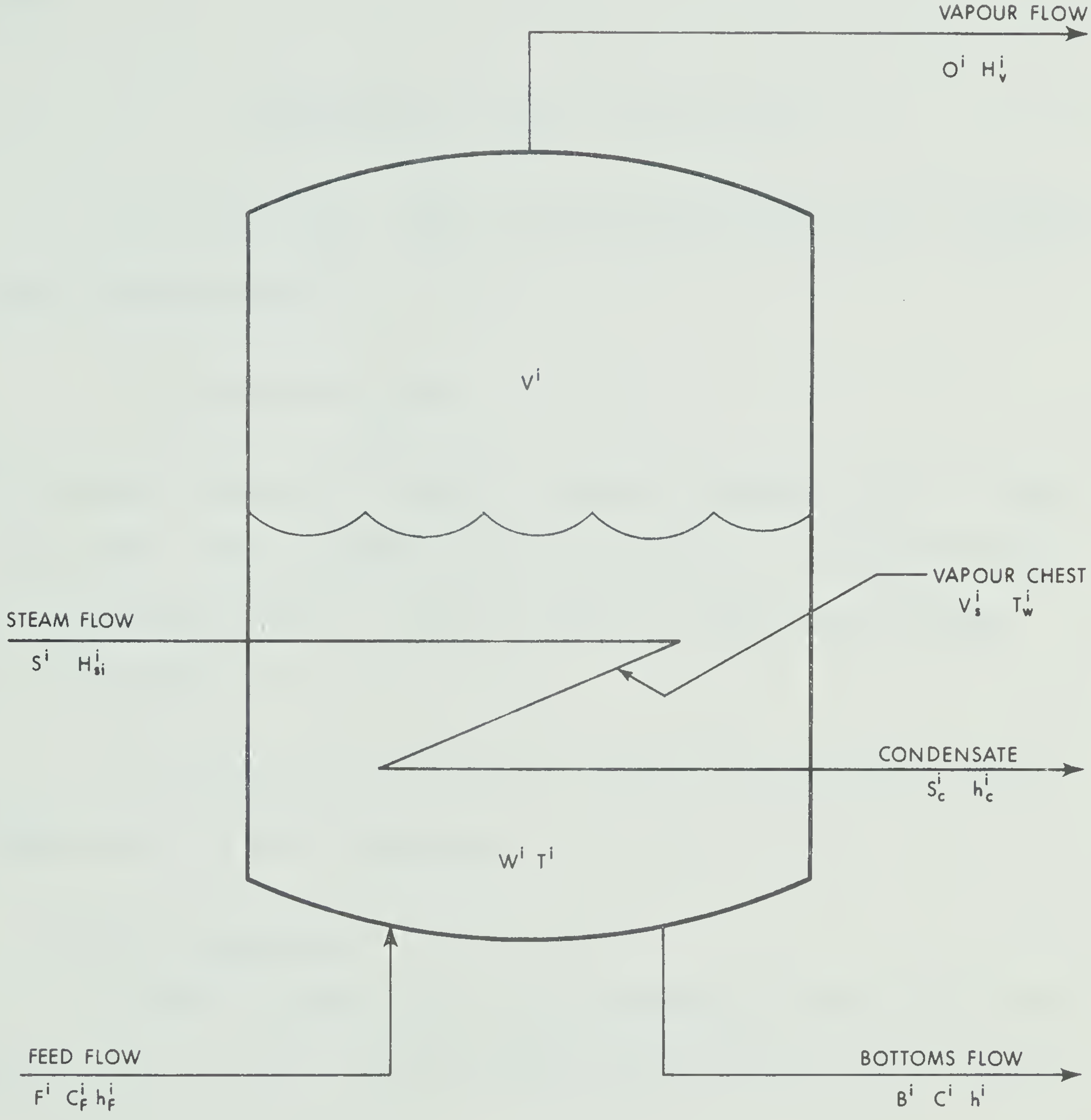


FIGURE 1. GENERAL EVAPORATOR EFFECT

$$V_s^i \frac{d}{dt} (\rho_s^i H_s^i) = S^i H_{si}^i - S_c^i h_c^i - Q_s^i - L_s^i \quad (2)$$

where

$$Q_s^i = U_v^i A^i (T_c^i - T_w^i) = S_c^i (\lambda_s^i + \delta_s^i) \quad (3)$$

$T_c^i = T_s^i - T_{bp}^j$; T_{bp}^j is the boiling point rise or superheat in the connected unit.

δ_s^i = subcooling term.

The constant volume V_s^i assumes a constant condensate level in the steam chest. When the steam chest is connected to the vapour space of the previous unit V_s^i should include the vapour space volume i.e. $(V_s^i + V^j)$.

An equation of state can be used to relate steam pressure, temperature, and other properties.

3.1.2. Tube Walls

With negligible temperature gradients in steam, tube walls, and solution a simple lumped-parameter energy balance may be written.

$$W_w^i C_{pw}^i \frac{dT_w^i}{dt} = U_v^i A^i (T_c^i - T_w^i) - U_s^i A^i (T_w^i - T^i) \quad (4)$$

The heat transfer coefficients may be assumed constant or expressed as general functions of temperature, concentration, and/or condensing or circulation rates. Use of an effective tube area would allow for heat transfer through downcomers and tube sheets.

3.1.3. The Solution

The balances on the solution in the evaporator assume perfect mixing. While this would generally be close to the case, large evaporator units with viscous solutions can develop significant concentration and temperature gradients within the solution. This would be particularly true where the circulation rate is low compared to the feedrate and where the vessel contains "dead" zones. The mass, solute, and energy balances follow.

$$\frac{dW^i}{dt} = F^i - B^i - O^i \tag{5}$$

$$\frac{d}{dt}(W^i C^i) = F^i C_F^i - B^i C^i \tag{6}$$

$$\frac{d}{dt}(W^i h^i) = F^i h_F^i - B^i h^i - O^i H_v^i + Q^i - L^i + \phi^i \tag{7}$$

where

$$Q^i = U_s^i A^i (T_w^i - T^i) \tag{8}$$

$$\phi^i = \text{heat of solution effects.}$$

3.1.4. The Vapour Space

The dynamics of the vapour space can generally be included with the steam chest of the next effect or the vapour side of a condenser. If this cannot be done the following mass and energy balances can be used to represent the dynamics, assuming negligible pressure gradients within the vapour.

$$V^i \frac{d\rho^i}{dt} = O^i - O^i_o \tag{9}$$

$$V^i \frac{d}{dt} (\rho^i H^i) = O^i H^i_v - O^i_o H^i - L^i_v \tag{10}$$

The vapour space pressure can be related to its density by an equation of state when required, for example in the case of pressure control of the vapour space.

This derivation has not considered boiling dynamics and assumes the vapour and solution to be in phase equilibrium at all times. In practice the effect of pressure changes can be exceedingly complex. An increase in pressure will increase the boiling point which could stop the boiling and hence decrease the heat transfer coefficient, particularly for natural circulation. This would cause the pressure and condensing temperature in the steam chest to rise above the normal steady state values. However, except in the event of direct pressure control on an effect, the pressure changes are dependent on heat transfer dynamics. Therefore, if the sensible heat in the solution is small compared to the heat load, the heating dynamics may be fast enough to prevent a total stop in boiling. This is dependent on the equipment in question. The present model will give the correct response to pressure changes provided that the attainment of phase equilibrium is more rapid than the pressure changes.

3.2. The Condenser

At least one effect in a series will be connected to a condenser which can be divided into three subsections. These are the vapour side, the tube walls, and the cooling water.

3.2.1. Vapour Side

Assuming negligible temperature gradients in the vapour and tube walls and a constant condensate holdup, the following balances may be written.

$$V_c^i \frac{d\rho_c^i}{dt} = O_v^i - O_c^i \tag{11}$$

$$V_c^i \frac{d}{dt} (\rho_c^i H_c^i) = O_v^i H_v^i - O_c^i h_c^i - Q_c^i - L_c^i \tag{12}$$

where

$$Q_c^i = U_{vc}^i A_c^i (T_{cc}^i - T_{wc}^i) \tag{13}$$

When the condenser is also used as a condensate accumulator the liquid dynamics of the condensate may have to be considered as well.

3.2.2. Tube Walls

Assuming negligible temperature gradients in the vapour and tube walls and using a mean temperature of the cooling water, the following energy balance may be written.

$$W_{wc}^i C_{pc}^i \frac{dT_{wc}^i}{dt} = U_{vc}^i A_c^i (T_{cc}^i - T_{wc}^i) - U_{sc}^i A_c^i (T_{wc}^i - T_m^i) \tag{14}$$

The heat transfer coefficients may be constant or functions of temperatures, cooling water flowrates, and condensing rates.

3.2.3. Cooling Water

Assuming a linear temperature profile in the cooling water the mean temperature is:

$$T_m^i = 0.5\ (T_{co}^i + T_{ci}^i) \tag{15}$$

and the heat load is:

$$Q_c^i = F_{cw}^i\ C_{pcw}^i\ (T_{co}^i - T_{ci}^i) \tag{16}$$

3.3. Multi-effect Model Building

With general model relations available for individual effects it is possible to build a model of a multi-effect process using a "cookbook" type of approach. The following information is necessary to complete a process model.

(a) For each evaporator effect, condenser, or other unit of equipment the following must be supplied.

(i) The general model dynamic relations, for example equations (1) to (10) for an evaporator effect. These will define ρ_s^i, H_s^i, T_w^i, W^i, C^i, h^i, and possibly ρ^i and H^i.

(ii) Property relations for the solution (for example defining h^i, ϕ^i, and T_{bp}^i) and the solvent vapour and liquid (or equations of state).

(iii) Physical parameters of the equipment (for example V^i, V_s^i, W_w^i, A^i, C_{pw}^i).

(iv) Operating variables (such as heat losses and heat transfer coefficients) which may be functions of the state of the unit.

(b) At least one algebraic configuration relationship must be written for each process stream connecting the units. At this stage the independent variables of the process will become apparent.

The complete model and configuration relations for a six effect evaporation process presented and modelled as two stirred tanks by Nisenfeld and Hoyle [2] are given in detail in Appendix A. A schematic flowsheet of the process appears in Figure 2 and an example of the configuration statements for the streams entering and leaving the second effect are given below.

$$\text{Stream C:} \quad S^2 = O^1, \quad H_{si}^2 = H_v^1 \tag{17}$$

$$\text{Stream E:} \quad S^3 = O^2, \quad H_{si}^3 = H_v^2 \tag{18}$$

$$\text{Stream D:} \quad F^2 = B^3, \quad C_F^2 = C^3$$

$$h_F^2 = (B^3 h^3 - L_D)/F^2 \tag{19}$$

$$\text{Stream B:} \quad F^1 = B^2 + \alpha B^1$$

$$C_F^1 = (B^2 C^2 + \alpha B^1 C^1)/F^1$$

$$h_F^1 = (B^2 h^2 + \alpha B^1 h^1 - L_B)/F^1 \tag{20}$$

where α is the fraction of the bottoms flow, B^1, which is recycled, and $L_{B,D}$ are the stream heat losses. Transport delays due to long

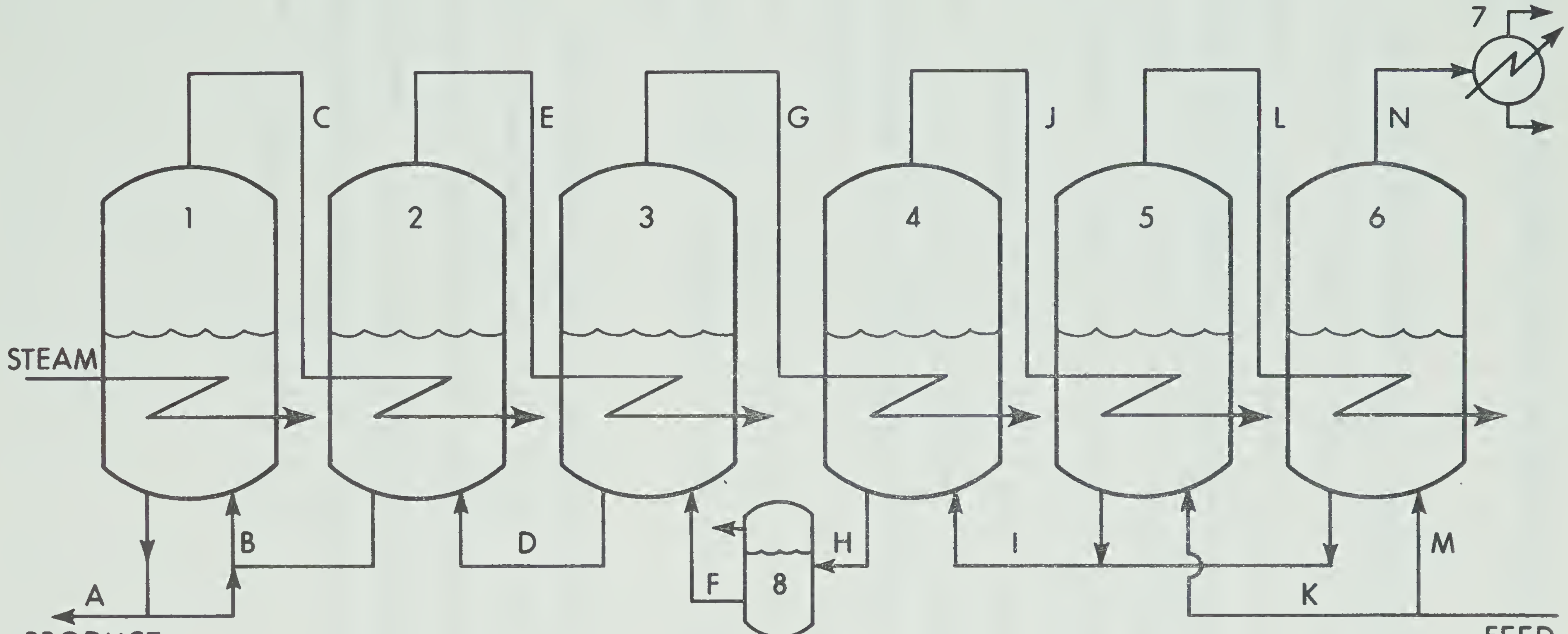


FIGURE 2. SIX EFFECT EVAPORATION PROCESS

pipes and/or low flowrates can be included in the configuration relations.

The profusion of differential equations such as equations (1) to (10) and the algebraic configuration equations above that result from such a process can be handled directly by such sophisticated digital simulation programs as the Continuous Systems Modelling Program (CSMP) on the IBM 360 series. No simplifying substitutions would be necessary although they would improve computational efficiency.

In order to arrive at a single set of differential equations in the form of a standard state space model the algebraic relations must be substituted into the differential relations, particularly if analytical expressions for the derivatives are desired. If the relations are to be linearized numerically by differences about the operating conditions (see Appendix C) then these substitutions are not necessary.

This "cookbook" approach demonstrates the power and flexibility of developing general unit models. However the order of resulting models is generally high and some form of model reduction becomes necessary. This is demonstrated in the following sections where a tenth order nonlinear model of an existing pilot plant is simplified and compared to experimental data.

4. MODELLING AND REDUCTION OF THE PILOT PLANT EVAPORATOR

The equipment on which this study is based is a pilot plant scale double effect evaporator in the Department of Chemical and Petroleum Engineering at the University of Alberta.

4.1. The Evaporator

The evaporator has a complex feed system to facilitate both operation of the equipment in a cyclic fashion and the introduction of load changes and disturbances in the feed conditions. Controlled flows of concentrated triethylene glycol solution and of water are temperature controlled by steam heaters and mixed in the proportions necessary to produce a feed of the desired flowrate, temperature, and concentration.

The first effect is a short tube vertical unit with natural circulation. The nine inch diameter unit has an operating holdup of two to four gallons and its eighteen inch 3/4 inch O.D. tubes give about ten square feet of heat transfer area. The second effect is a long tube vertical effect set up for either natural or forced circulation. It has five square feet of heat transfer surface made up of three six foot long, one inch OD tubes. The capacity of its circulating system is about three gallons.

Figure 3 shows the major features of the process equipment including the level controllers used in this study. Details of the equipment are available in theses by Andre [15], Wilson [16], Fehr [17], and Jacobson [18]. The equipment is fully instrumented and can be controlled either by Foxboro electronic controllers or under Direct

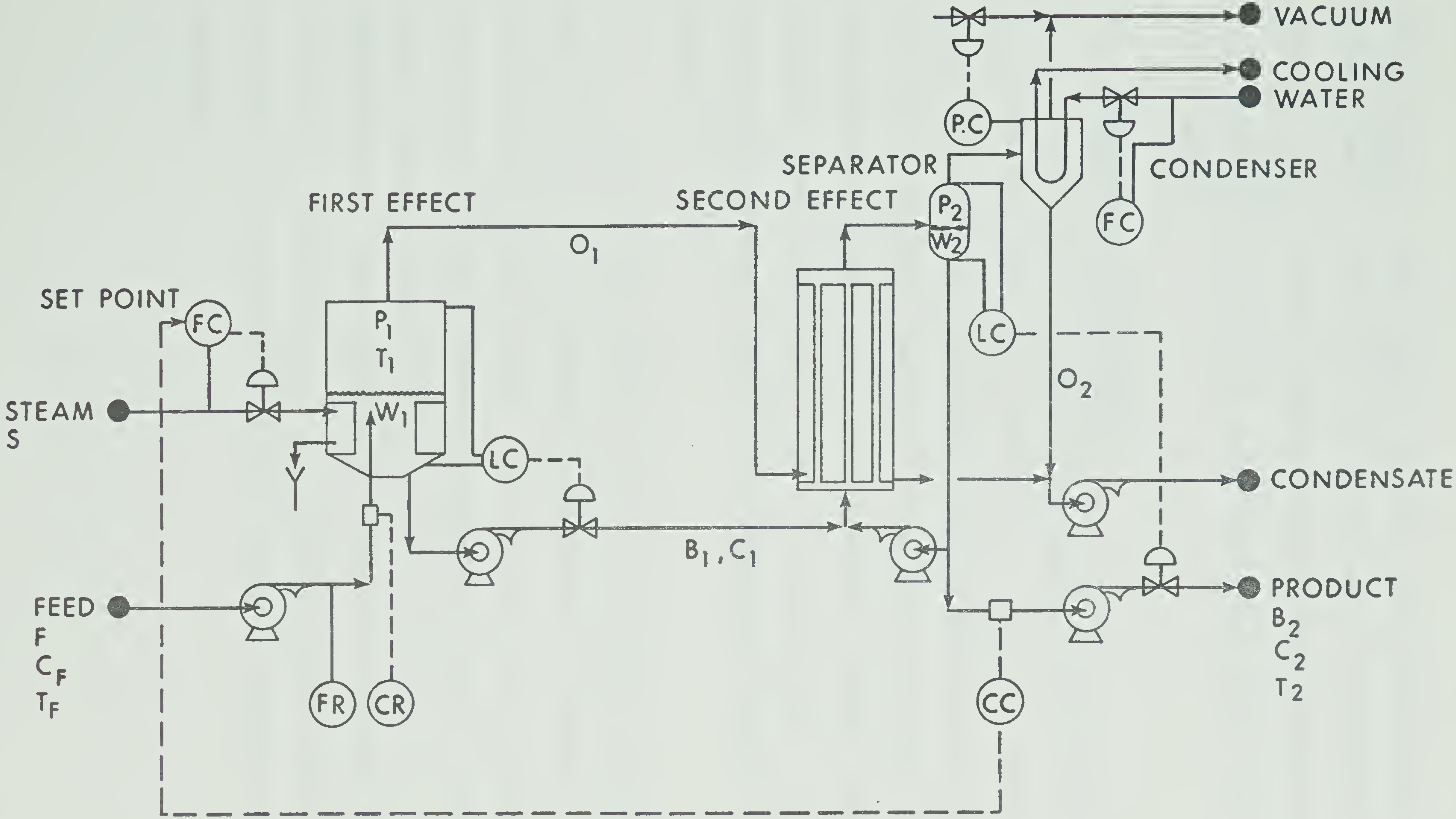


FIGURE 3. PILOT PLANT DOUBLE EFFECT EVAPORATOR

Digital Control (DDC) from an IBM 1800 Data Acquisition and Control Computer. The sophisticated and complete interfacing between plant and computer enables implementation of complex multivariable control schemes using the on-line computing capabilities of the digital machine.

4.2. Tenth-Order Nonlinear Model

The "cookbook" procedure for producing general model relations was used for the double effect evaporator in Figure 3. The derivation was made assuming negligible heat of solution effects and boiling point rises, saturated steam in the first effect steam chest and the vapour spaces of both effects, and constant heat transfer coefficients. Transport delays between the effects were also neglected. The resulting differential equations and configuration relations are detailed in Appendix B.

To facilitate further reduction the algebraic relations were substituted into the differential equations and ten nonlinear differential equations resulted. The model was then linearized numerically by central differences (Appendix C) to give a standard linear state space model. The ten equation nonlinear model was simulated using CSMP on an IBM 360 model 67 (see results plotted in Figure 4). However for its use in design by modern control techniques, it is advantageous to have a state space model of lower dimension.

4.3. Fifth-Order (State Space) Model

The ten equation nonlinear model is reduced by the more intuitive approach of making physically justifiable approximations which reduce dynamic relations to their steady state form. This is similiar to the model reduction technique of Marshall [7].

Consider the following points relative to the equipment and its operation.

(a) Because of the relatively small heat capacity of the three sets of tube walls and the small volume of the first effect steam chest, the dynamics of the wall and steam temperatures are fast compared with those of the solution. As a result the steady state relations can be used in these areas and represent a fair approximation.

(b) Since the evaporator is operated with very tight control on the pressure in the second effect, the second effect temperature can be considered constant.

As a result of these approximations the model relations reduce to five nonlinear first-order differential equations.

Steam Chest:
The steam temperature is related to flowrate by the following equation:

$$S^1 \lambda^1_s = U^1 A^1 (T^1_s - T^1) \tag{21}$$

First Effect:

$$\frac{dW^1}{dt} = F^1 - B^1 - O^1 \tag{22}$$

$$W^1 \frac{dC^1}{dt} = F^1 (C_F^1 - C^1) + O^1 C^1 \tag{23}$$

$$W^1 \frac{dh^1}{dt} = F^1 (h_F^1 - h^1) - O^1 (H_V^1 - h^1) + Q^1 - L^1 \tag{24}$$

where

$$Q^1 = U^1 A^1 (T_s^1 - T^1) \tag{25}$$

$$O^1 = (Q^2 + L^2)/(H_V^1 - h_C^2) \tag{26}$$

$$Q^2 = U^2 A^2 (T^1 - T^2) \tag{27}$$

Second Effect:

$$\frac{dW^2}{dt} = B^1 - B^2 - O^2 \tag{28}$$

$$W^2 \frac{dC^2}{dt} = B^1 (C^1 - C^2) + O^2 C^2 \tag{29}$$

where

$$O^2 (H_V^2 - h^2 + \frac{\partial h^2}{\partial C^2} C^2) = Q^2 - L^2 + B^1 (h^1 - h^2) + \frac{\partial h^2}{\partial C^2} B^1 (C^2 - C^1) \tag{30}$$

These relations make up the basic model which is used in much of the research involved with the evaporator unit. Andre and Ritter [6] obtained the same set of relations by a direct derivation. These nonlinear relations can now be linearized and arranged to form a linear state equation model.

A number of factors make it desirable to work with linear differential equations. They are principally that a large number of flexible procedures exist for solving simultaneous linear equations and the bulk of modern control and optimal control theory can only reasonably be applied to linear systems. A set of five simultaneous linear differential equations with variables in normalized perturbation form were derived [19] by analytical linearization (see Appendix C). The resulting model is presented in state space form in Equation (31) and the steady states listed in Table 1.

A FORTRAN program [19] was written which will generate the linearized perturbation equations for any given set of steady state conditions. These calculations are normally done offline but the necessary data could also be read directly from the process, checked for validity by a material and energy balance program such as MEBOL [20], and the linearized model generated on-line. This would allow a type of adaptive modelling over the full range of steady states at which the process can operate.

Another possible approach to obtaining a linear reduced model would be to use Davison's [9] technique for eliminating large eigenvalues. Approximation (b) reduces the dimension of the model to nine. These relations can then be linearized and the large eigenvalues associated with the first effect steam chest and the three sets of tube walls can be eliminated to give a linear five equation model.

TABLE 1

EVAPORATOR STEADY STATE

PROCESS VARIABLE	STEADY STATE
W^1	30 lbs.
C^1	4.85 % glycol
h^1	194 Btu./lb.
W^2	35 lbs.
C^2	9.64 % glycol
S^1	1.9 lb./min.
B^1	3.3 lb./min.
B^2	1.66 lb./min.
F^1	5 lb./min.
C_F^1	3.2 % glycol
h_F^1	162 Btu./lb.

$$\begin{bmatrix} \dot{W}^1 \\ \dot{C}^1 \\ \dot{h}^1 \\ \dot{W}^2 \\ \dot{C}^2 \end{bmatrix} = \begin{bmatrix} 0 & -.00156 & -.1711 & 0 & 0 \\ 0 & -.1419 & .1711 & 0 & 0 \\ 0 & -.00875 & -1.102 & 0 & 0 \\ 0 & -.00128 & -.1489 & 0 & .00013 \\ 0 & .0605 & .1489 & 0 & -.0591 \end{bmatrix} \begin{bmatrix} W^1 \\ C^1 \\ h^1 \\ W^2 \\ C^2 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & -.143 & 0 \\ 0 & 0 & 0 \\ .392 & 0 & 0 \\ 0 & .108 & -.0592 \\ 0 & -.0486 & 0 \end{bmatrix} \begin{bmatrix} S^1 \\ B^1 \\ B^2 \end{bmatrix}$$

$$+ \begin{bmatrix} .2174 & 0 & 0 \\ -.074 & .1434 & 0 \\ -.036 & 0 & .1814 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F^1 \\ C^1_F \\ h^1_F \end{bmatrix}$$

$$\begin{bmatrix} W^1 \\ W^2 \\ C^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} W^1 \\ C^1 \\ h^1 \\ W^2 \\ C^2 \end{bmatrix}$$

Equation (31)

4.4. Second and Third-Order Models

Consideration was given to even lower order models. Although a five dimensional model can be readily handled by a digital computer, one of lower order can be implemented more easily and will very often be just as effective for control purposes.

Since the five equation model has eigenvalues of the same order of magnitude and it is required to preserve the physical significance of the variables, further reduction of order can only be achieved by making assumptions regarding the equipment's operating conditions. As might be expected these assumptions are less valid than those made previously. The basic assumptions used to further reduce the model were:

(a) constant holdup in the second effect (due to physical constraints it must be within $\pm$ 10 percent of the steady state value, so the assumption is reasonable in this case),

(b) constant first effect temperature;

(c) insignificant enthalpy dependance on concentration,

and (d) no heat losses.

Further assuming that the change in solution enthalpy between effects is small compared to the heat load, the following relation follows from equations (26), (28), and (30).

$$k^2 = \frac{O^2}{O^1} = \frac{\bar{H}^1_v - \bar{h}^2_c}{\bar{H}^2_v - \bar{h}^2} \tag{32}$$

Similiarly a constant k^1 can be defined using equations (21), (24),

and (25) and assuming the feed temperature is close to that of the first effect.

$$k^1 = \frac{O^1}{S^1} = \frac{\bar{\lambda}^1_s}{\bar{H}^1_v - \bar{h}^1} \tag{33}$$

Analytical linearization of the remaining differential equations, (22), (23), and (29), results in the following model.

$$\frac{dW^1}{dt} = F^1 - B^1 - k^1 S^1 \tag{34}$$

$$\bar{W}^1 \frac{dC^1}{dt} = (\bar{C}^1_F - \bar{C}^1)F^1 + \bar{F}^1 C^1_F + (k^1 \bar{S}^1 - \bar{F}^1)C^1 + k^1 \bar{C}^1 S^1 \tag{35}$$

$$\bar{W}^2 \frac{dC^2}{dt} = (\bar{C}^1 - \bar{C}^2)B^1 + \bar{B}^1 C^1 + (k^1 k^2 \bar{S}^1 - \bar{B}^1)C^2 + k^1 k^2 \bar{C}^2 S^1 \tag{36}$$

An alternative approach to finding a simple model is to make basically the same assumptions but develop the model relations directly from mass balances. In this case the direct approach would give the following expressions for the "k^i" constants.

$$k^1 = \frac{O^1}{S^1} = \frac{\bar{F}^1 - \bar{B}^1}{\bar{S}^1} \tag{37}$$

$$k^2 = \frac{O^2}{O^1} = \frac{\bar{B}^1 - \bar{B}^2}{\bar{F}^1 - \bar{B}^1} \tag{38}$$

The basic model equations, (34) to (36), remain the same.

The difference in the derivations, which appear in detail in Appendix D, is the means of evaluating the "k^i" constants. The values calculated from enthalpies (equations (32) and (33)) cause up to 16 percent steady state imbalance in the model equations because of the approximations made. The "k^i" values from consistent steady state data close the balances and differ in value from the "theoretical" values by up to 10 percent.

Further reduction in model order is possible if the first effect level can be considered constant (by being under tight level control). The following model relations are developed in Appendix D.

$$\bar{W}^1 \frac{dC^1}{dt} = (\bar{C}^1_F - \bar{C}^1)F^1 + \bar{F}^1 C^1_F + (k^1 \bar{S}^1 - \bar{F}^1)C^1 + k^1 \bar{C}^1 S^1 \tag{39}$$

$$\bar{W}^2 \frac{dC^2}{dt} = (\bar{C}^1 - \bar{C}^2)F^1 + \bar{B}^1 C^1 + (k^1 k^2 \bar{S}^1 - \bar{B}^1)C^2 + (k^1 k^2 \bar{C}^2 - k^1 (\bar{C}^1 - \bar{C}^2))\, S^1 \tag{40}$$

It follows from these model equations (see Appendix D) and the steady state values in Table 1 that the transfer function between product concentration C^2 and steam S^1 is approximately

$$G(s) = \frac{C^2(s)}{S^1(s)} = \frac{10.2\ (6.7s + 1)}{(9.1s + 1)\ (21.1s + 1)} \simeq \frac{10.2}{(\ \tau s\ + 1)} \tag{41}$$

which indicates approximately first-order behaviour.

5. MODEL EVALUATION

All experimental open loop tests were made with levels under DDC proportional-plus-integral control because of the physical limitations on liquid levels in the equipment. Hence all simulated results from the evaporator models were obtained with simulated P + I control of levels to enable direct comparisons with real data.

Nonlinear models were simulated by CSMP on an IBM 360 Model 67 and linear models by the state difference equations. With a time interval of one second, compatable with the DDC time base of one second, both the nonlinear and linear simulations took less than half a minute of computer time for a simulated three hour transient.

5.1. Simulated Evaluation

Figures 4a and 4b make a comparison of the tenth-order nonlinear (10NL), fifth-order nonlinear (5NL), and fifth-order linearized (5L) models for a ten percent step up in feed flowrate. The triangles on the axes indicated the initial steady state. The C2 response calculated from the two fifth-order models can be seen to lead the response of the tenth-order model by one to two minutes. This illustrates the effect of neglecting the small time constants. It is also noticeable that the C2 steady state predicted by the linearized model is lower than that predicted by the nonlinear models. The responses of the other variables calculated from the 10NL model are similar to those from the 5NL model and are not plotted. Figure 5 shows the opposite effect for a ten percent decrease in feed flowrate. This error in predicting steady state gains caused by linearizing a model with an appreciable nonlinearity can require nonlinear gain corrections when the model is required to predict accurate steady states.

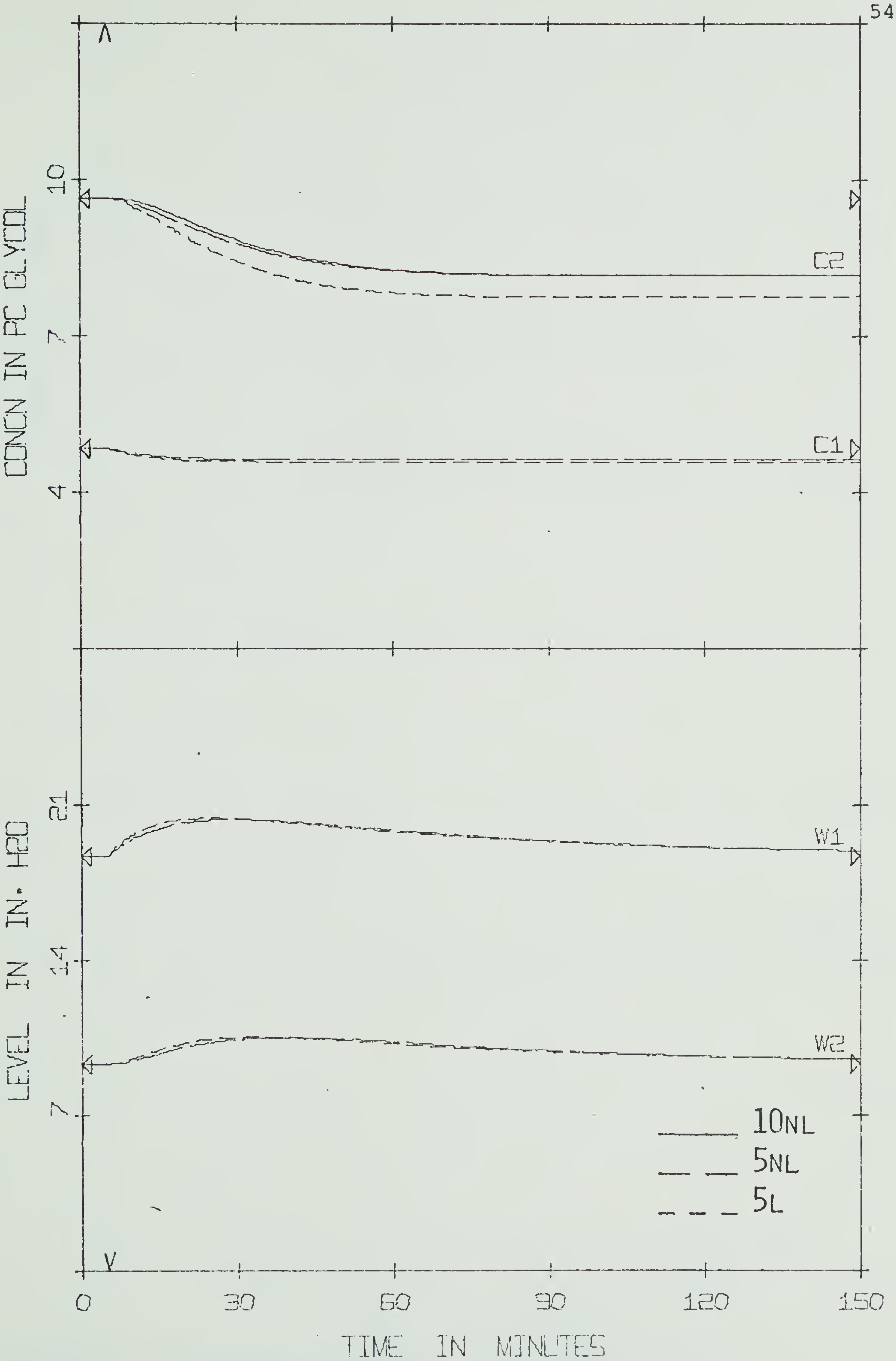


FIGURE 4a. SIMULATED COMPARISON OF HIGHER ORDER MODELS I (10NL, 5NL, 5L/+10%F/OL)*

*Run condition codes are defined in the Nomenclature (p. 370)

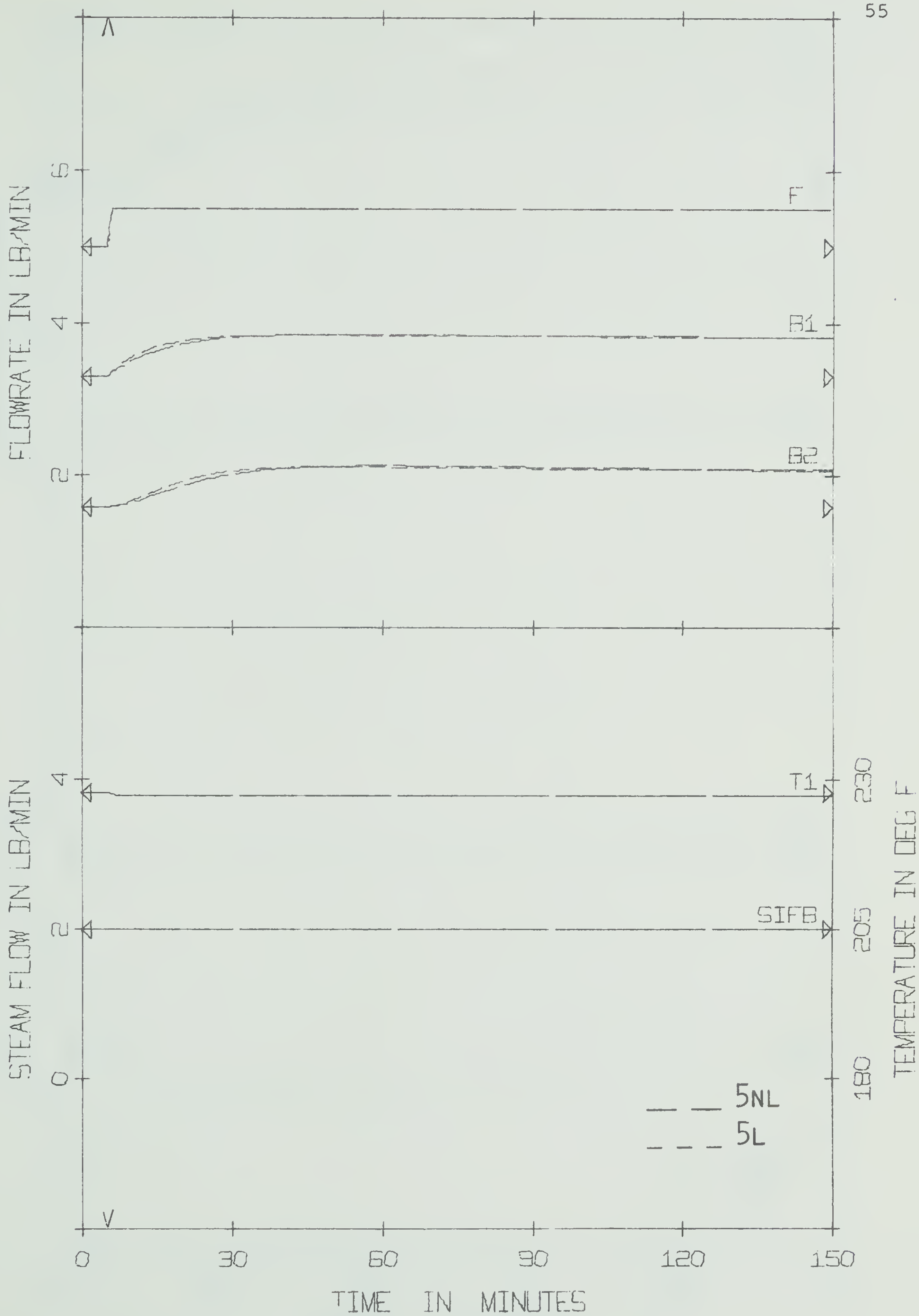


FIGURE 4b. SIMULATED COMPARISON OF HIGHER ORDER MODELS I (10NL, 5NL, 5L/+10%F/OL)

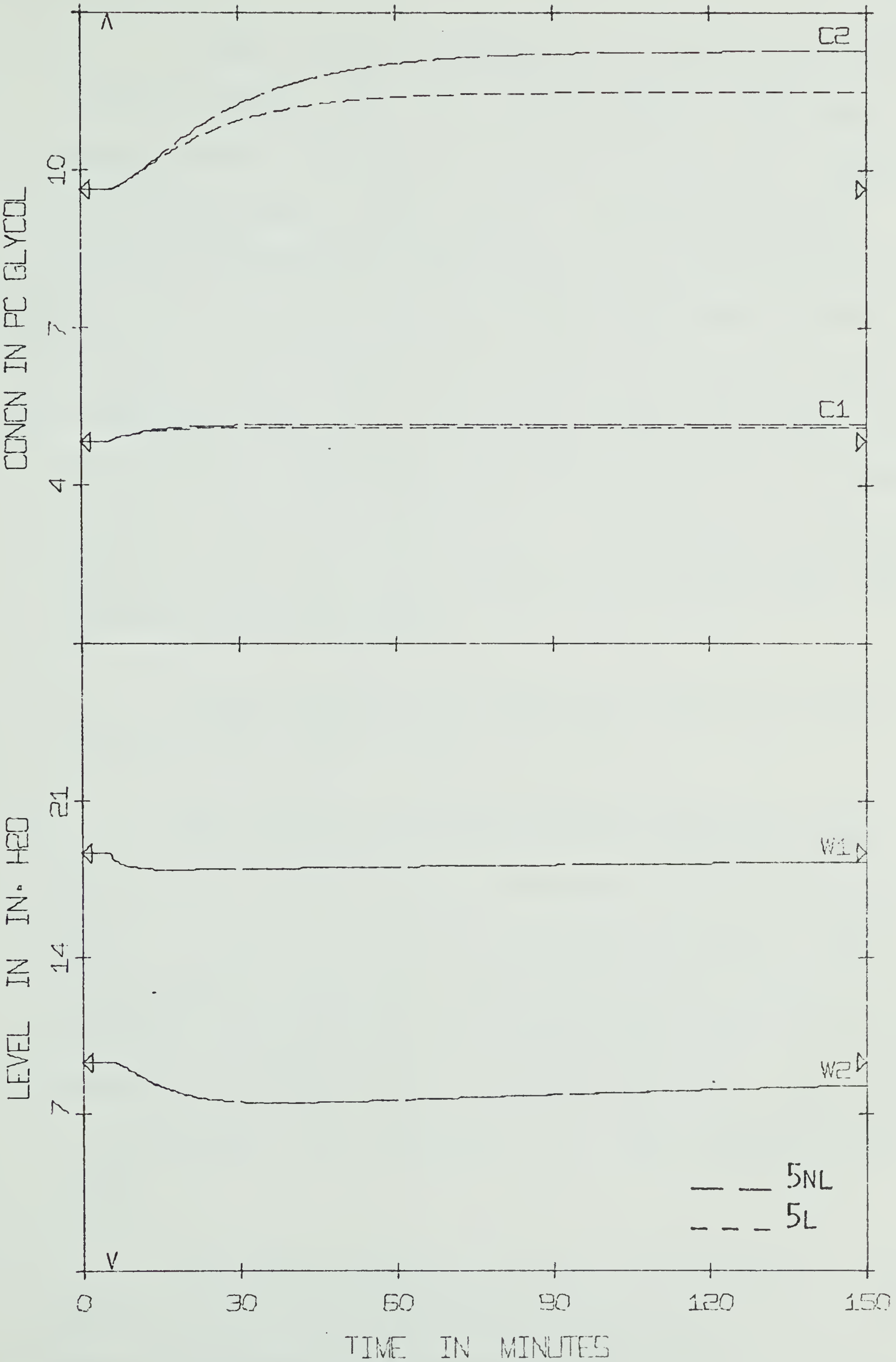


FIGURE 5. SIMULATED COMPARISON OF HIGHER ORDER MODELS II
(5NL,5L/-10%F/OL)

Figure 6 shows a comparison of the responses of the five equation linearized (5L), three equation reduced (3LR), three equation derived (3LD), and two equation derived (2LD) models to a ten percent increase in feed flowrate. All models accurately represented first effect concentration C^1 and first effect level W^1 with the exception of the second-order model which assumes W^1 constant. The fifth-order was the only one including the dynamics of second effect holdup W^2. The third and fifth-order models, despite some variation in predicted steady states, gave the correct dynamic response of the product concentration. The second-order model gave an essentially first-order type response rather than the higher order responses of the other models.

Table 2 tabulates the steady state offsets and balance closures for the fifth and lower order models.

The linearized model gave generally good steady states falling between those of the up and down steps with the nonlinear model. The second and third-order models gave reasonable steady states except for steam steps. This was due to the assumption of the evaporation being proportional to heating vapour (the "k" constants). The third-order reduced model did not give good agreement with respect to steady states. The "theoretical" "k" values account for this effect.

The linearized model was the only one which did not give good balance closures because the majority of its coefficients are heavily dependent on theoretically predicted parameters as opposed to being calculated from steady state values.

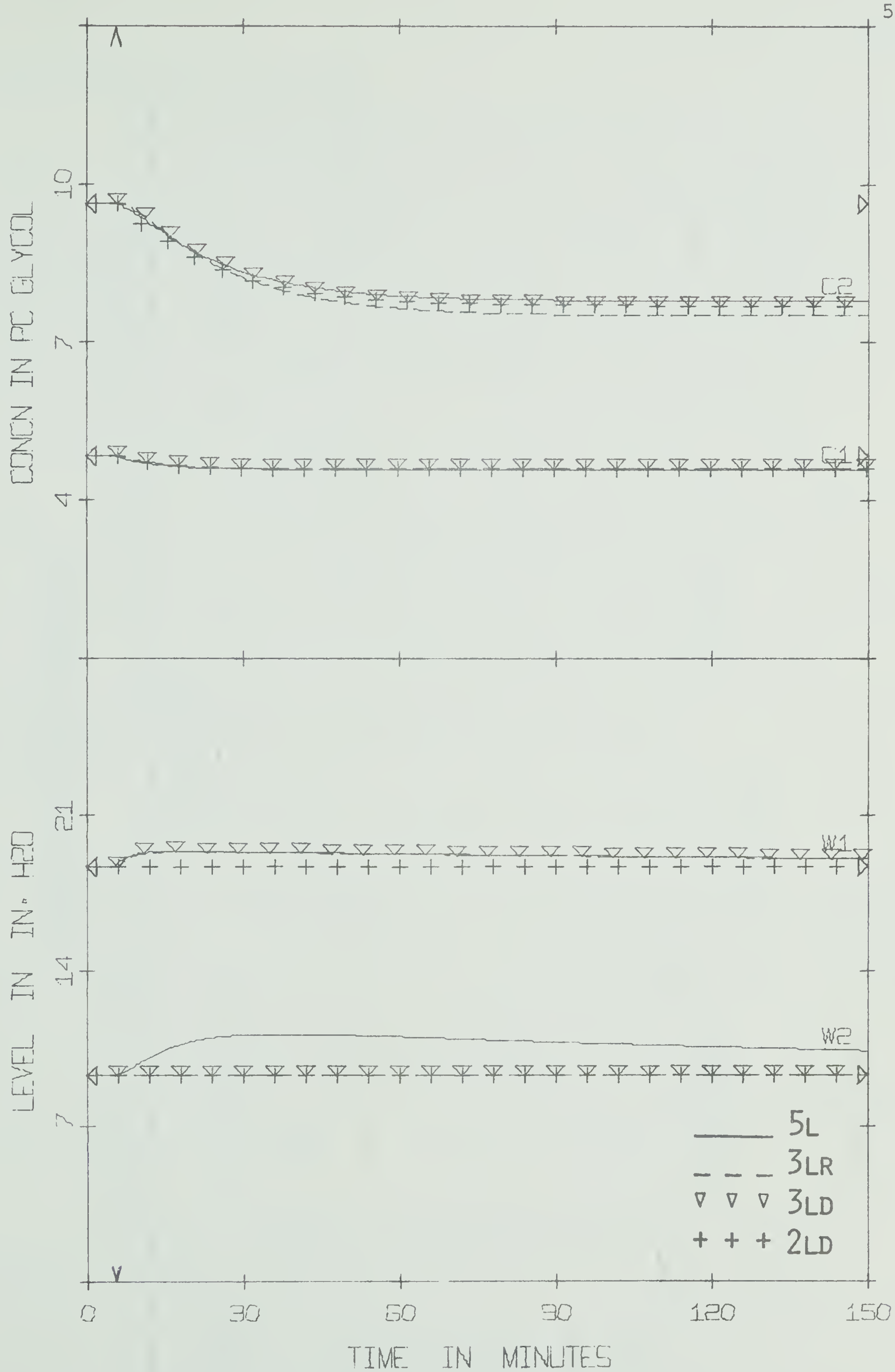


FIGURE 6. SIMULATED COMPARISON OF LOWER ORDER MODELS
(5L, 3LR, 3LD, 2LD/+10%F/OL)

TABLE 2

STEADY STATE MODEL COMPARISONS

Concentrations: percent glycol; closures: percent of average balance term

Model	Disturbance: 10% step in F^1					10% step in C_F^1					10% step in S^1				
	ΔC^1	ΔC^2	1	2	3	ΔC^1	ΔC^2	1	2	3	ΔC^1	ΔC^2	1	2	3
5NL+	-.23	-1.42	0.0	0.0	0.0	+.49	+ .98	0.0	0.0	0.0	+.20	+1.75	0.0	0.0	0.0
5NL-	+.32	+2.61	0.0	0.0	0.0	-.49	- .96	0.0	0.0	0.0	-.19	-1.31	0.0	0.0	0.0
5L +	-.27	-1.85	0.0	0.8	3.9	+.49	+ .97	0.1	0.2	0.1	+.19	+1.51	0.0	0.2	1.5
3LR+	-.26	-2.13	0.0	0.1	0.1	+.41	+1.10	0.0	0.0	0.0	+.29	+2.32	0.0	0.0	0.0
3LD+	-.15	-1.94	0.0	0.1	0.2	+.49	+ .96	0.0	0.1	0.4	+.25	+1.94	0.0	0.0	0.2
2LD+	-.15	-1.94	-	0.0	0.1	+.48	+ .96	-	0.1	0.1	+.25	+1.94	-	0.0	0.2

Steady States: C^1 = 4.85% glycol

C^2 = 9.64% glycol

Closures:
1. 1st effect mass balance
2. 1st effect solute balance
3. 2nd effect solute balance

Disturbance: + step up

\- step down

5.2. Experimental Evaluation

The evaporator models have been used in both the design and actual implementation of conventional multiloop, inferential, feed-forward, and optimal multivariable control schemes. In addition to the open loop study these implemented control schemes will be discussed relative to the performance of the models.

5.2.1. Open Loop Study*

The bottoms flowrates, B^1 and B^2, were used to control the liquid levels in the first and second effects respectively so that only four independent variables were left for use as forcing functions. These were steam and feed flowrates, feed concentration and feed enthalpy. Experimental runs were carried out with positive and negative steps of ten and twenty percent of steady state in all four variables.

The results showed reasonable agreement between the experimental and the linearized five equation model (Equation (31)) for step changes of up to ten percent in feed flowrate and twenty percent in the other three variables. These results are exemplified by Figures 7, 8 and 9 showing the evaporator's response to twenty percent step disturbances in steam and feed concentration and a ten percent step disturbance in feed flowrate respectively.

It was apparent in all the results (for example Figures 7 through 11) that the models responded faster than the equipment. This was most probably due to the neglect of several small time constants (Figure 4a) in the initial model reduction and the neglect of the small time delays between the effects and equivalent time delays

*All open loop experimental data are taken from Jacobson [18].

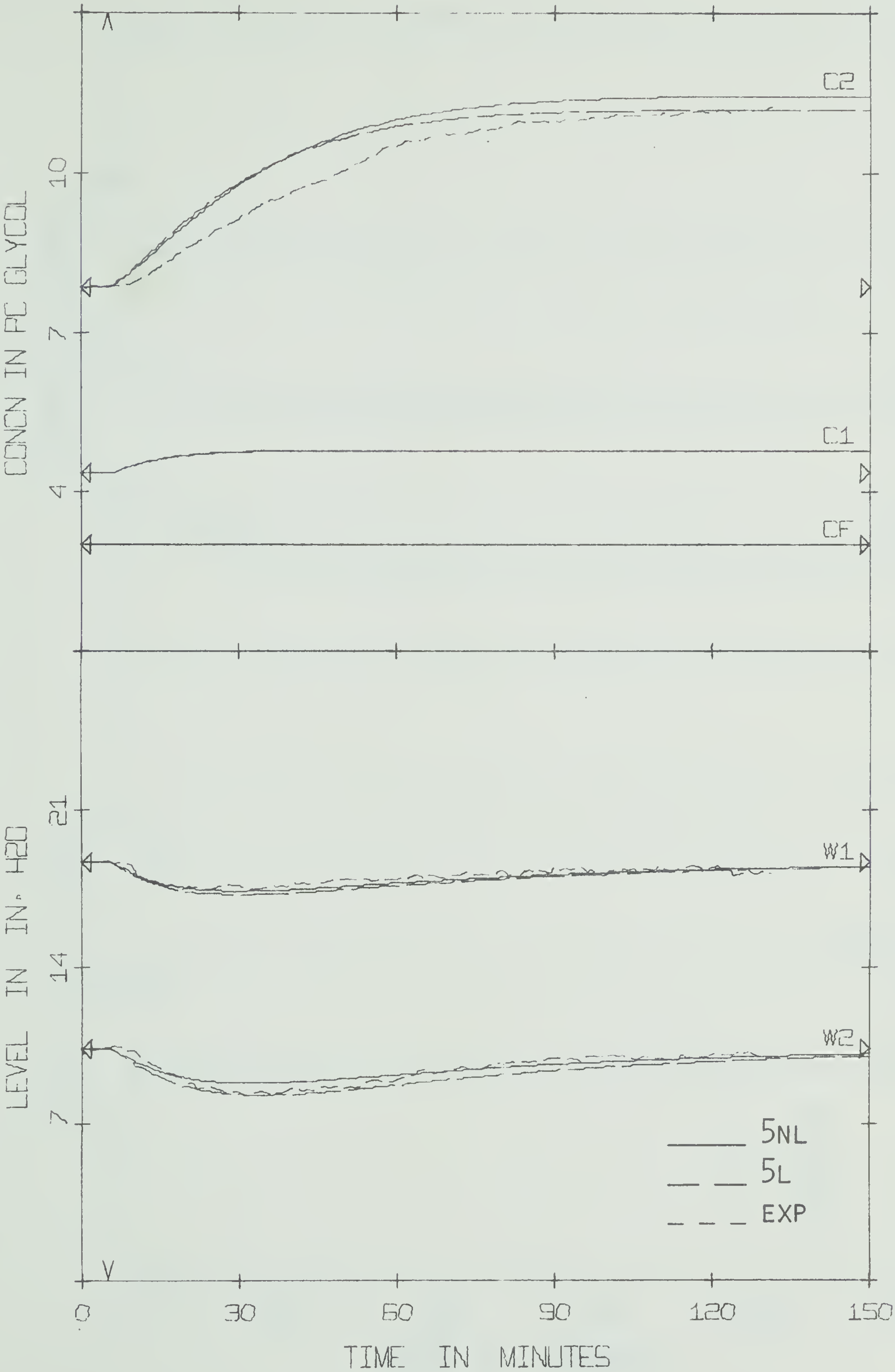


FIGURE 7a. EXPERIMENTAL EVAPORATOR RESPONSE, RUN OL16
(5NL, 5L, EXP/+20%S/OL)

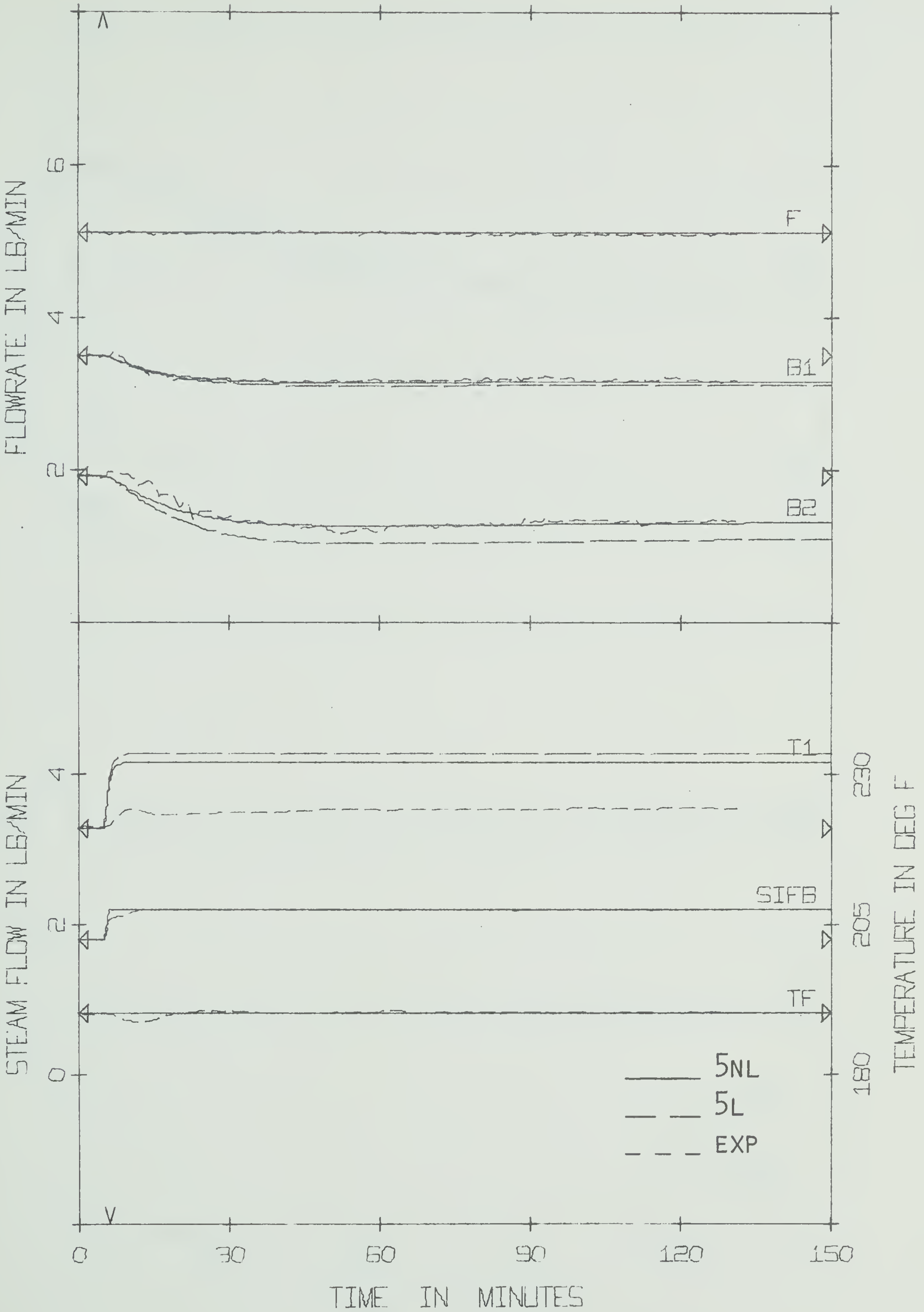


FIGURE 7b. EXPERIMENTAL EVAPORATOR RESPONSE, RUN OL16
(5NL, 5L, EXP/+20%S/OL)

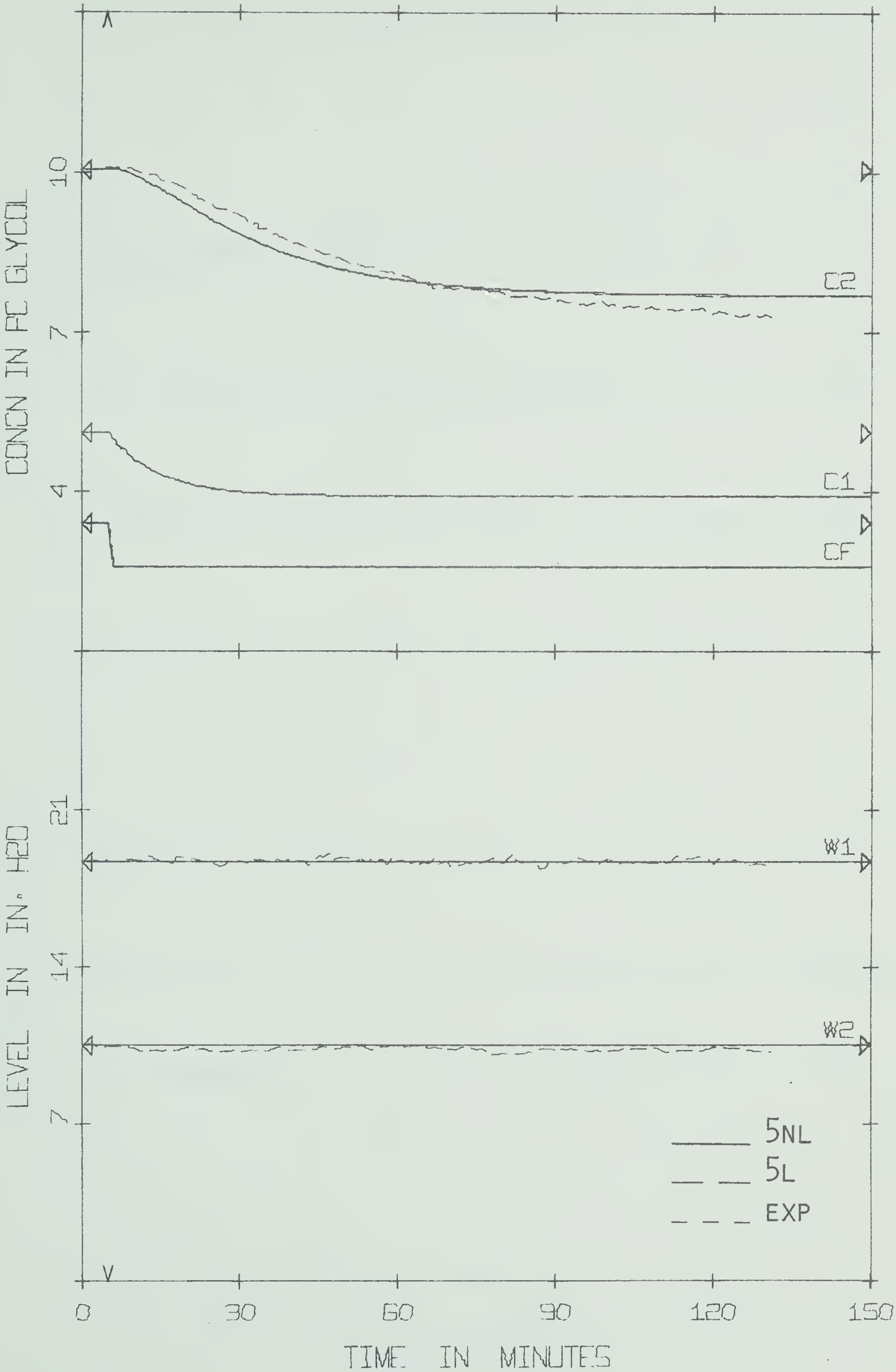


FIGURE 8a. EXPERIMENTAL EVAPORATOR RESPONSE, RUN OL8
(5NL, 5L, EXP/-20%CF/OL)

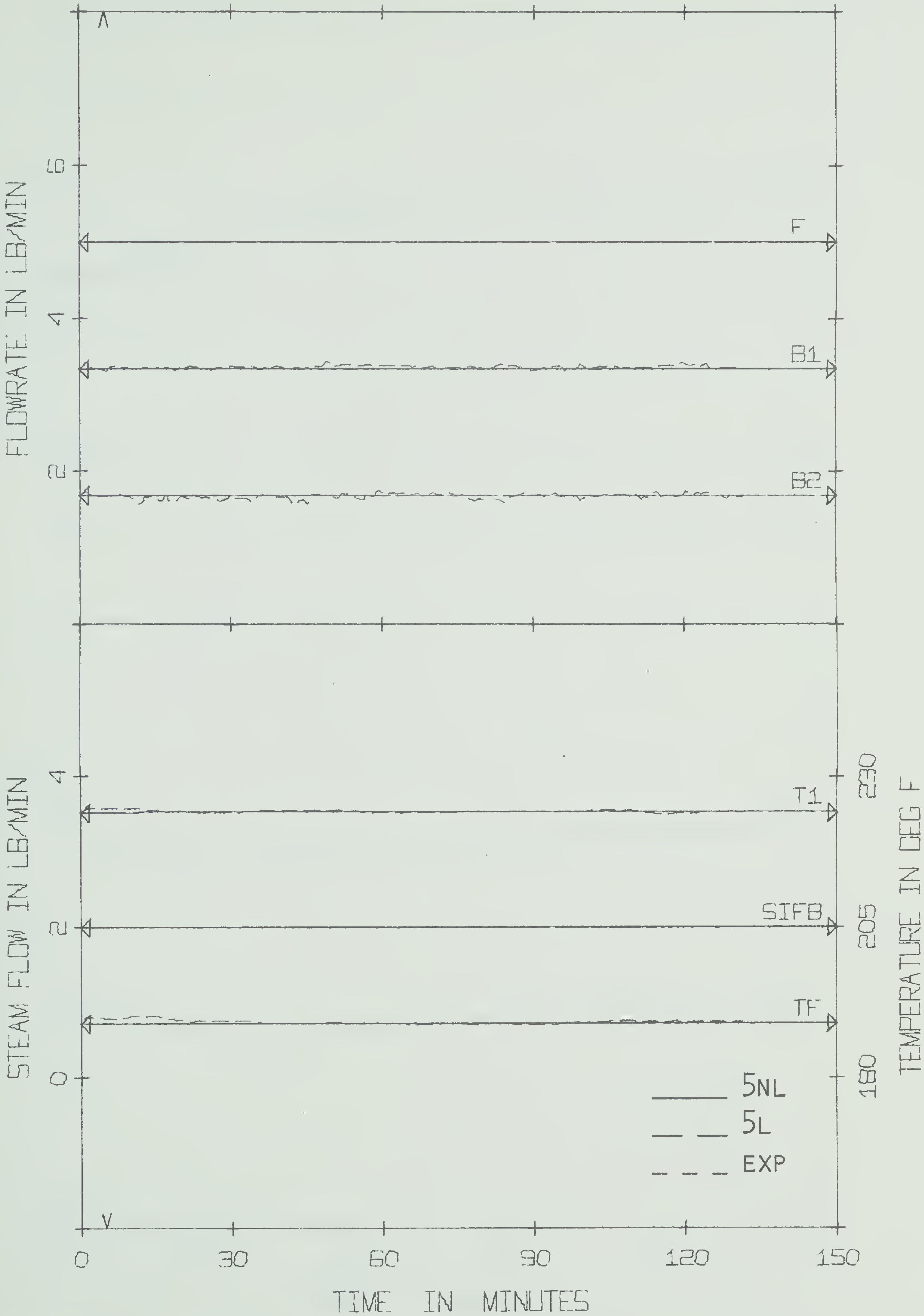


FIGURE 8b. EXPERIMENTAL EVAPORATOR RESPONSE, RUN OL8
(5NL, 5L, EXP/-20%CF/OL)

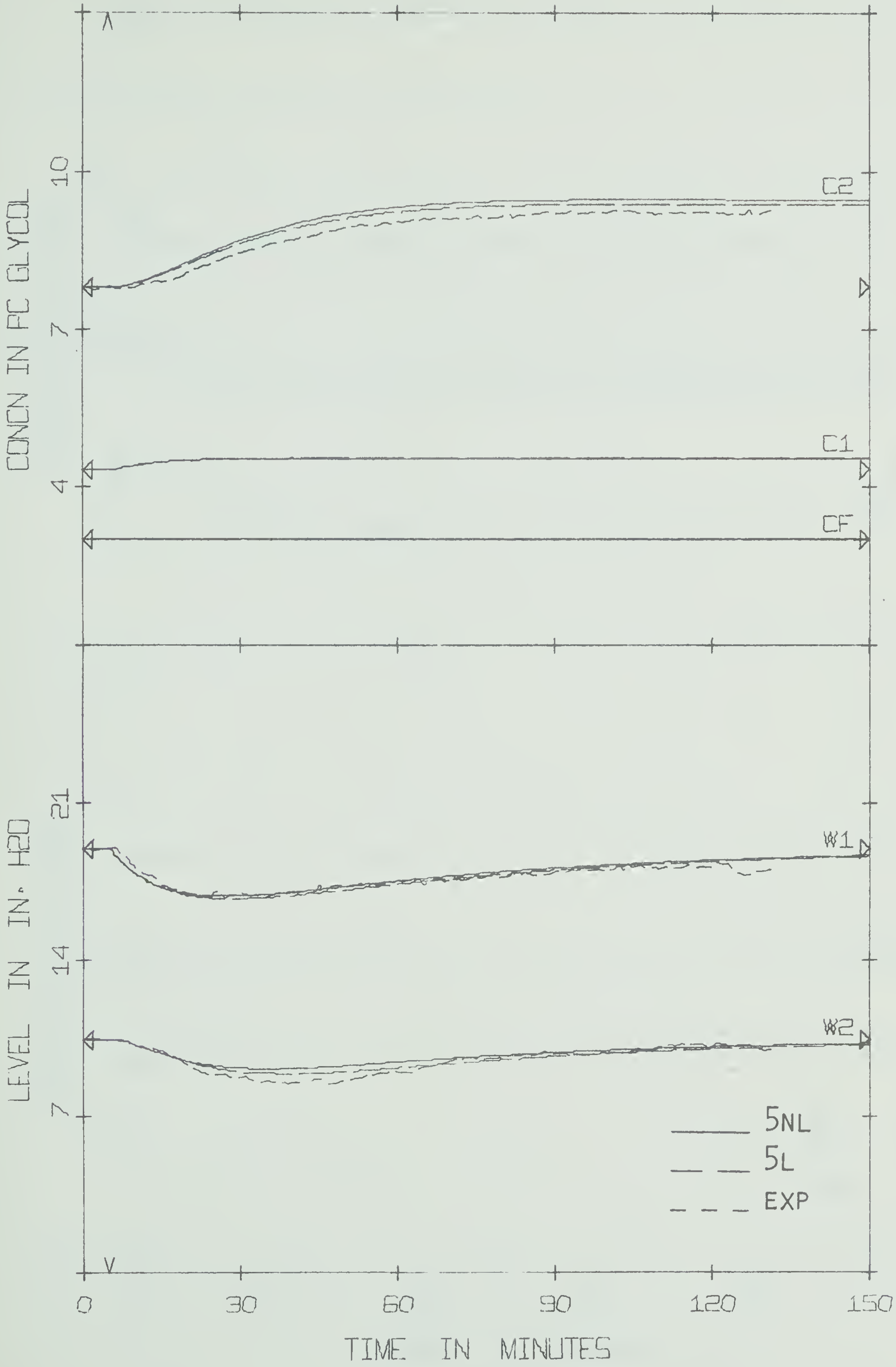


FIGURE 9a. EXPERIMENTAL EVAPORATOR RESPONSE, RUN OL12
(5NL, 5L, EXP/-10%F/OL)

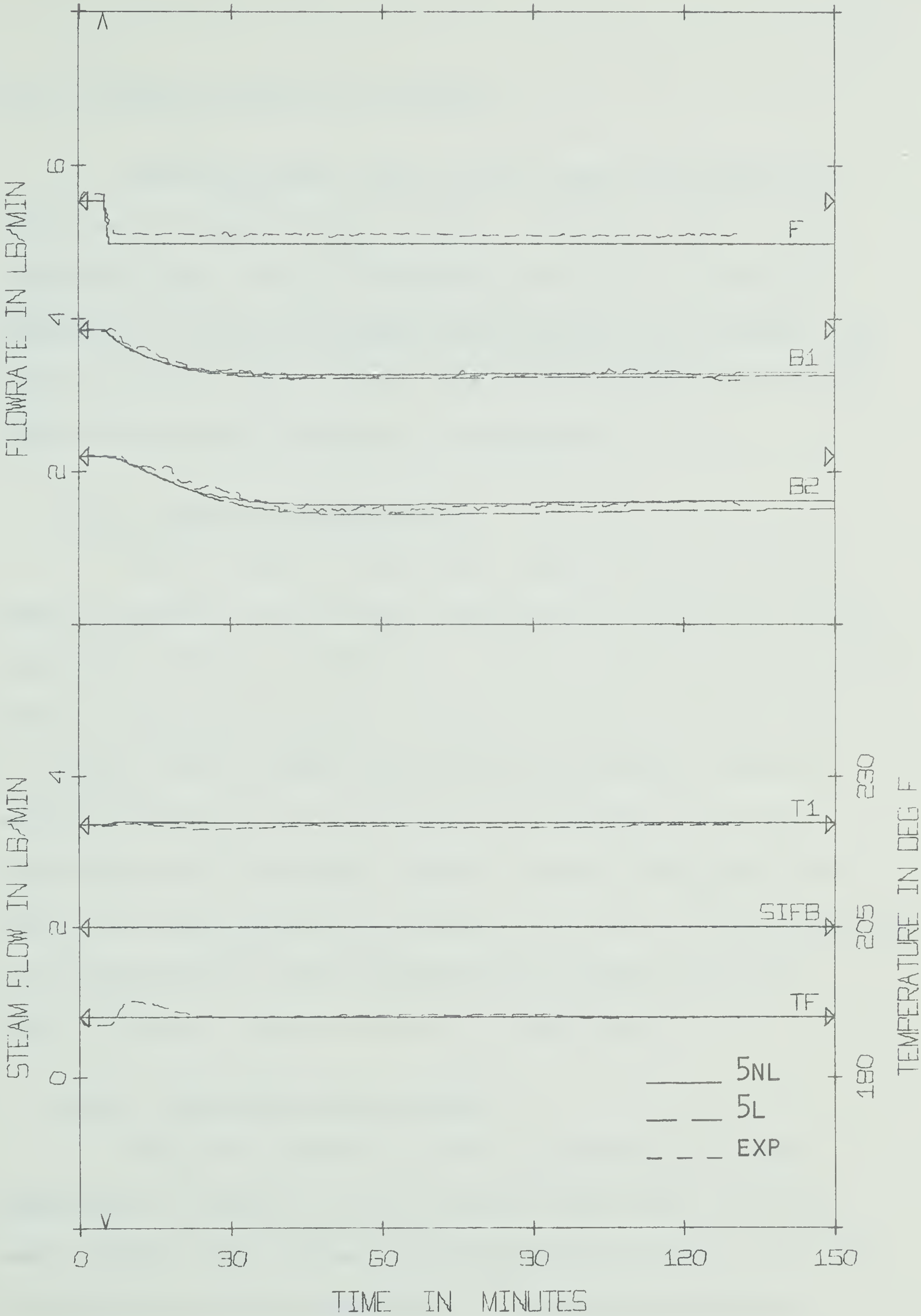


FIGURE 9b. EXPERIMENTAL EVAPORATOR RESPONSE, RUN OL12
(5NL, 5L, EXP/-10%F/OL)

due to incomplete mixing in the effects.

The other notable feature of the results was apparent for twenty percent changes in feed flowrate. Figures 10 and 11 show responses to such a step in the positive and negative directions respectively. While the equipment and linearized model differed with respect to steady state gains the gains of the nonlinear model showed reasonable agreement with those of the equipment.

5.2.2. Multiloop Control System

The five equation state space evaporator model is used to design a successful DDC multiloop configuration of controllers using a sensitivity technique in Chapter 4. However, proportional-plus-integral feedback controller constants obtained by simulating the system on an EAI 580 analog computer and obtaining satisfactory responses produced instability when implemented experimentally. This was attributed to the time delays, dynamics, and non-linearities of the process that were neglected by the model. The same multiloop scheme was then implemented and tuned experimentally by Jacobson [18] and Figure 12A shows a typical response to a load change.

5.2.3. Inferential Control System

Inferential control of product concentration has been implemented by Jacobson [18], who used the fifth-order state space model, and Fehr [17], who used a transfer function model, to estimate the unmeasured product concentration. Figure 12B illustrates an evaporator response while under inferential control. The improved control over normal DDC is a result of the model leading the process

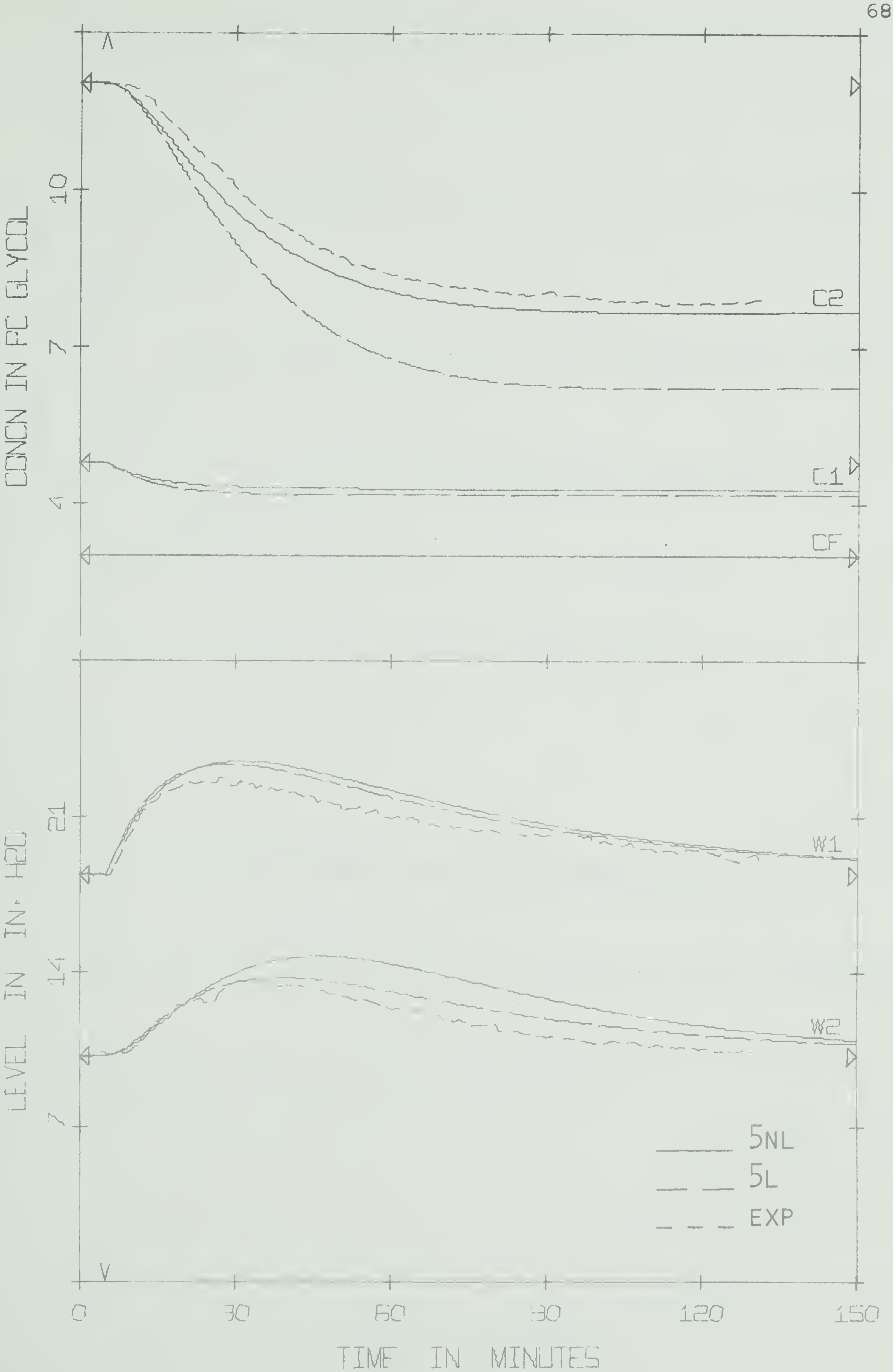


FIGURE 10a. EXPERIMENTAL EVAPORATOR RESPONSE, RUN OL11
(5NL, 5L, EXP/+20%F/OL)

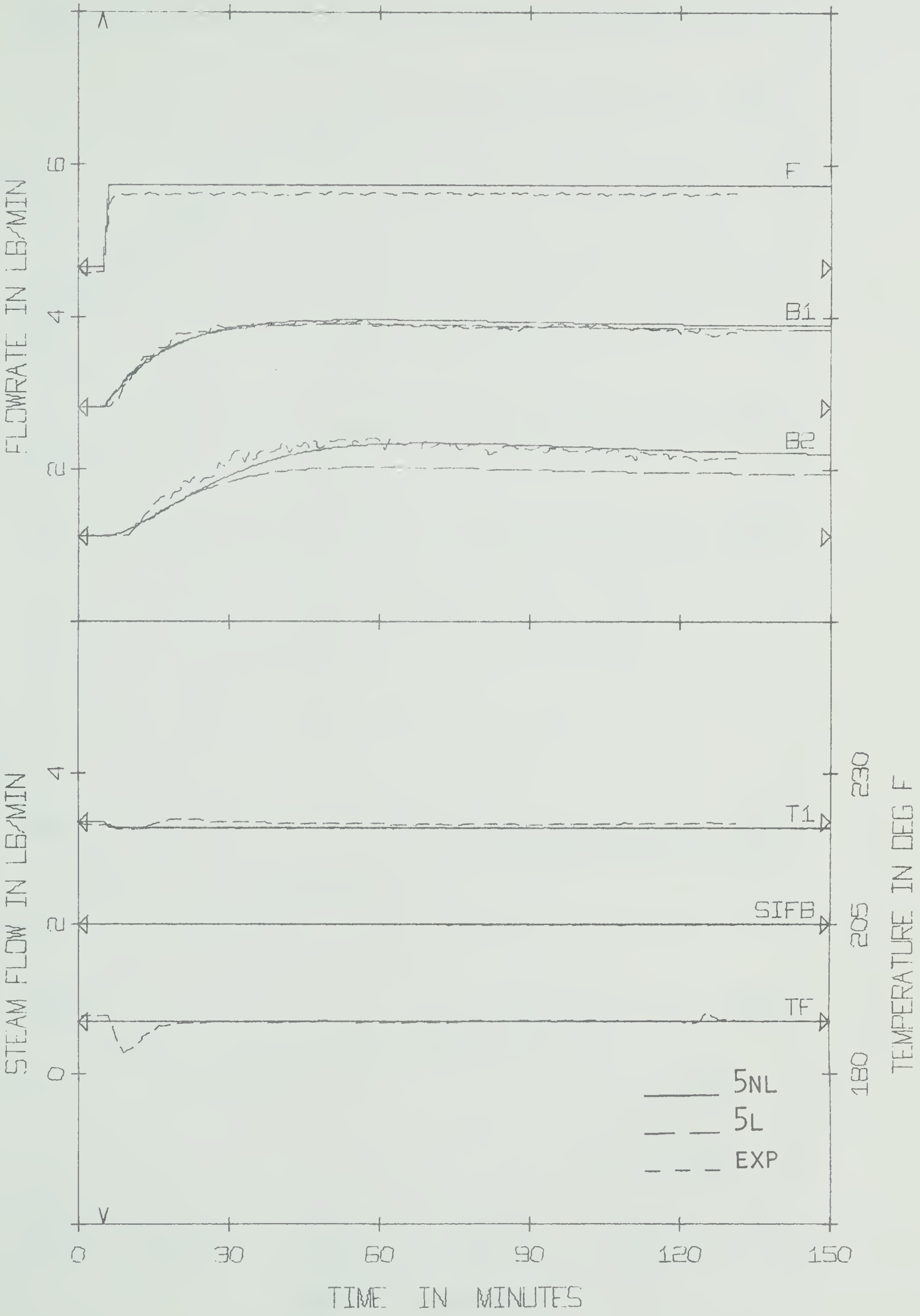


FIGURE 10b. EXPERIMENTAL EVAPORATOR RESPONSE, RUN OL11
(5NL, 5L, EXP/+20%F/OL)

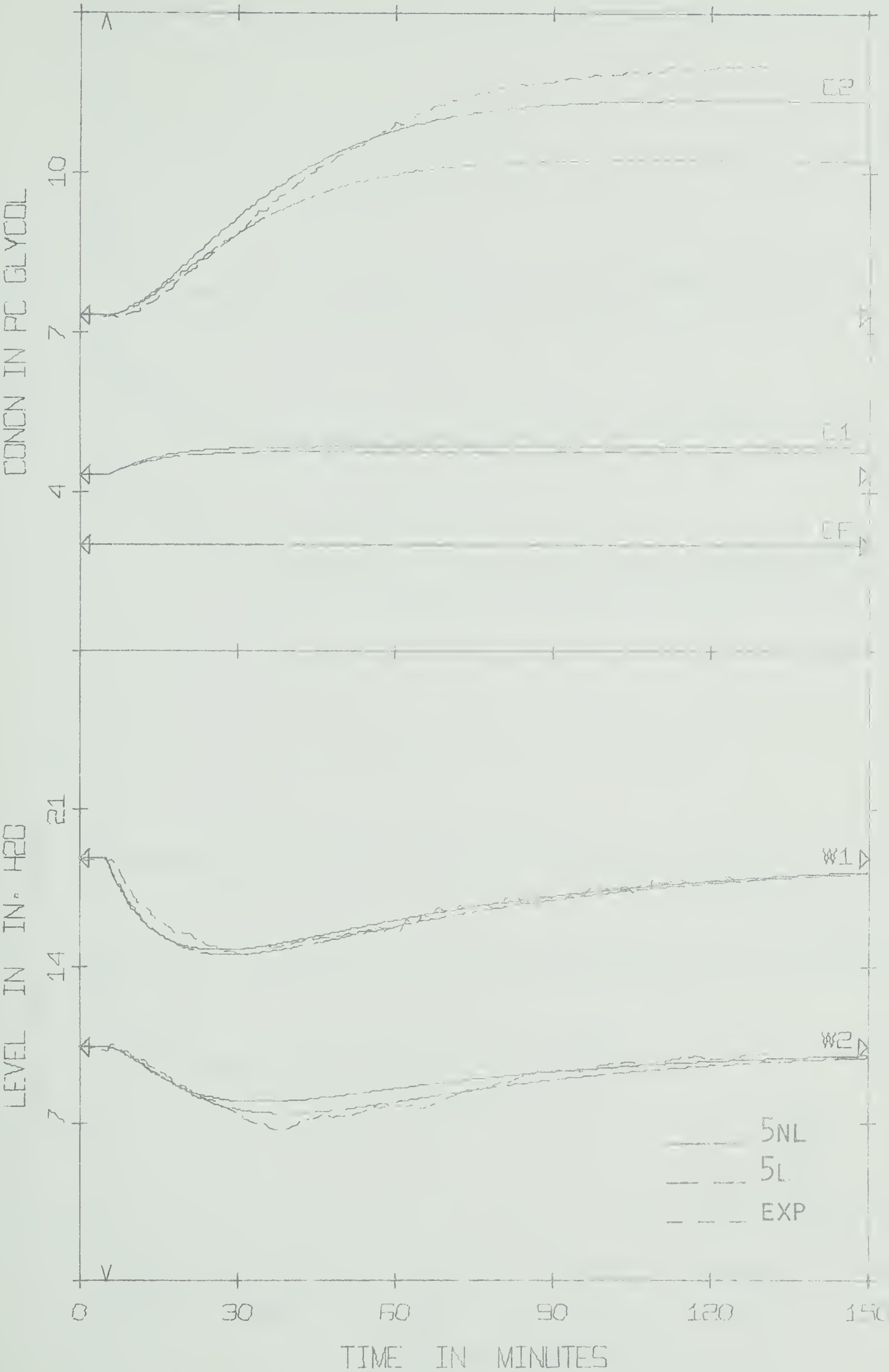


FIGURE 11a. EXPERIMENTAL EVAPORATOR RESPONSE, RUN OL10 (5NL, 5L, EXP/-20%F/OL)

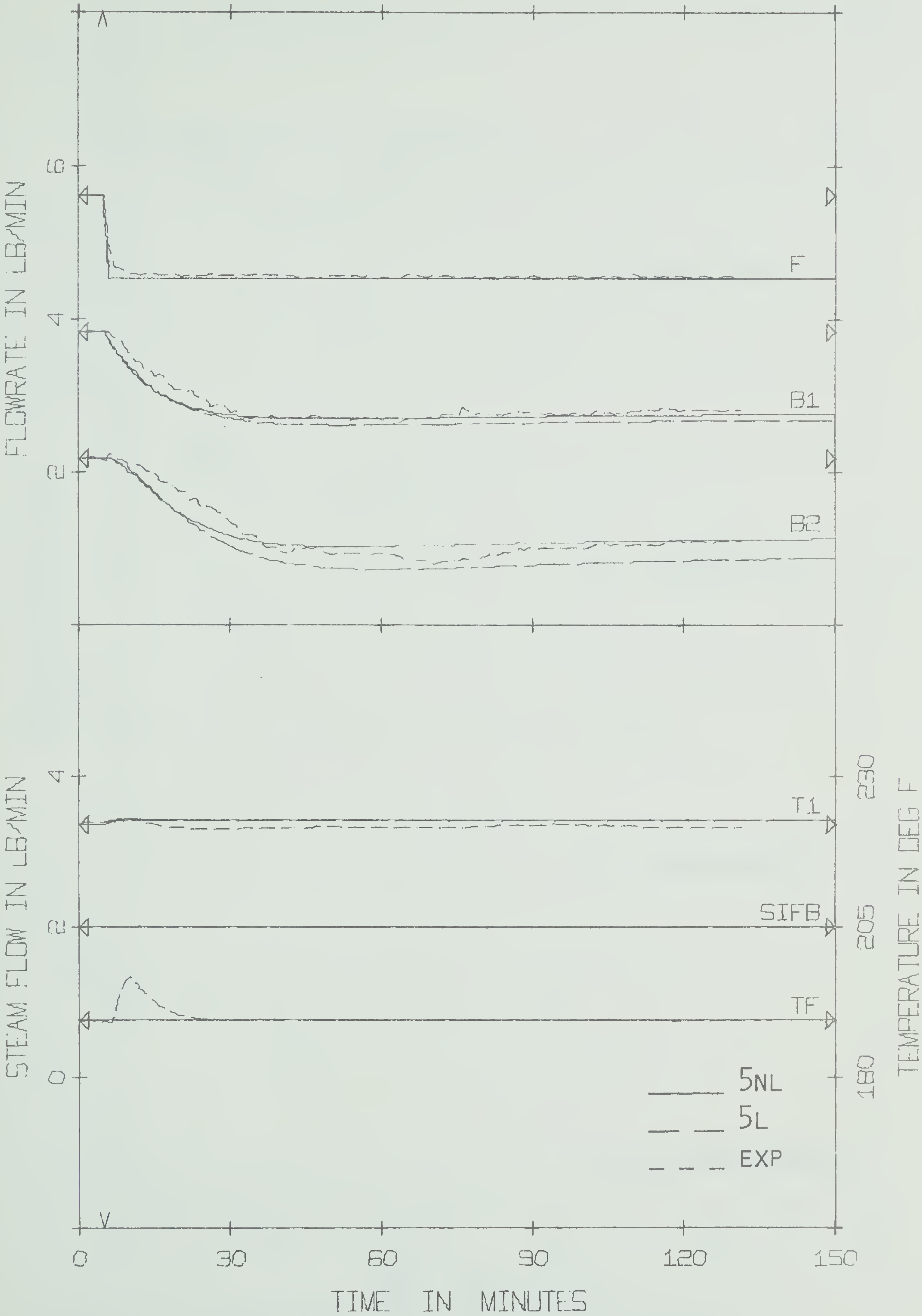


FIGURE 11b. EXPERIMENTAL EVAPORATOR RESPONSE, RUN OL10
(5NL, 5L, EXP/-20%F/OL)

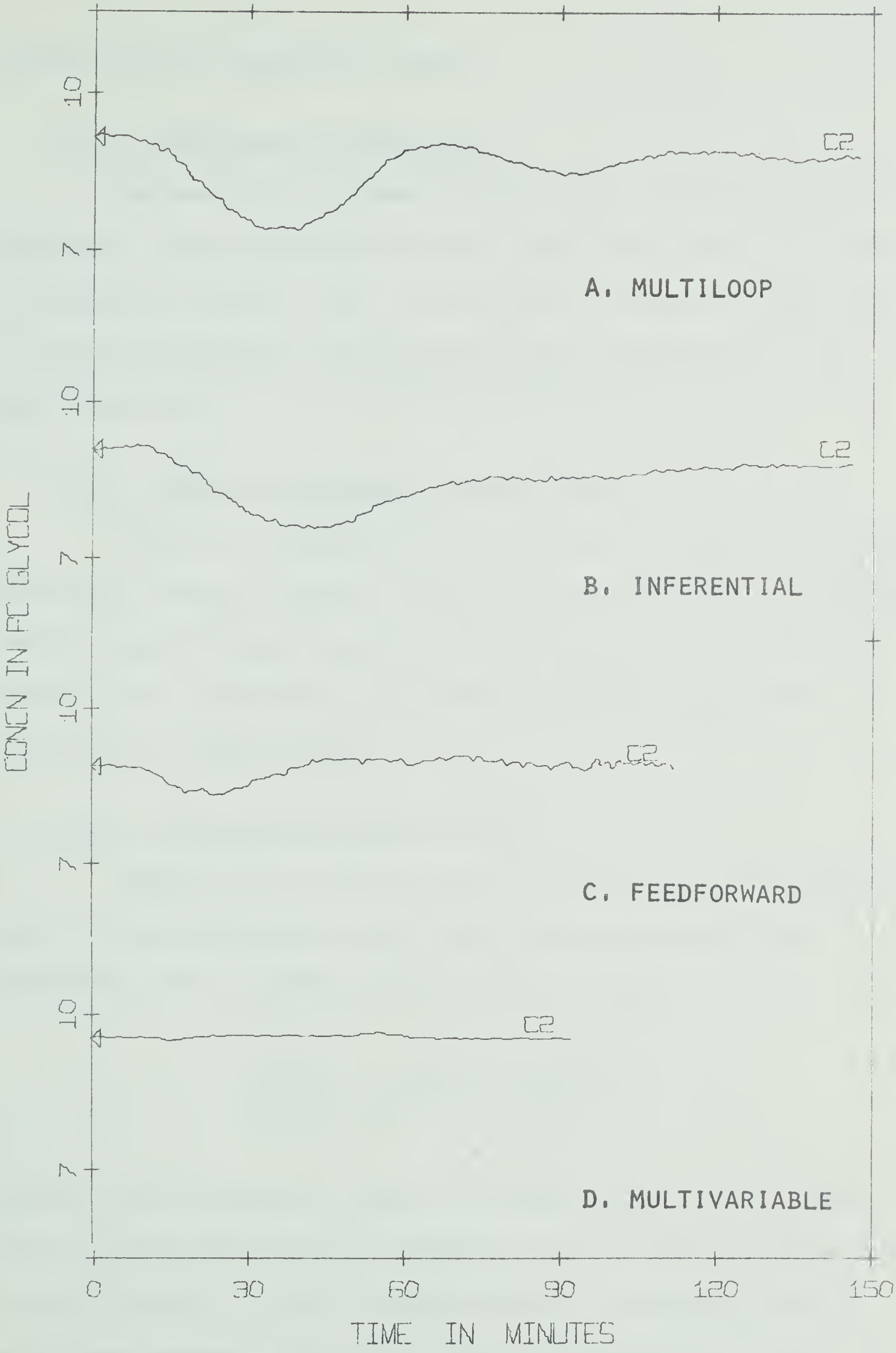


FIGURE 12. EXPERIMENTAL COMPARISON OF CONTROL SYSTEMS (+20%F/DDC, INF, FF, FB)

and hence giving "predictive" control.

5.2.4. Feedforward Control System

The evaporator has been operated under feedforward control by Jacobson [18] using the third-order linear model (Figure 12C) and by Wilson [16] and Fehr [17]. The evaporator's response (Figure 12C) is vastly improved but suffers from the approximations made in the model reduction.

5.2.5. Optimal Multivariable Control System

Dynamic programming and the fifth-order linearized model was used in this project to design an optimal proportional regulator [21]. Experimental results exemplified by Figure 12D show excellent results despite the nonlinearities, time delays, and dynamics neglected in the model derivation and simplification.

5.2.6. State Driving Control System

Optimal state driving control has also been implemented [21] using the fifth-order state space model, and second-order models equivalent to the following transfer function form.

$$\frac{C^2(s)}{S^1(s)} = \frac{K \exp(-\tau_d S)}{(\tau_1 S + 1)(\tau_2 S + 1)} \tag{42}$$

Determination of parameter values by fitting several sets of experimental response data gave time constants of 10 and 20 minutes with no time delay and 7 and 22 minutes when a time delay of three minutes was assumed. Both models fitted the data with the same "goodness of fit" but as shown in Figure 13 produced significantly

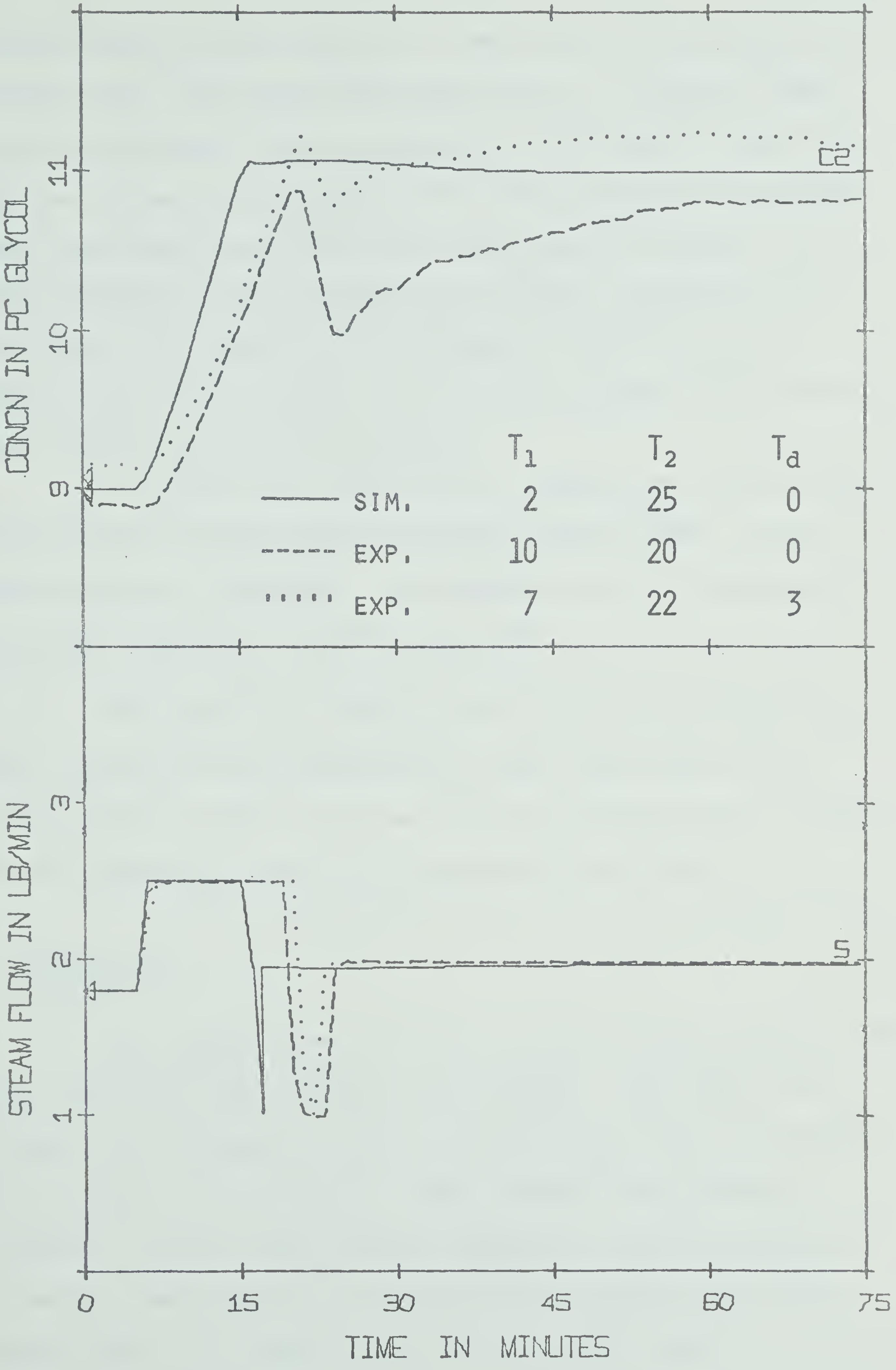


FIGURE 13. EFFECT OF MODEL ACCURACY ON STATE DRIVING

different optimal control policies. These policies were analytically evaluated [21]. The overcorrection was due to τ_1 being too large and it is significant that an examination of the numerical values in Equation (41) and an examination of the poles and zeros of the fifth-order linear model would both lead to the choice of a smaller τ_1. Time constants of 2 and 25 minutes were obtained when Equation (42) was fitted to the response of the 5NL model. The solid curve in Figure 13 shows the simulated response of the 5NL model using switching times calculated using these parameters.

The value of the steam flow to be applied at the end of the state driving transient must be calculated from the model. It was found that use of a constant, K, in Equation (42) was inadequate and a nonlinear algebraic function was used [21].

The optimal state driving control policy based on the fifth-order linearized model produced better results than shown by the broken curves in Figure 13 but was still not accurate enough to handle the large bang-bang changes in the manipulated variables.

6. DISCUSSION

The evaporator models present a number of features which make them interesting from a control study viewpoint. The liquid levels introduce two integrating states and consequently two zero eigenvalues into the model. While this is a common feature in most chemical processes, the zero eigenvalues are considered in much of the theory as special cases which are not fully treated and which many methods cannot handle. For example a ficticious very small eigenvalue must be

added in order to avoid singular matrices in the evaluation of a quadratic Liapunov function. The process is also such that it cannot be decoupled by state variable feedback so that control design techniques must handle the interactions or ignore them and suffer the consequences. Other special features include the controllability and observability of the system, and the nonlinearities, particularly to feed flowrate and steam rate. For these reasons the evaporator models developed here are recommended as test problems for other control studies.

The evaluation of the model from experimental results of open loop tests and the various control systems has also raised the question of what defines a "good" model. It would appear that a model can only be defined as "good" if it performs the task for which it is required and this certainly cannot always be determined from open loop tests alone.

7. CONCLUSIONS

This paper illustrates a typical theoretical modelling approach to a piece of process equipment and some of the inherent dangers in simplifying such a model.

A generalized approach to the modelling of multi-effect evaporators was developed and proved effective and convenient to use. The tenth-order non-linear dynamic model developed for a double effect evaporator was simplified and linearized to produce a fifth-order state space model which gave generally good comparisons with experimental data. Lower order models were also developed but gave satisfactory results only in specific applications such as feedforward

control.

The models were used in the design and implementation of various control schemes and showed that the suitability of the model depends on the end application. For example, the state space model gave excellent results in optimal, multivariable, regulatory control applications but only fair results when used to determine optimal state driving policies.

CHAPTER FOUR

A MULTILOOP APPROACH TO MULTIVARIABLE CONTROL

ABSTRACT

Since the increasing numbers of real-time computers now make the demonstrated advantages of implementing optimal process control techniques feasible, it becomes necessary for the practising engineer to develop a feel for state space design and analyses. By designing a standard multiloop control system from the state space viewpoint, the paper illustrates that the conventional design approach involving both mathematical tools and intuitive reasoning can be applied from a state space model with some advantages over transfer functions.

A seventh-order state space model of an evaporator is partitioned into subsystems introducing the concepts of dynamic and control independence. A five equation subsystem is controlled by three single loops whose configuration follows from a sensitivity analysis and qualitative examination of the dynamics. The state space model predicted interactions between control loops which aided in the controller design and tuning.

Experimental results from a computer controlled pilot plant sized double effect evaporator demonstrated the validity of the design procedure and the existence of the predicted control loop interactions.

1. INTRODUCTION

Engineers have long been familiar with, and made use of, classical transfer functions in the design and analyses of control systems. However, recent research has evolved almost entirely in the "state space" domain which is particularly convenient with more and more analyses and even design being computerized. Also, there is an increasing number of real-time computer installations [1] and work such as that done at the University of Alberta [2,3], has demonstrated the potential of and advantages of optimal control techniques. As a result, it will become important for the practising engineer to be familiar with the state space approach and its relation to the classical domain of transfer functions.

The aim of the paper is to present, in state space, some of the "physical" interpretations which are used in designing multiloop control systems from transfer functions. Designing a multiloop control system from the state space viewpoint is illustrated with a seventh-order state space model of a double effect evaporator and its feed system.

The pilot plant sized evaporator is computer controlled by an IBM 1800 control computer in the Data Acquisition, Control, and Simulation Centre [4] in the Department of Chemical and Petroleum Engineering at the University of Alberta.

Experimental results demonstrated the validity of the conclusions based on the model reduction, mathematical guides, and qualitative reasoning used in the design.

2. LITERATURE SURVEY

The use of a sensitivity analysis to select control loops has long been used under the guise of "experience". It has been only recently that attempts to present the analyses on a mathematical footing have appeared.

Bristol [5] defined a measure of interaction which subsequently led to a sensitivity design technique. Assuming a model relation between output and manipulated variables of the form

$$\underline{y} = \underline{f}(\underline{u}) \qquad (1)$$

the interaction measure was defined as

$$\mu_{ij} = \frac{\left.\dfrac{\partial y_i}{\partial u_j}\right|_{\substack{u_k = \text{const} \\ k \neq j}}}{\left.\dfrac{\partial y_i}{\partial u_j}\right|_{\substack{y_k = \text{const} \\ k \neq i}}} \qquad (2)$$

The "μ" matrix can be interpreted in the form

$$\underline{y} = \underline{\mu}\ \underline{u} \qquad (3)$$

with the elements of $\underline{y}$ and $\underline{u}$ (and hence the elements of $\underline{\mu}$) arranged such that the diagonal μ_{ii} elements are the largest positive numbers. Then the corresponding y_i and u_i are paired into control loops. Large positive "off-diagonal" elements in $\underline{\mu}$ indicate strong interactions between loops and hence difficulties in control. If the "off-diagonal" elements are all zero then the system would be non-interacting.

The design procedure does suffer from some disadvantages:

(a) Although independent of the system of units the values of μ_{ij} are dependent on the actual units of $\underline{u}$ and $\underline{y}$.

(b) The analysis of "non-square" systems, where it is desired to choose between a larger number of manipulated variables than the number of output variables, are complex. This is because the partial derivative in the denominator of equation (2) is difficult to evaluate directly, although it can be mathematically derived from the numerator for "square" systems [6].

(c) In general the partial derivatives are a function of time but Bristol used steady state values. Also , in the case of "integrating" outputs such as liquid level these steady state values do not exist.

Nevertheless, the technique is a useful one and has been recommended by Shinskey[7] and used by Nisenfeld and Stravinski [8] in the analyses of a distillation column.

Davison and Man [9] have recently defined a dynamic interaction index. It was used for proportional state driving control and the index evaluated mathematically. The interaction index was defined as a measure of the relative change in the control of an output when all outputs are controlled compared to when that output alone is controlled.

$$I_j = \frac{J_j^* - J_j}{J_j} \qquad (4)$$

J_j indicates an integral criterion on output y_j when y_j alone is controlled and J_j^* the same criterion but with all the outputs controlled.

The measure does introduce the dynamics but suffers a number of disadvantages when applied to a multiloop design problem:

(a) It assumes an inherently controllable process. Many processes would be out of control with only one output controlled.

(b) It assumes a configuration and controller form. It can therefore only be used to evaluate a design after the fact.

(c) It is strongly dependent on the disturbance or load if used to evaluate a regulatory control system.

3. PROCESS MODELLING

For any form of quantitative design a model of the process is necessary whether it be empirical or theoretically derived. A state space model can follow from a set of differential equations in the time domain or from transfer functions, a procedure which can be handled routinely by a computer [10]. A derived model was used for the evaporator example.

A simplified flow diagram of the double effect evaporator is shown in Figure 1. The two effects are a calandria type unit with an eight inch tube bundle and a long tube vertical type with external forced circulation. The effects are connected in a "forward feed" configuration, and with the second effect under vacuum it is able to use the vapour from the first effect as heating medium.

The feed system mixes streams of aqueous triethylene glycol and water to obtain the correct feed, a nominal 5 lb./min. at 3 percent producing a 10 percent product. Both the water and solution streams are preheated by steam exchangers.

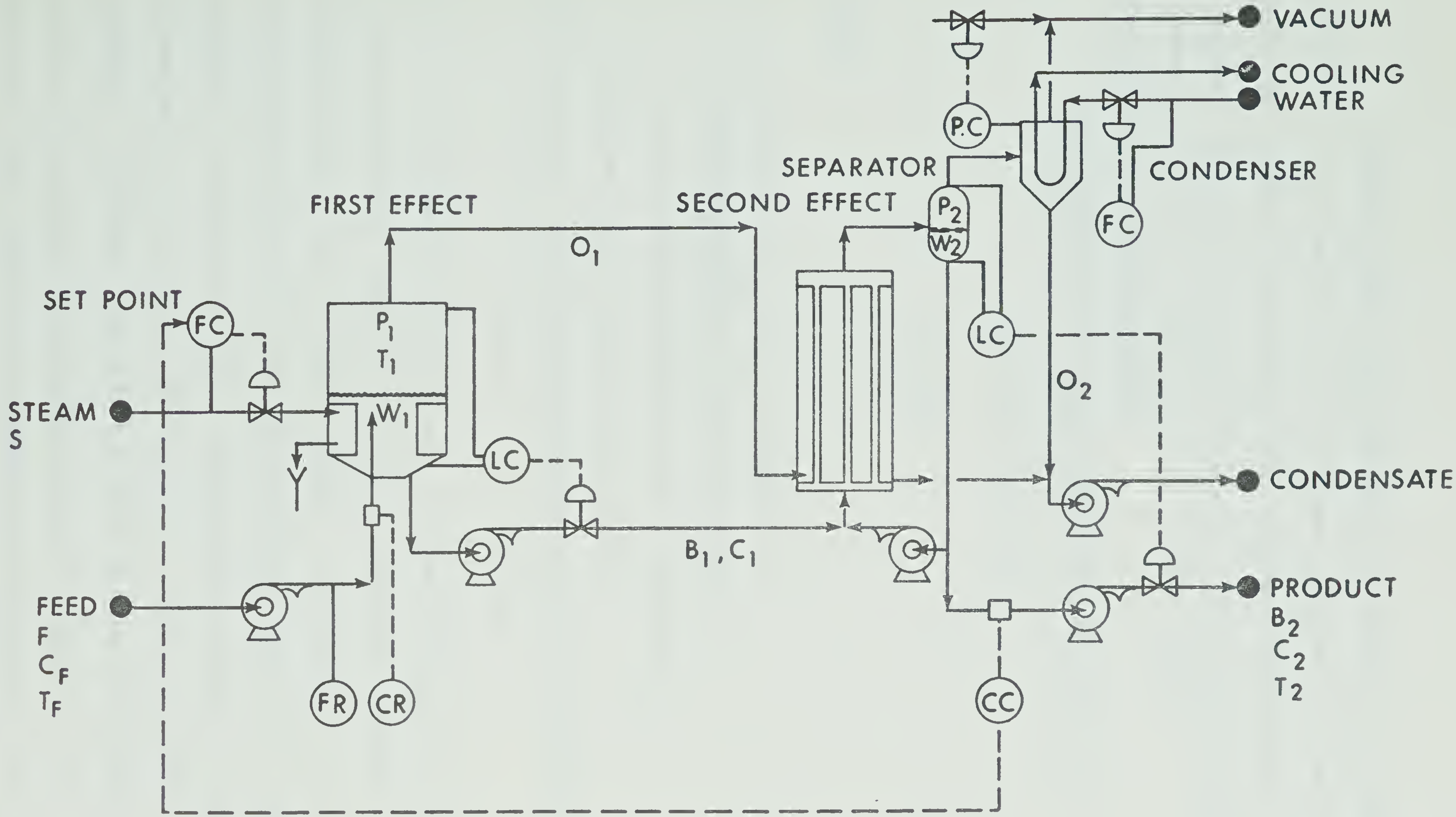


FIGURE 1. PILOT PLANT DOUBLE EFFECT EVAPORATOR

Previous work in the Department has resulted in a model [11,12] in the form of a set of nonlinear differential equations. This, and a model of the feed system, completed the model used in this chapter. Note that the process steady state and hence the model parameters differ from those used in the other chapters.

3.1. State Space Representation

The standard form of the state space equation can be written

$$\underline{\dot{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} + \underline{\underline{D}}\,\underline{d} \tag{5}$$

$$\underline{y} = \underline{\underline{C}}\,\underline{x} \tag{6}$$

where

$\underline{x}$ is the state vector of dimension n,

$\underline{u}$ is the manipulated vector of dimension m,

$\underline{d}$ is the load/disturbance vector of dimension p,

$\underline{y}$ is the output vector of dimension q,

and $\underline{\underline{A}}$, $\underline{\underline{B}}$, $\underline{\underline{D}}$, $\underline{\underline{C}}$ are constant coefficient matrices of dimension $n \times n$, $n \times m$, $n \times p$, and $q \times n$ respectively.

In order to present a set of linear differential equations in this form it is necessary to classify the process variables. This classification is not in general unique. In the present case with first-order equations the state vector becomes simply those variables appearing in a time derivative. Manipulatable variables are generally determined by the design specifications and control objectives for the process. The control objective for the evaporator was defined as control of output concentration accepting any changes in feed conditions. This left steam rates to the feed heaters, steam rate to the first

effect, and bottom rates from the two effects as possible manipulatable variables. The remaining variables are then classified together as loads or disturbances. Output variables, as is the case with manipulatable variables, depend to a large extent on physical and policy matters. The control objective requires output concentration from the second effect to be constant and physical considerations make it necessary to control holdups, or levels, in the two effects, at least between limits.

These assumptions lead to the classification of variables shown in equation (7). The two holdups, W1 and W2, and the output concentration, C2, constitute the output vector.

The state space model in equation (7) follows from a linearization of the nonlinear differential equations about a steady state operating condition. A computer program [13] has been written to produce the coefficient matrices from any consistent steady state. Measurements from the process can be adjusted to provide consistency either by the program or before entry by a statistical adjustment program such as MEBOL [14].

3.2. Model Reduction

Reducing the dimensionality of a model is especially important when large systems of equations are encountered. The complexity of the design and the numerical computations when using the model can be reduced. In this paper the partitioning of a large problem into smaller partially or completely independent problems is considered based on the following concepts:

(a) Dynamic Independence
    Complete or Sequential

$$
\begin{bmatrix} \dot{HOF1} \\ HOF2 \\ \hline W1 \\ C1 \\ H1 \\ \hline W2 \\ C2 \end{bmatrix}
=
\left[\begin{array}{cc|ccc|cc}
-16.28 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -24.22 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & -.002 & -.087 & 0 & 0 \\
0 & 0 & 0 & -.043 & +.087 & 0 & 0 \\
.010 & .015 & 0 & -.014 & -.571 & 0 & 0 \\
\hline
0 & 0 & 0 & -.003 & -.125 & 0 & -.0001 \\
0 & 0 & 0 & +.039 & +.125 & 0 & -.036
\end{array}\right]
\begin{bmatrix} HOF1 \\ HOF2 \\ \hline W1 \\ C1 \\ H1 \\ \hline W2 \\ C2 \end{bmatrix}
$$

$$
+
\left[\begin{array}{cc|ccc}
5.46 & 0 & 0 & 0 & 0 \\
\hline
0 & 7.58 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & -.045 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & +.169 & 0 & 0 \\
0 & 0 & 0 & +.061 & -.036 \\
0 & 0 & 0 & -.024 & 0
\end{array}\right]
\begin{bmatrix} S1 \\ S2 \\ S \\ B1 \\ B2 \end{bmatrix}
+
\begin{bmatrix}
-5.45 & 0 & +10.84 & 0 & 0 \\
0 & 0 & 0 & -7.62 & +16.6 \\
+.026 & 0 & 0 & +.039 & 0 \\
+.019 & +.045 & 0 & +.015 & 0 \\
-.031 & 0 & 0 & -.034 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} F1 \\ CF1 \\ HF1 \\ F2 \\ HF2 \end{bmatrix}
$$

Equation (7)

(b) Control Independence

Complete or Sequential

Complete dynamic independence of two subsystems implies that neither subsystem is dependent on the other's state vector, that is in a partitioned form of the state equation

$$\begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \end{bmatrix} = \begin{bmatrix} \underline{\underline{A}}_{11} & \underline{\underline{A}}_{12} \\ \underline{\underline{A}}_{21} & \underline{\underline{A}}_{22} \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + \begin{bmatrix} \underline{\underline{B}}_{11} & \underline{\underline{B}}_{12} \\ \underline{\underline{B}}_{21} & \underline{\underline{B}}_{22} \end{bmatrix} \begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \end{bmatrix} \tag{8}$$

the partitioned matrices $\underline{\underline{A}}_{12}$ and $\underline{\underline{A}}_{21}$ are zero.

Sequential dynamic independence of two subsystems implies that the first subsystem in the sequence is independent of the state of the other but the second subsystem in the sequence is dependent on the state of the first. That is for the sequence S_1S_2 the partitioned matrix $\underline{\underline{A}}_{12} = 0$ while for the sequence S_2S_1 then $\underline{\underline{A}}_{21} = 0$. From a study of the state matrix in equation (7) it becomes obvious that the evaporator could be partitioned into three "sequential dynamically independent" subsystems, the feed preparation section, the first effect, and the second effect, this also being intuitively obvious. It also becomes plain that the feed section can be further broken down into two "completely dynamically independent" subsystems, that is the two feed heaters.

This method of reduction is "obvious" if the state variables are ordered so as to produce an $\underline{\underline{A}}$ matrix that can be partitioned directly as was done with equation (7). However, in general it may be necessary to change the order of the state variables (i.e. interchange

corresponding rows and columns of $\underline{\underline{A}}$ and $\underline{\underline{B}}$) to determine the interdependence of different subsystems. The proper ordering of the $\underline{x}$ vector cannot be predicted a priori although in general it is better to group the variables associated with physical subsystems as opposed to grouping them into flows/levels/etc.

While dynamic independence is sufficient to reduce computational order for open loop studies, there is another aspect if the reduced subsystems are to be studied separately for control and design. Dynamically independent subsystems can be considered completely control independent if neither system is dependent on the manipulated vector of the other, that is $\underline{\underline{B}}_{12}$ and $\underline{\underline{B}}_{21}$ in equation (8) are null matrices. Sequential control independence is defined in an analogous manner to sequential dynamic independence. Any single one of the number of possible combinations may or may not present a simplification in the design or computation. A brief examination of the structure will decide this point.

Reduction methods such as those of Davison [15] and Chidambara [16] could now be used to eliminate the less dominant modes (or time constants) of the subsystems.

In the evaporator model, equation (7), an examination of the $\underline{\underline{A}}$ matrix shows that there are four subsystems with dynamic independence, but the two feed heaters and the combined process constitute the only three control independent subsystems in matrix $\underline{\underline{B}}$. These three subsystems may be studied separately with regard to designing control systems.

All subsequent work in this paper will deal with the five state variable subsystem representing the two effects together. This reduced model is shown in equation (9).

Experimental results [12] have shown that the model is a good representation of the evaporator for load and manipulated variable changes of up to ten percent. The correspondence between model and process responses deteriorates somewhat for larger perturbations.

4. MULTILOOP DESIGN

In designing a multiloop system of feedback controllers for a process, there are three important steps; choosing the manipulated and output variables, pairing the chosen variables to produce the "best" configuration, and determining the controller constants in the resulting control loops. The configuration can be chosen by selecting a scheme involving the least number of interactions as presented here, or by synthesizing a more complex set of compensators to produce a completely noninteracting system [17]. Gilbert and Pivnichny [18] have produced a computer algorithm for such a design in state space.

4.1. Manipulated and Output Variables

These variables follow to some extent from the system itself and the objectives of the system. This was discussed in Section 3.1 and for the evaporator resulted in three output and three manipulated variables.

In general, such a fortuitous arrangement $(m = q = 3)$ may not occur. If there is an excess of manipulated variables then the

$$\begin{bmatrix} \dot{W1} \\ \dot{C1} \\ \dot{H1} \\ \dot{W2} \\ \dot{C2} \end{bmatrix} = \begin{bmatrix} 0 & -.002 & -.087 & 0 & 0 \\ 0 & -.043 & +.087 & 0 & 0 \\ 0 & -.014 & -.571 & 0 & 0 \\ 0 & -.003 & -.125 & 0 & -.0001 \\ 0 & +.039 & +.125 & 0 & -.036 \end{bmatrix} \begin{bmatrix} W1 \\ C1 \\ H1 \\ W2 \\ C2 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & -.045 & 0 \\ 0 & 0 & 0 \\ +.169 & 0 & 0 \\ 0 & +.061 & -.036 \\ 0 & -.024 & 0 \end{bmatrix} \begin{bmatrix} S \\ B1 \\ B2 \end{bmatrix}$$

$$+ \begin{bmatrix} +.065 & 0 & 0 \\ -.020 & +.045 & 0 \\ -.040 & 0 & +.025 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F \\ CF \\ HF \end{bmatrix}$$

$$\begin{bmatrix} W1 \\ W2 \\ C2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} W1 \\ C1 \\ H1 \\ W2 \\ C2 \end{bmatrix}$$

Equation (9)

Reduced Model State Equation

excess can be held constant and the choice between them made on the basis of the sensitivity analyses presented in the following section. The "degrees of freedom" for pairing manipulated and controlled variables is the number of manipulated variables. If there is an excess of output variables then their number must be reduced either by neglecting those least important or combining numbers of them into single control objectives.

4.2. Configuration Design

This paper presents a simply defined and evaluated sensitivity ratio which offers some advantages over Bristol's interaction measure. However, a multiloop configuration cannot be designed solely on the basis of a sensitivity evaluated at steady state or some arbitrary time. Such important factors as dynamics, the range over which a manipulatable variable can vary, and the "cost" of changing the manipulatable variables must be considered. These effects are at best semi-quantitative but some possible procedures for taking them into account are presented. An attempt at designing a feedforward control configuration using sensitivities led to a multivariable scheme which is presented with other optimal schemes in Chapter 6.

4.2.1. Sensitivity Analysis

Consider the system model defined in state space form, equations (5) and (6), with the variables in perturbation form and normalized by their steady state values. This removes possible effects of units. For defining a sensitivity ratio assume no loads or disturbances and step changes in manipulated variables, then:

$$\underline{y}(t) = \underline{\underline{C}} \int_{0}^{t} e^{\underline{\underline{A}}(t-\tau)} d\tau \; \underline{\underline{B}} \; \underline{u}$$

$$= \underline{\underline{F}}(t) \; \underline{u} \tag{10}$$

A time-varying sensitivity ratio is defined by

$$R_{ij}(t) = \frac{|f_{ij}(t)|}{\sum_{k=1}^{m} |f_{kj}(t)|} \tag{11}$$

where the columns of the $\underline{\underline{R}}(t)$ matrix sum to unity.

Note: $f_{ij}(t)$ is equivalent to Bristol's partial derivative with $u_k = \text{const}, \; k \neq j$.

The values of $R_{ij}(t)$ can be evaluated at steady state ($t = \infty$) or at some arbitrary time. An integrating output, such as the liquid levels in the evaporator example, has no steady state value under a sustained disturbance and open loop conditions. In examples where there are integrating outputs an arbitrary time of three or four major time constants could be used. (The system's major time constant is approximately the inverse of the smallest non-zero diagonal element of the $\underline{\underline{A}}$ matrix.)

Then the output and manipulated variables should be grouped so that their corresponding sensitivity ratios are the largest. As with Bristol's interaction measure, other significant elements indicate interactions between loops and possible control difficulties, and if the chosen elements are unity and the others correspondingly zero then the control loops are noninteracting.

4.2.2. Dynamic Considerations

It is also desirable to consider the dynamic relationships between the paired output and manipulated variables. In the extreme case of very slow dynamics, relative to the influence of disturbances, this consideration could overrule the sensitivity analysis. Also, where two configurations have similar "steady state" sensitivities it would be desirable to select the configuration involving the "fastest" responses. It is also necessary to pay attention to process time delays which may or may not have been included in the model formulation.

Three possible approaches from the state space viewpoint are suggested:

(a) an integrated average sensitivity ratio defined as

$$R_{ij} = \frac{|m_{ij}|}{\sum_{k=1}^{m} |m_{kj}|} \tag{12}$$

where

$$m_{ij} = \int_{o}^{T} f_{ij}(t)dt \tag{13}$$

and T is an arbitrary time suggested to be three or four major time constants.

For similar "steady state" sensitivities the integral values should give an indication of the speeds of response, faster responses giving larger integral values.

(b) a derivative sensitivity ratio defined as

$$R_{ij} = \frac{\left|\frac{df_{ij}}{dt}\right|_{t=0}}{\sum_{k=1}^{m} \left|\frac{df_{kj}}{dt}\right|_{t=0}} \tag{14}$$

This measure, when compared with the "steady state" sensitivity values, distinguishes between first-order and higher order responses or those involving time delays. A disadvantage is that in the latter cases the initial slopes are small irrespective of subsequent behaviour.

(c) a more detailed examination of the dynamics of the different configurations can be made by a study of the state space equation (see Section 4.3).

4.2.3. Manipulated Variable Weighting

It is often desirable to "weight" the manipulated variables either because they are costly or often because there are restrictions to the amount by which they can be changed. Weighting can be easily accomplished by postmultiplying the sensitivity matrix by a diagonal weighting matrix. If required, this should be done before selecting the loops. The diagonal elements of the weighting matrix could be expressed as

$$(\text{cost factor}) * \frac{(\text{desired or actual span of manipulated variable})}{(\text{steady state value})}$$

4.2.4. Evaporator Example

Equation (15) presents the state space model solution in the form of equation (9). From this equation, which includes two integrating outputs W1 and W2, the following sensitivity ratio matrix was evaluated at a time of two hours (about four times the major time constant of 28 minutes).

$$\begin{bmatrix} W1 \\ W2 \\ C2 \end{bmatrix} = \begin{bmatrix} 0.338 & \underline{0.404} & 0 \\ 0.493 & 0.547 & \underline{1.0} \\ \underline{0.169} & 0.049 & 0 \end{bmatrix} \begin{bmatrix} S \\ B1 \\ B2 \end{bmatrix} \tag{16}$$

The underlined sensitivity ratios indicate the control loop configuration which gives the highest ratios in each row:

(i) First effect holdup W1 controlled by the outflow B1

(ii) Second effect holdup W2 controlled by the outflow B2

(iii) Product concentration C2 controlled by the steam S

Figure 2 illustrates the sensitivity ratios as a function of time. The crossing of the W2 and C2 sensitivity curves within the first four minutes indicates that care must be exercised in choosing a time at which to evaluate the sensitivities.

Equations (17) and (18) present the integral and derivative ratios. The difference in magnitude of the initial slopes of the responses between the C2 sensitivities (third row) and the others, obvious in Equation (18), indicates the higher order dynamics involved, since it is known that time delays are not incorporated.

$$\begin{bmatrix} W1 \\ W2 \\ C2 \end{bmatrix} = \begin{bmatrix} (-.026t + .041 - .045e^{-.569t} + .004e^{-.045t}) & (-.045t) & 0 \\ (-.037t - .048 - .009e^{-.036t} + .065e^{-.569t} - .008e^{-.045t}) & (.061t) & (-.036t) \\ (1.582 + .067e^{-.569t} + 2.388e^{-.045t} - 4.037e^{-.036t}) & (-.667 + .667e^{-.036t}) & 0 \end{bmatrix} \begin{bmatrix} S \\ B1 \\ B2 \end{bmatrix}$$

Equation (15)

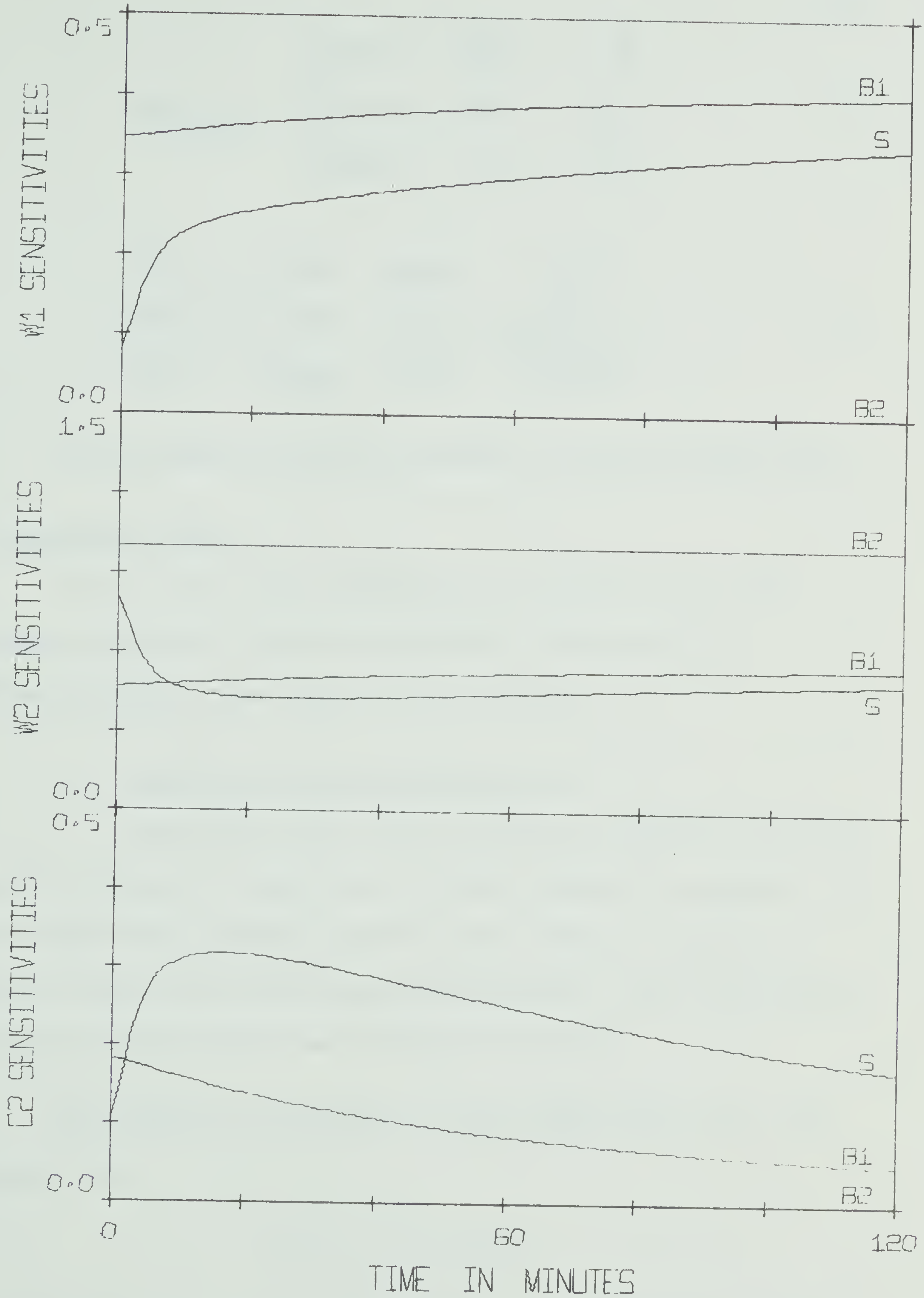


FIGURE 2. EVAPORATOR OUTPUT SENSITIVITY RATIOS

$$\underset{=}{R}_{integral} = \begin{bmatrix} 0.311 & \underline{0.392} & 0 \\ 0.464 & 0.534 & \underline{1.0} \\ \underline{0.225} & 0.074 & 0 \end{bmatrix} \tag{17}$$

$$\begin{bmatrix} \dot{W}1 \\ \dot{W}2 \\ \dot{C}2 \end{bmatrix} = \begin{bmatrix} 0.403 & \underline{0.423} & 0 \\ 0.574 & 0.574 & \underline{1.0} \\ \underline{0.023} & 0.003 & 0 \end{bmatrix} \begin{bmatrix} S \\ B1 \\ B2 \end{bmatrix} \tag{18}$$

These sensitivity matrices support the choice already made.

4.3. Controller Design

Once a control loop configuration has been decided upon it becomes necessary to choose constants for the controllers. There are basically three attacks on the problem.

(a) Stability Based Design Techniques

Procedures such as Bode or Nyquist analyses [19,20] can be used to design both controllers and their parameters *a priori*. The methods require the dynamics, usually in the form of a transfer function, between output and manipulated variable. This can be simply extracted from the state space coefficient matrices.

The system is represented in state space form by the model (equation (5)):

$$\underline{\dot{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} + \underline{\underline{D}}\,\underline{d}$$

The elements of the columns of b_{ji} and d_{ji} indicate direct gains between the corresponding manipulated variable u_i or load d_i and the corresponding state variable x_j. The off-diagonal elements of

$\underline{\underline{A}}$ are the gains for the process interactions between state variables. The diagonal elements of $\underline{\underline{A}}$ are related to the time constants of the corresponding state nodes by the relation:

$$\tau_i = \frac{-1}{a_{ii}} \tag{19}$$

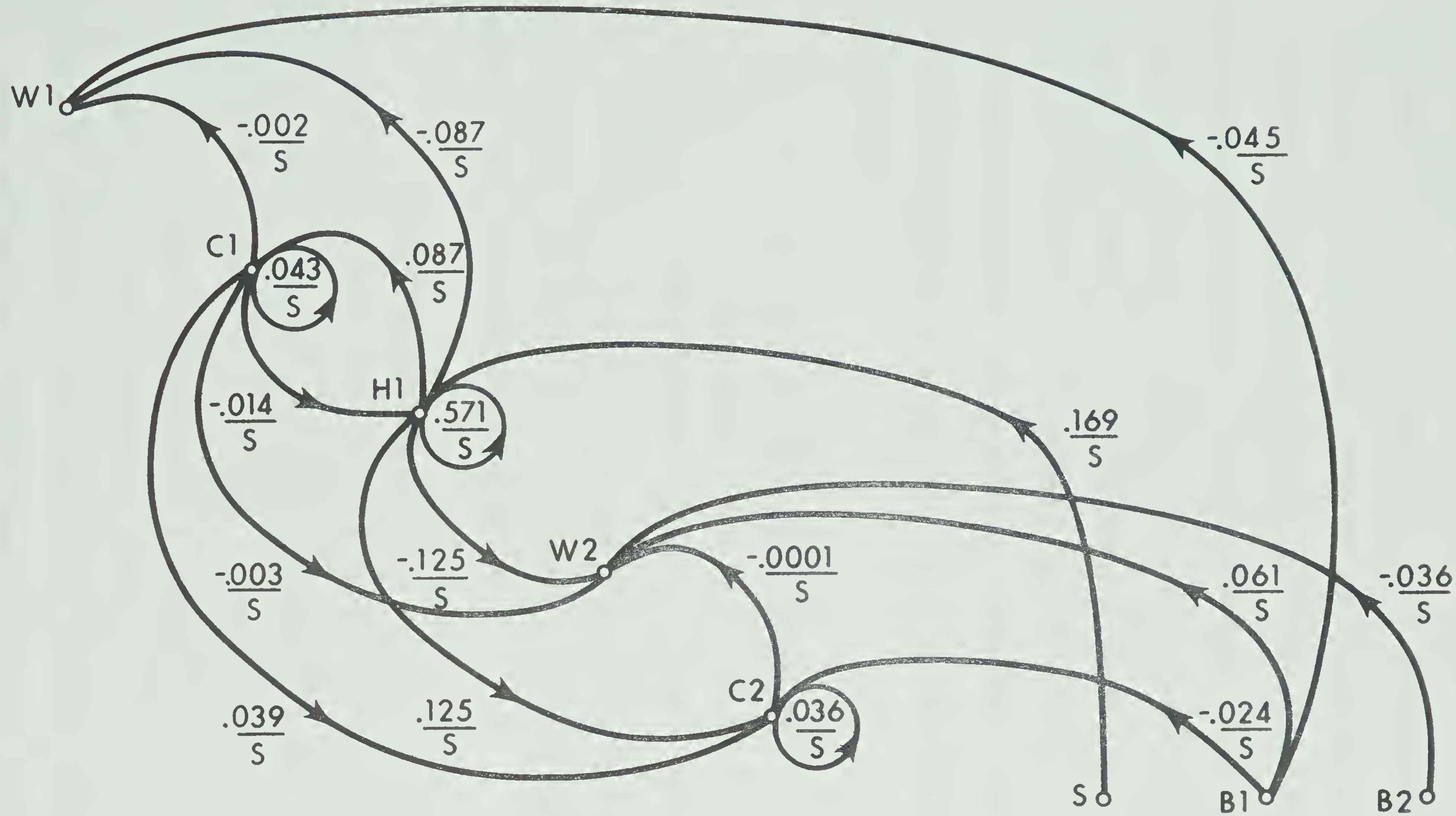
($a_{ii} = 0$ indicates an integrating state node)

These time constants are not necessarily equal to the system time constants due to the process interactions but are approximately so for a system with distinct eigenvalues and exactly so for a noninteracting process (diagonal $\underline{\underline{A}}$ matrix).

The dynamics can best be extracted by considering the state equation as a signal flow graph and using Mason's Formula [20]. The diagonal elements of $\underline{\underline{A}}$ are nodes representing the states with self-loops (of transmittance a_{ii}/s), the upper off-diagonal elements are feedback branches (of transmittance a_{ij}/s), the lower off-diagonal elements are forward branches, and the elements of $\underline{\underline{B}}$ and $\underline{\underline{D}}$ are branches of transmittance b_{ij}/s or d_{ij}/s from the corresponding manipulated variable or load to the corresponding node or state. With some experience the dynamics can be read from the state equation matrices without drawing an intermediate flow graph (Figure 3).

(b) Simulation Techniques

Control constants can be evaluated a priori by simulating the process and control loops on an analogue computer and adjusting the constants on-line to obtain a good response.



$\underline{\underline{A}}$ MATRIX

$\underline{\underline{B}}$ MATRIX

FIGURE 3. EVAPORATOR SIGNAL FLOW DIAGRAM (WITHOUT LOADS)

This was done for the evaporator problem and the constants predicted for the C2-S loop were significantly higher than those under which the pilot plant would successfully operate. This was attributed to nonlinearities, small time delays, and dynamics which the model did not contain. The evaporator has a small time delay in the solution stream between effects due to poor mixing in the bottom of the first effect and a piping delay. This was not included in the model and would have contributed to the unsuccessful prediction of controller parameters. These factors point to the care which must be exercised when estimating constants either by Bode type analyses or by simulation.

(c) Empirical On-line Tuning Techniques

These procedures, such as the Zeigler-Nichols method, are perhaps the most reliable in choosing suitable controller constants for a multiloop control configuration which neglects the process interactions. However, they do not permit or facilitate the design and setup of controllers prior to operation.

Interactions between control loops which often give trouble in tuning multiloop systems can be predicted from the state equation. In the evaporator example a strong interaction between B1 and C2, through gain element b_{25} and the C2 node, indicated possible tuning problems. A dynamic study of the $\underline{\underline{A}}$ matrix showed that, considering the most direct forward paths, B1 acted on C2 through the C2 node while corrective action from steam S acted through the H1 and C2 nodes and was thus a little slower. Obviously it was necessary to tune the W1 - B1 loop so that B1 changed gradually and smoothly,

termed averaging control on a level loop, so as not to unduly disturb product concentration with its slow control loop.

In tuning the first effect level loop, controller settings used by Andre [12] and others were found to produce considerable overshoot in B1 and about one and a half cycles (Trial 1 in Table 1). In order to reduce the overshoot and cycling the usual intuitive action is to reduce the gain (or increase proportional band). However, as shown by Trial 2, this increases overshoot and cycling. Deeper investigation of averaging liquid level control in Buckley [[19] Chapter 18] suggested a smaller proportional band, and very little integral action. Trial 3 gave much more satisfactory results and constants for future use.

5. RESULTS

A number of experimental runs were carried out on the pilot plant process to support the conclusions arrived at in studying the state space model.

Figure 4 shows the evaporator's response to a 20 percent change in feed flowrate using the control configuration chosen by the sensitivity analyses and averaging level controls. Tight level control with the same load change is shown in Figure 5 and the unfavourable interaction between B1 (Figure 5b) and C2 (Figure 5a) which was predicted becomes obvious.

Evaporator control under an alternative configuration (W2-B2, W1-S, and C2-B1 loops) was found to be poor and did not give stable enough control for a steady state to be reached.

TABLE 1

LEVEL CONTROLLER TUNING

Trial	Prop. Band %	Int. time mins.	B2 overshoot %	C2 criterion ISE
1	80	6	69	1.51
2	320	6	73	3.79
3	87	50	12	1.13

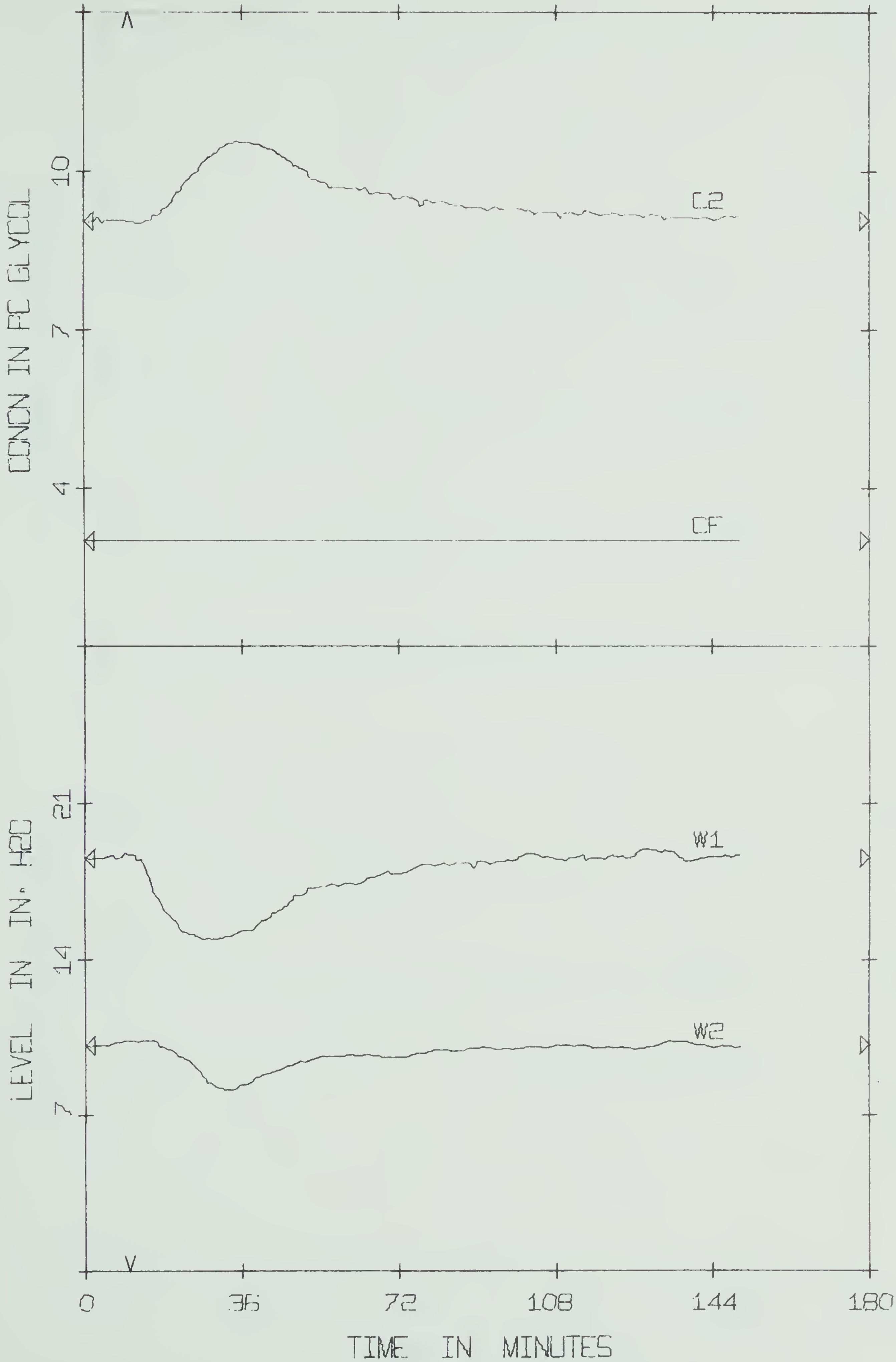


FIGURE 4a. EXPERIMENTAL MULTILOOP CONTROL RUN I
(EXP/-20%F/DDC/T1/DDC15)

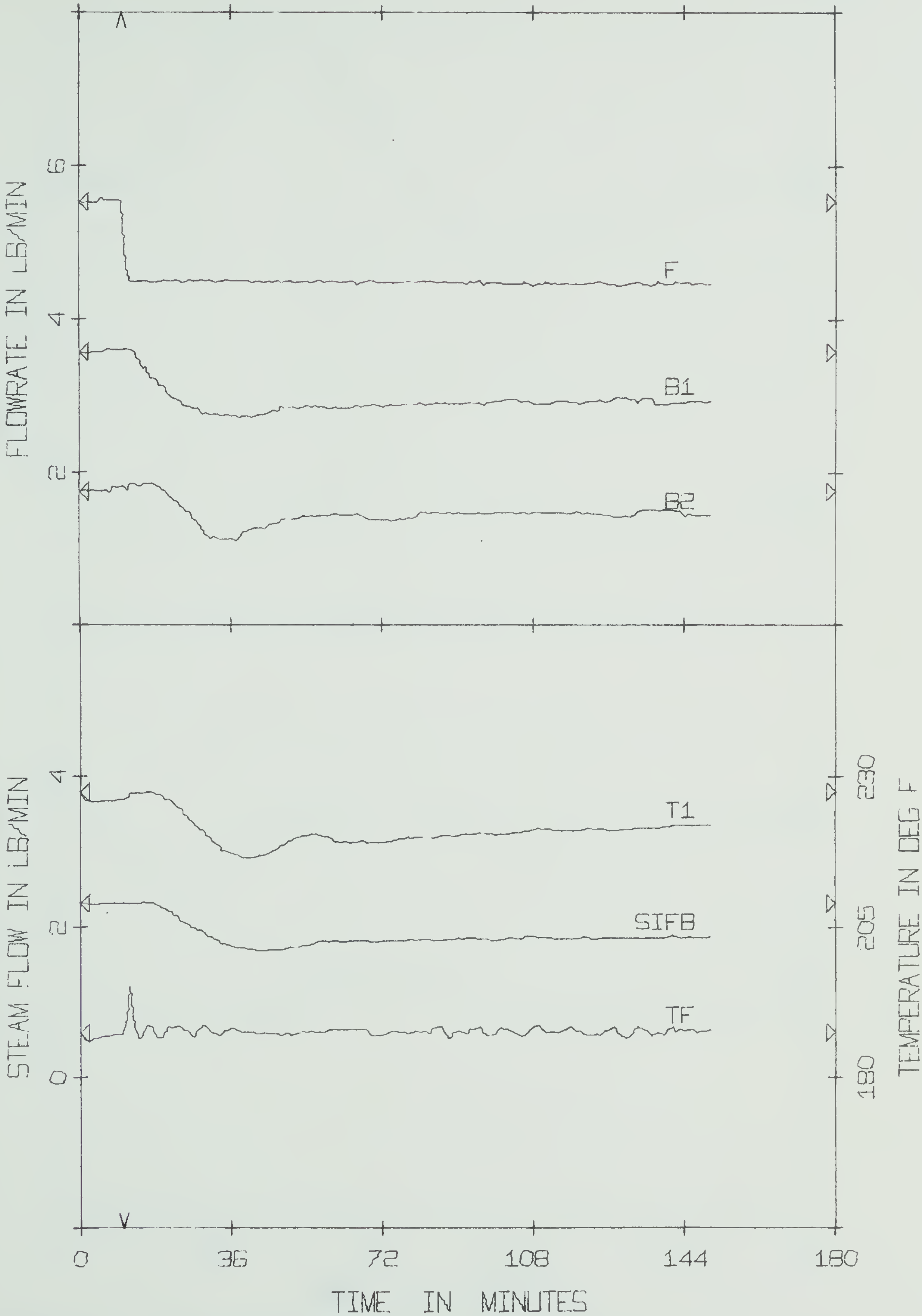


FIGURE 4b. EXPERIMENTAL MULTILOOP CONTROL RUN I
(EXP/-20%F/DDC/T1/DDC15)

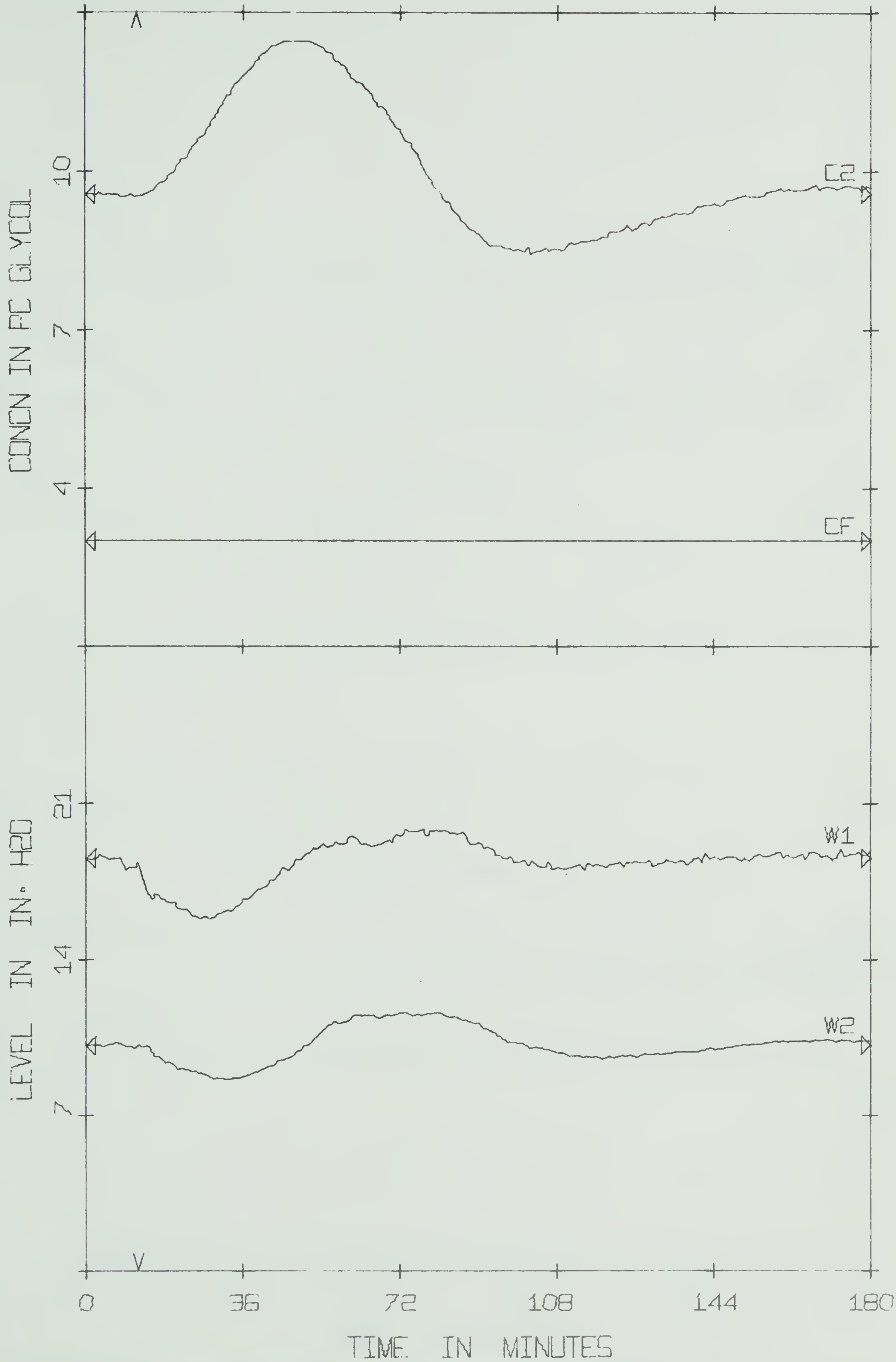


FIGURE 5a. EXPERIMENTAL MULTILOOP CONTROL RUN II
(EXP/-20%F/DDC/T2/DDC4)

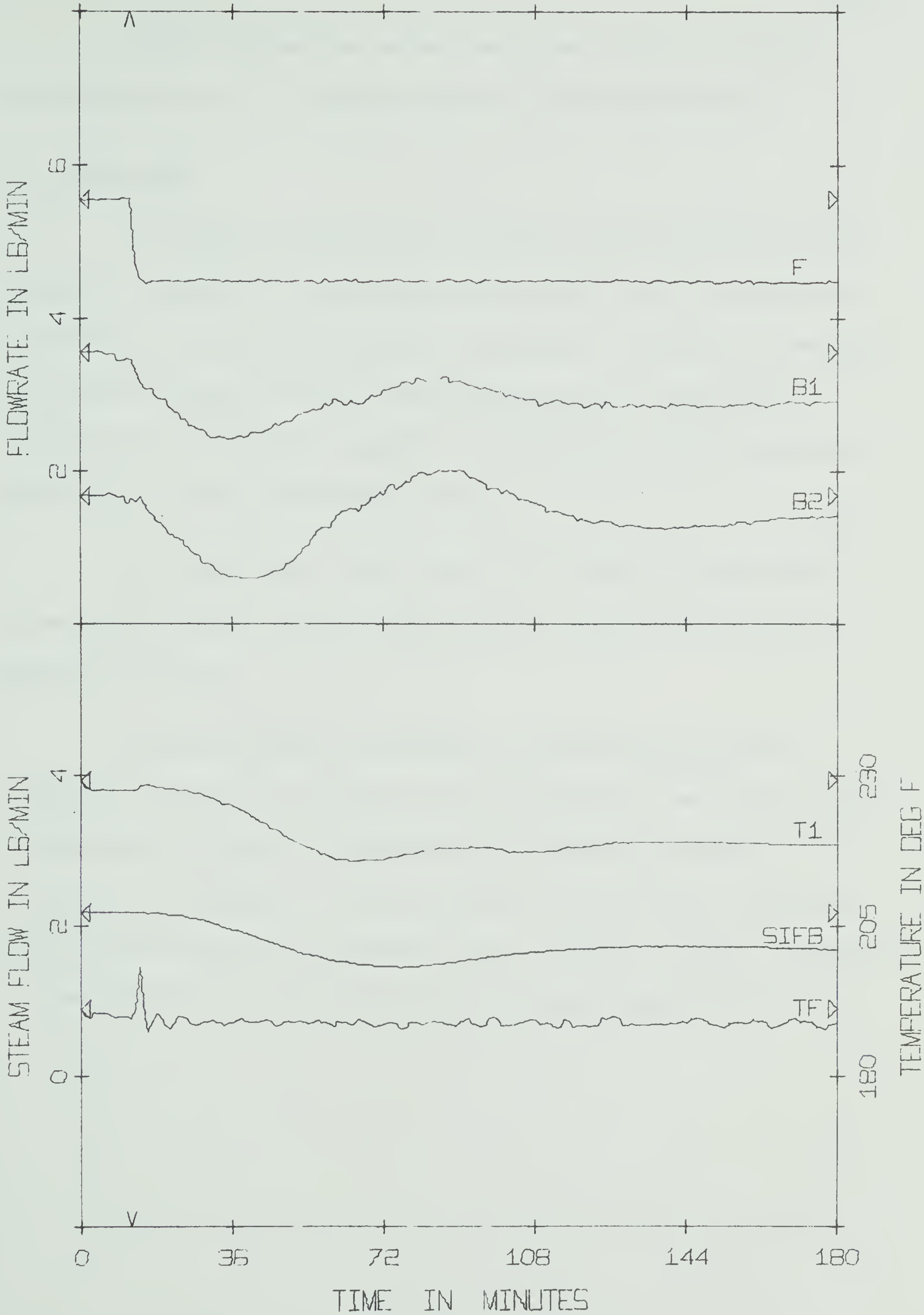


FIGURE 5b. EXPERIMENTAL MULTILOOP CONTROL RUN II (EXP/-20%F/DDC/T2/DDC4)

Figure 6 shows the control responses under the designed configuration to a 30 percent change in feed concentration.

6. CONCLUSIONS

The paper has shown that successful multiloop design can be carried out based on a state space process model using a combination of mathematical tools and intuitive reasoning. A sensitivity analysis was used to produce a control configuration which a qualitative consideration of dynamics supported. Experimental results showed the superiority of the configuration chosen.

The state space model proves to be useful in predicting interactions between control loops which can affect the controller design and tuning.

While the model was adequate for predicting open loop response for small perturbations and for designing a successful control configuration, it was not good enough for predicting controller constants by simulation. This illustrates the fact that a model may not be good enough for all possible uses, although it can be considered a "good" model if it adequately handles the task required of it.

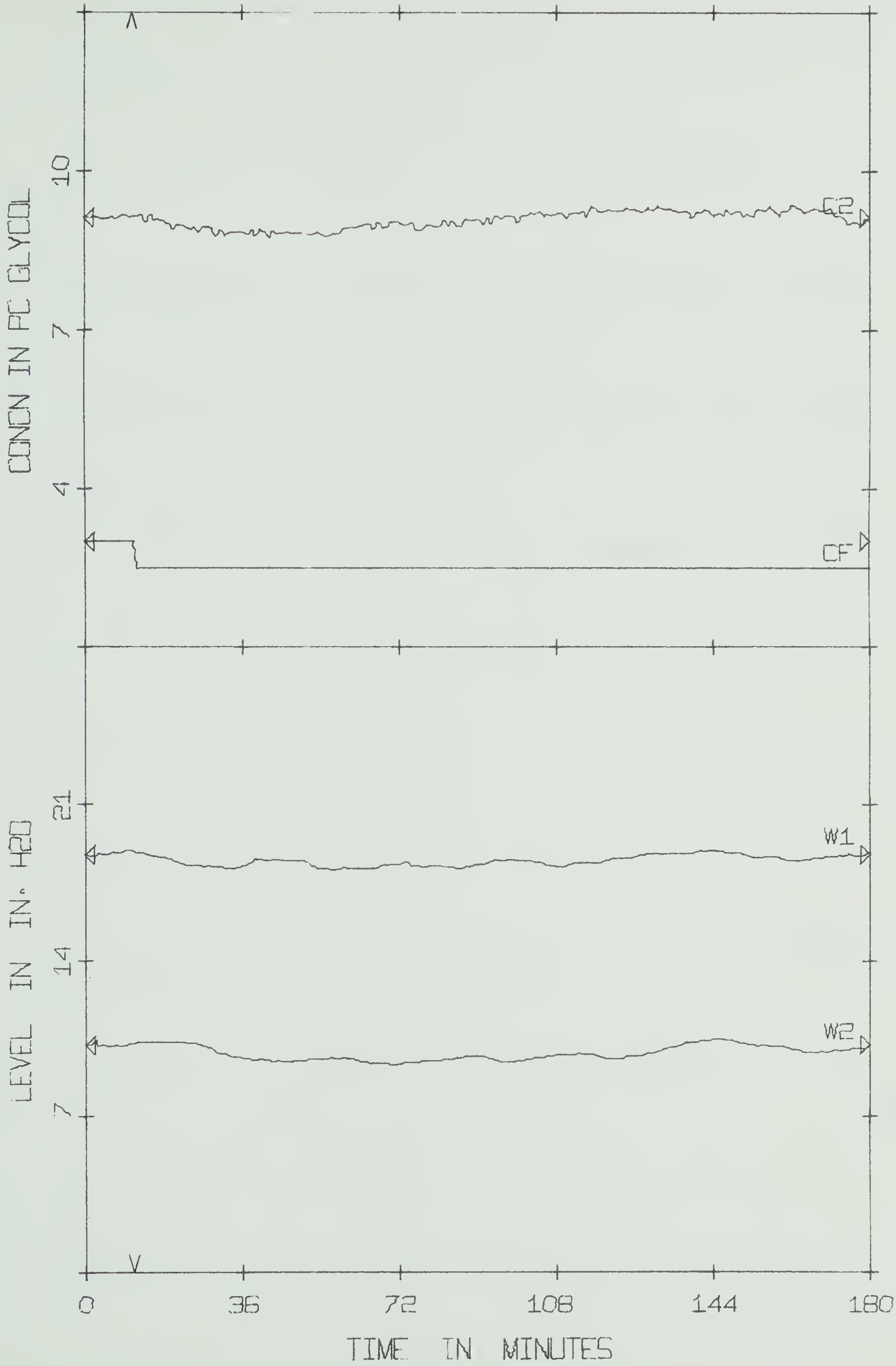


FIGURE 6a. EXPERIMENTAL MULTILOOP CONTROL RUN III
(EXP/-17%CF/DDC/T1/INF2)

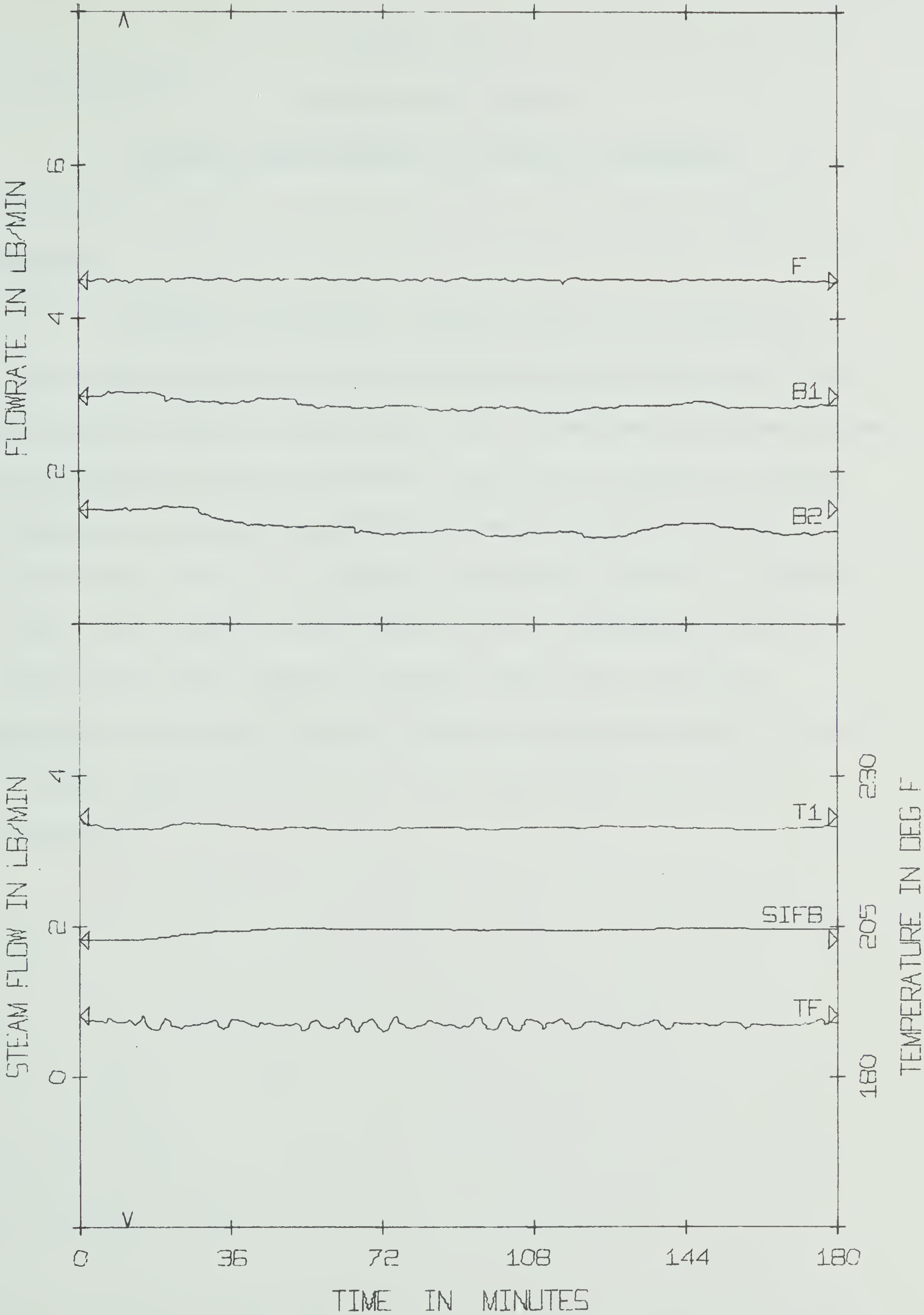


FIGURE 6b. EXPERIMENTAL MULTILOOP CONTROL RUN III
(EXP/-17%CF/DDC/T1/INF2)

CHAPTER FIVE

MULTIVARIABLE CONTROL

OPTIMAL DESIGN APPROACH VIA DYNAMIC PROGRAMMING

ABSTRACT

Optimal, multivariable computer control was applied to a pilot plant evaporator and produced significantly better control than conventional Direct Digital Control (DDC). The generalized proportional plus feedforward control algorithm, which also accounts for setpoint changes, was generated by applying dynamic programming techniques to a formulation based on a quadratic performance index and a standard linear, time invariant, state space model. Experimental results showed that longer sampling times and lower gains (produced by appropriate choice of weighting matrices) were more desirable than the "optimum" values obtained from a simulation study of the design parameters.

1. INTRODUCTION

The aim of this work was to design and implement an optimal multivariable regulator for a pilot plant double effect evaporator. The control is implemented by a digital process control computer which by its nature defines a discrete time system.

It is possible to extend classical design techniques by specifying a control law and optimizing the parameters. However many of the more flexible and efficient optimization techniques in modern control theory produce both the form of the control law and the parameter values. Some of these formulations produce complex and even physically unrealizeable control laws. However it has been shown that a linear, discrete, time invariant model optimized according to a summed quadratic criterion produces a proportional feedback control law [[1], page 155].

This paper will examine this formulation for processes subject to load disturbances and will extend the dynamic programming solution to include these loads and the desired values of some states. The resulting control law contains feedback, feedforward, and set-point modes. The control law derivation will be based on a general summed quadratic criteria with exponential time weighting. The effect of the design parameters is investigated by simulations and some general guidelines for their choice presented.

The feedback mode of the control law is implemented by a FORTRAN program in conjunction with a Direct Digital Control system on an IBM 1800. Experimental results obtained in controlling the

pilot plant evaporator are presented to confirm the guidelines that were based on simulated data.

2. LITERATURE SURVEY

The linear discrete regulator problem can be solved by most optimal control techniques including the calculus of variations, which produces a discrete Ricatti equation [[2] page 130], and dynamic programming [[1] Chapter 3]. The dynamic programming solution based on a quadratic index and a formulation of a model without load variables has been used by Kalman and Koepcke [3] for state driving and Anderson [4] and Nicholson [5] for regulatory control. None of the authors discussed the implications of load variables.

The commonly used quadratic criterion involves weighted terms in state variables and control variables and has been extended to include forms of time weighting. Despite its wide application little has been done to enable a priori determination of the weighting matrices.

The state weighting matrix has received the most attention. Nicholson [5] used a Liapunov function weighting matrix which guarantees stability but is not necessarily the "best" weighting. Chant and Luus [6] suggested procedures to obtain the weighting matrix for state driving by "brute force" optimization with a minimum time criterion. Tyler and Tuteur [7] related the system poles to the elements of a diagonal weighting matrix and obtained the required root loci by trial-and-error. Chen and Shen [8] improved the pole assignment approach with a sensitivity analysis which was developed into a computer algorithm. These approaches assume the designer knows where he wants

the system poles.

In continuous optimal control the control weighting matrix must be non-zero and positive definite to prevent infinite control action. However in the discrete case the control weighting matrix need not be non-zero or positive definite and has been examined in Lapidus and Luus [[1] page 190]. Hassan [9] estimated the elements of a diagonal weighting matrix in order to prevent control saturation by an iterative procedure. This would be time consuming for real-time regulatory control.

The standard quadratic criterion has been extended to include time weighting. Kalman and Koepcke [3] mentioned exponential time weighting but did not discuss its use. Man and Smith [10] introduced time weighting as integer powers of time, equivalent to ISE (power of zero), ITSE (power of one), and ISTSE (power of two).

At the present time the industrial implementation of optimal multivariable control has not received wide acceptance. Noton and Choquette [11,12] have reported on an application presently being studied. They claim to have found it necessary to update the process model during large transients, basically during state driving, and use different weighting matrices at different times during the transient. No further explanation was given.

3. CONTROL LAW DERIVATION

The common derivation of the optimal regulator problem [[1] page 155] was based on the model,

$$\underline{\dot{x}} = \underline{\underline{A}}_c \, \underline{x} + \underline{\underline{B}}_c \, \underline{u} \tag{1}$$

and criterion $J(\underline{X}, \underline{U})$, producing the control law,

$$\underline{u} = \underline{\underline{K}}_{FB} \, \underline{x} \tag{2}$$

which implied that the state of the process was optimally driven from its present value to the origin $\underline{x} = 0$.

If this control law was applied to a system subject to load changes,

$$\underline{\dot{x}} = \underline{\underline{A}}_c \, \underline{x} + \underline{\underline{B}}_c \, \underline{u} + \underline{\underline{D}}_c \, \underline{d} \tag{3}$$

then the closed loop system would be of the following form.

$$\underline{\dot{x}} = (\underline{\underline{A}}_c + \underline{\underline{B}}_c \, \underline{\underline{K}}_{FB})\underline{x} + \underline{\underline{D}}_c \, \underline{d} \tag{4}$$

This system would suffer from an offset $\underline{x}_o$ rather than being driven to the origin.

$$\underline{x}_o = -(\underline{\underline{A}}_c + \underline{\underline{B}}_c \, \underline{\underline{K}}_{FB})^{-1} \underline{\underline{D}}_c \, \underline{d} \tag{5}$$

Furthermore it can be shown that, with the change of variables $\underline{X} = \underline{X} - \underline{X}_o$ and $\underline{U} = \underline{U} - \underline{\underline{K}}_{FB} \, \underline{X}_o$, equation (3) is equivalent to equation (1), where $\underline{X}_o$ is defined by equation (5). The control law given by equation (2) still applies but the system is driven optimally, relative to the criterion $J\,(\underline{X} - \underline{X}_o, \underline{U} - \underline{\underline{K}}_{FB}\underline{X}_o)$, to its offset, $\underline{X}_o$, rather than to the origin.

It is therefore desirable to reduce the offset by such techniques as the feedforward action included in the extended derivation. Further work has also been carried out on feedforward control [13] and on integral control [14] with the aim of minimizing the offsets.

3.1. System Definition

Many physical processes can be adequately represented by a linear state equation and output equation in the following form.

$$\dot{\underline{x}} = \underline{\underline{A}}_c\, \underline{x} + \underline{\underline{B}}_c\, \underline{u} + \underline{\underline{D}}_c\, \underline{d} \tag{6}$$

$$\underline{y} = \underline{\underline{C}}\, \underline{x} \tag{7}$$

where $\underline{x}$ is the state vector of dimension n

$\underline{u}$ is the control vector of dimension m

$\underline{d}$ is the load vector of dimension p

$\underline{y}$ is the output vector of dimension q

$\underline{\underline{A}}_c$, $\underline{\underline{B}}_c$, $\underline{\underline{D}}_c$, $\underline{\underline{C}}$ are constant coefficient matrices of appropriate dimensions.

The differential state equation can be expressed in discrete form with the coefficient matrices evaluated by standard techniques [15].

$$\underline{x}_{n+1} = \underline{\underline{A}}\, \underline{x}_n + \underline{\underline{B}}\, \underline{u}_n + \underline{\underline{D}}\, \underline{d}_n \tag{8}$$

This expression is mathematically exact at the sampling instants if $\underline{u}$ and $\underline{d}$ are constant during the sampling intervals, Δt. With control implemented by process control computer the assumption is correct for the control vector $\underline{u}$. However the load vector $\underline{d}$ is not

normally constant during control intervals. This would result in an effective time delay of approximately $\Delta t/2$ [15] in $\underline{d}$ if it was measured from the process.

The model is assumed to be in perturbation form with steady state represented by $\underline{x} = \underline{u} = 0$. While this is not a necessary condition for the derivation it has been found to simplify the convergence of the resulting recursive relations [[1] page 163].

3.2. Control Criterion

The general summed quadratic criterion to be used in this derivation can be expressed as follows.

$$
\begin{aligned}
J = {} & \beta^N (\underline{x}_N - \underline{\underline{C}}^T \underline{y}_d)^T \underline{\underline{S}} (\underline{x}_N - \underline{\underline{C}}^T \underline{y}_d) \\
& + \sum_{k=1}^{N} \beta^k \left((\underline{x}_k - \underline{\underline{C}}^T \underline{y}_d)^T \underline{\underline{Q}} (\underline{x}_k - \underline{\underline{C}}^T \underline{y}_d) + \underline{u}_{k-1}^T \underline{\underline{R}}\, \underline{u}_{k-1} \right)
\end{aligned}
\tag{9}
$$

where $\underline{y}_d$ is the desired output or output setpoint vector,

β is the time weighting factor,

$\underline{\underline{Q}}$ is the state weighting matrix,

$\underline{\underline{R}}$ is the output weighting matrix,

$\underline{\underline{S}}$ is the final state weighting matrix.

It should be noted that the inclusion of the desired output as $\underline{\underline{C}}^T \underline{y}_d$ is only valid if the output $\underline{y}$ is a subset of the state $\underline{x}$. In the special case, where the dimensions of $\underline{x}$ and $\underline{y}_d$ are the same and $\underline{\underline{C}}$ is nonsingular, $\underline{\underline{C}}^{-1} \underline{y}_d$ would be used. The degrees of freedom of a control system as far as driving variables to set values [14] is the dimension of the control vector. As a result the dimension of $\underline{y}_d$ should be less than or equal to the dimension of the control vector $(q \leq m)$ for effective setpoint control.

3.3. Derivation

The derivation makes use of the dynamic programming concept of breaking the system into subsystems, in this case by time into control intervals. The relevant terms of the criterion can then be optimized with respect to the control vector at each interval (in each subsystem), beginning at the (N-1)th interval and working backwards. This approach results in a control law of the following form.

$$\underline{u} = \underline{\underline{K}}_{FB}\, \underline{x} + \underline{\underline{K}}_{FF}\, \underline{d} + \underline{\underline{K}}_{SP}\, \underline{y}_d \qquad (10)$$

The control matrix $\underline{\underline{K}}_{FB}$ in equation (2) converges to a constant and unique value [[1] p.160] as the number of control intervals increases and it has been found that the same is true for the control matrices in equation (10). The control matrices can be evaluated from independent recursive relations derived in Appendix A. Since regulatory control has no definite time origin the constant matrices at infinite time (numerically seldom more than 5n to 10n intervals) are used in the implementation.

The only assumptions made in the derivation are symmetrical weighting matrices and constant load $\underline{d}$ and setpoint $\underline{y}_d$ over the period $N\Delta t$ of the derivation. The latter assumption is necessary in view of the backwards derivation in time and since prediction of values is impractical. The assumption in no way restricts the implementation of the control law. A detailed examination of the feedforward and setpoint modes of control is the subject of further work in Chapters 6 and 8.

The performance of this optimal regulatory control design is evaluated by applying it to the pilot plant evaporator described in the

next section.

4. EVAPORATOR MODEL

The equipment (Figure 1) on which the work was evaluated is a pilot plant scaled double effect evaporator in the Department of Chemical and Petroleum Engineering at the University of Alberta.

The first effect is a natural circulation calandria type unit heated with a nominal 2 lb./min. of fresh steam and fed with a nominal 5 lb./min. of three percent triethylene glycol by weight. First effect vapour is used to heat the second effect, an externally forced-circulation long tube vertical unit, which concentrates first effect product to about ten percent. The second effect is kept under tight pressure control by a vacuum system and condenser.

The equipment can be controlled either by Foxboro electronic controllers or a Direct Digital Control (DDC) monitor system operating on an IBM 1800 digital process control computer [17].

A tenth-order nonlinear dynamic model of the evaporator was derived in Chapter 3. By neglecting fast dynamics in the vapour spaces and tube walls and assuming constant second effect pressure this model was reduced and then linearized to present the fifth-order state space model in Equation (11). The process variables are defined in the Nomenclature.

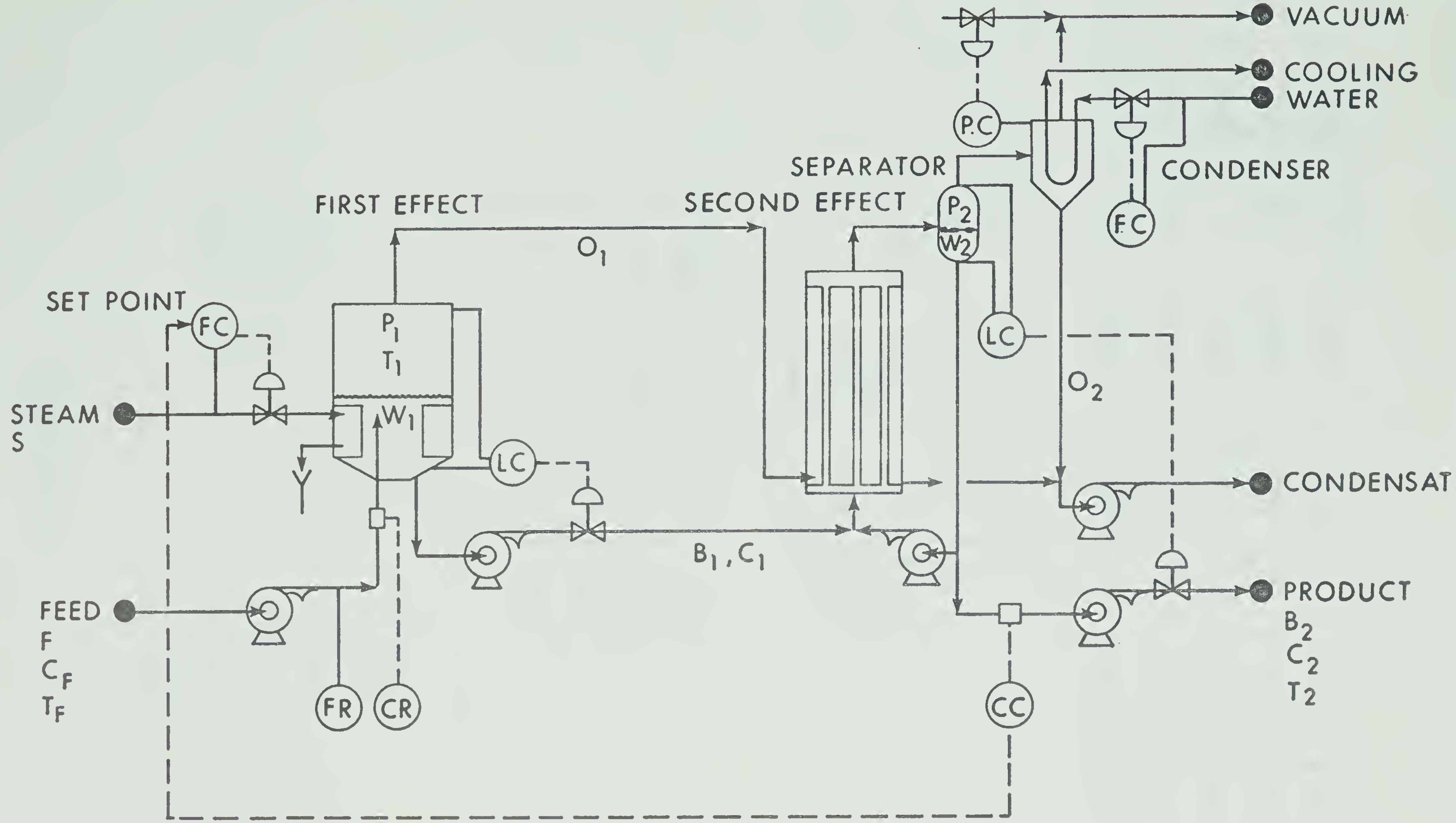


FIGURE 1. PILOT PLANT DOUBLE EFFECT EVAPORATOR

$$\begin{bmatrix} \dot{W1} \\ C1 \\ H1 \\ W2 \\ C2 \end{bmatrix} = \begin{bmatrix} 0 & -.00156 & -.1711 & 0 & 0 \\ 0 & -.1419 & .1711 & 0 & 0 \\ 0 & -.00875 & -1.102 & 0 & 0 \\ 0 & -.00128 & -.1489 & 0 & .00013 \\ 0 & .0605 & .1489 & 0 & -.0591 \end{bmatrix} \begin{bmatrix} W1 \\ C1 \\ H1 \\ W2 \\ C2 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & -.143 & 0 \\ 0 & 0 & 0 \\ .392 & 0 & 0 \\ 0 & .108 & -.0592 \\ 0 & -.0486 & 0 \end{bmatrix} \begin{bmatrix} S \\ B1 \\ B2 \end{bmatrix}$$

$$+ \begin{bmatrix} .2174 & 0 & 0 \\ -.074 & .1434 & 0 \\ -.036 & 0 & .1814 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F \\ CF \\ HF \end{bmatrix}$$

$$\begin{bmatrix} W1 \\ W2 \\ C2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} W1 \\ C1 \\ H1 \\ W2 \\ C2 \end{bmatrix}$$

Equation (11)

5. DESIGN PARAMETERS

There are a number of parameters which must be specified by the designer in order to obtain both the desired quality of control and practical implementation. These are the control interval Δt, the time weighting parameter β, and the weighting matrices $\underline{\underline{Q}}$, $\underline{\underline{R}}$, and $\underline{\underline{S}}$.

The effects of these parameters have been evaluated by simulation results using the evaporator model and a distillation column model derived by Rafal and Stevens [18]. The simulated evaporator results are presented in the text and those from the distillation column model (Chapter 10) support the conclusions drawn.

5.1. Control Interval

The effects of changing the control interval are intuitively reasonable. At very short intervals the system should approach the continuous case with the best control quality while with a long control interval the process has more time to react before the control action is applied and worse control would be expected. Simulated control runs with both models verified this prediction.

The gains in the feedback control matrix tend to increase with a decreasing control interval and the offsets in the output variables decrease as a result. This is illustrated in Table 1. At large control intervals (i.e. greater than 96 seconds for the evaporator) significant overshoots, as high as 40 percent, occur due to the time before control action can take effect. At very small intervals overshoot is also apparent, due to the high gains in the feedback control matrix.

TABLE 1

EFFECT OF CONTROL INTERVAL

Run Number	Δt (Seconds)	Product Concentration		
		Steam Gain*	Offset (% of Steady State)	$\overline{ISE}_N$**
11110	4	252	.25	1.005
11115	16	61	.26	1.004
11112	64	14.5	.29	.995
11113	96	9.6	.31	.983
11114	256	3.1	.56	1.010
11116	448	1.4	1.13	1.021

* Element k_{15} of feedback matrix $\underline{\underline{K}}_{FB}$.

** The criterion $\overline{ISE}_N$ is the sum of squared errors divided by N multiplied by the offset squared. Since the rise times of the multivariable responses with overshoot were generally fast a value of $\overline{ISE}_N$ greater than unity indicates overshoot and less than unity an overdamped rise to the offset. The criterion is not satisfactory when the offset approaches zero.

In choosing a control interval the following points should be considered.

(a) Small intervals and the subsequent high gains combine with process noise (Figure 4) and available significant figures in the measurements to cause undesirable oscillations (Figure 2b).

(b) Real time computer usage increases when small control intervals are implemented.

(c) Large control intervals suffer from large offsets and overshoot.

The effects have been confirmed by experimental results in Figures 4 to 7.

5.2. Time Weighting Parameter

The weighting parameter β in the control criterion is equivalent to exponentially weighting deviations from the desired state with time. That is, deviations are weighted more heavily as the time since the disturbance increases.

Simulated control runs with β equal to 1.5 and 5 are compared to no time weighting ($\beta = 1$) in Table 2. Increasing β tended to decrease offsets because some elements of the control matrix increased. The faster rise times can be explained by the increases in gain elements which were generally restricted to those corresponding to first effect variables W1 and C1 which would result in faster reaction by the control variables.

TABLE 2

EXPONENTIAL TIME WEIGHTING

Run	Weighting β	W1			W2			C2		
		Gain	Offset	Rise-Time	Gain	Offset	Rise-Time	Gain	Offset	Rise-Time
11610	1.0	3.95	3.13	3.90	15.8	.02	.90	-14.5	-.29	3.30
11611	1.5	4.10	3.01	2.40	15.8	.02	.90	-16.2	-.26	2.30
11629	5.0	13.2	2.08	.95	15.8	.02	.90	-69.7	-.17	1.35

However the improvements in response have to be traded off against increased oscillations due to higher gains acting on process noise.

5.3. State Weighting

As indicated in the Literature Survey little work has been done towards obtaining an *a priori* estimate of $\underline{\underline{Q}}$, probably the most important parameter in the commonly used quadratic criterion. A Liapunov function can be used as the criterion or as Anderson [4] recommends the states can be weighted so that each makes an equal contribution to the criterion. It would appear desirable however to choose the weighting parameters so that the states which it is desired to control closely contribute most. The only conditions the derivation places upon $\underline{\underline{Q}}$ are symmetry and positive definiteness.

In an attempt to find some general effects a diagonal $\underline{\underline{Q}}$ has been specified and a number of simulation runs made using both the evaporator and distillation models. Some of the trends exhibited by both models are discussed here relative to the evaporator. The overall control policy of the evaporator is close control on product concentration C2 and enough control on the levels W1 and W2 to keep them within reasonable limits. The remaining states C1 and H1 are "interior" and their values are not important.

First consideration was given to the weighting of product concentration C2 . Table 3 shows three series of simulations in which the C2 weighting was varied while the others remained constant. The first two series show reasonably predictable trends with

decreasing offsets in C2 and increasing offsets in levels (see also Figure 2). Note that W2 varies little since it is principally controlled by B2 which does not interact with any of the other states. It can be seen that where levels are more heavily weighted in the second series there is a relatively smaller increase in their offsets as the C2 weighting increases. However the offset in C2 is higher than for the previous series with the same weighting on C2. Other points of interest include the trend from "overdamped" responses to overshoot as C2 is weighted more heavily and the gains increase. Also there is a relatively marginal improvement in control when the weighting parameter is increased from 10^4 to 10^6 suggesting that the useful range of weights may be limited. The larger difference in magnitude between weights also required more computational time to evaluate the control matrices. Experimental results in Figures 8 and 9 confirm these observations.

The third series in Table 3 where the weighting is removed from the states H1 and C1 did not show any significant change in gains, offsets, or criteria irrespective of the weighting on C2. Since the control variable S (Steam) acts directly through H1 (note the elements of the $\underline{\underline{B}}$ matrix in equation (11)) this case could be considered as a removal of an indirect weighting on a control variable. This is also supported by a sharp increase in magnitude of the elements of the feedback control matrix. It is reasonable that the "optimum" optimal control should occur with no direct or indirect weighting on the control variables, allowing an approach to "infinite" control action.

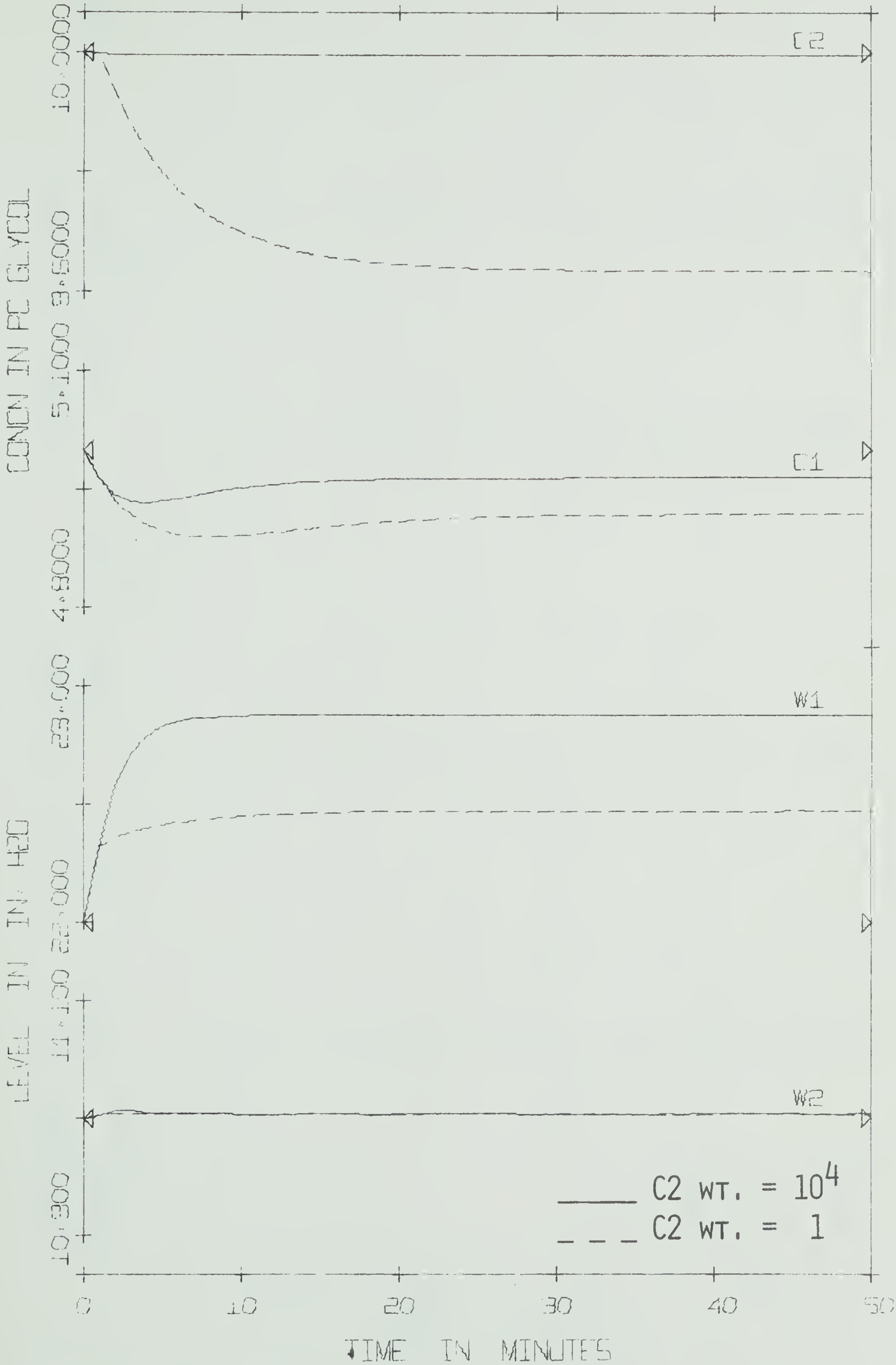


FIGURE 2a. SIMULATED EFFECT OF WEIGHTING C2
(5L/+10%F/FB/Q2, Q3/R1/D1)

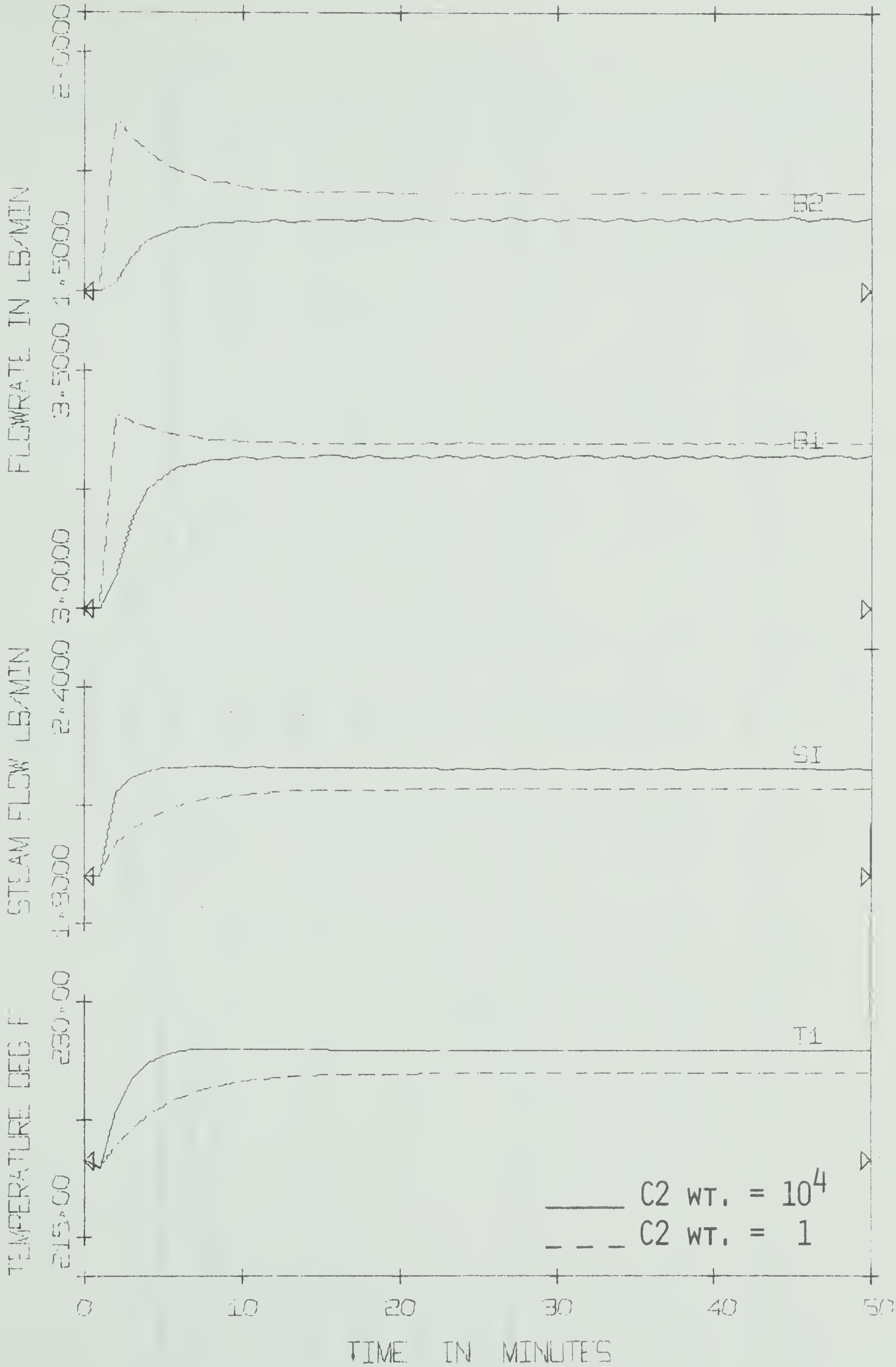


FIGURE 2b. SIMULATED EFFECT OF WEIGHTING C2
(5L/+10%F/FB/Q2, Q3/R1/D1)

TABLE 3

WEIGHTING C2

*Responses plotted in Figure 2.

Run	State Weighting W1	C1	H1	W2	C2	W1 Gain	W1 Offset	W1 ISE_N	W2 Gain	W2 Offset	W2 ISE_N	C2 Gain	C2 Offset	C2 ISE_N
11517*	1	1	1	1	1	5.92	3.29	.984	15.8	.02	1.505	- 2.44	-3.66	.955
11518	1	1	1	1	10	3.61	4.96	.986	15.8	.02	1.493	- 6.23	- .92	.981
11511	1	1	1	1	100	1.75	5.92	.986	15.8	.02	1.515	- 8.16	- .16	1.056
11513*	1	1	1	1	10000	1.38	6.09	.987	15.8	.02	1.471	- 8.49	- .05	1.219
11521	10	1	1	10	10	5.62	2.69	.998	15.8	.02	1.475	- 7.89	-1.18	.989
11510	10	1	1	10	100	3.95	3.13	.998	15.8	.02	1.464	-14.5	- .29	1.022
11514	10	1	1	10	10000	2.85	3.38	.997	15.8	.02	1.520	-17.4	- .05	1.209
11516	10	1	1	10	10^6	2.84	3.39	.999	15.8	.02	1.535	-17.5	-.045	1.201
11522	1	0	0	1	1	4.75	2.34	1.001	15.8	.02	1.650	-29.2	-.045	1.255
11512	1	0	0	1	100	4.75	2.34	1.001	15.8	.02	1.657	-29.2	-.045	1.253
11523	1	0	0	1	10000	4.75	2.34	1.003	15.8	.02	1.689	-29.2	-.045	1.273

Changing the weights on liquid levels with a constant weight on concentration C2 exhibits a similiar trend, reducing offsets on levels and increasing them on concentration (see Table 4 - first series, and experimental results in Figures 10 and 11). The second series in Table 4 shows that without C1 and H1 weighted the level weighting also has no significant effects.

A numerical problem arose when the weighting was removed entirely from W2. Because the control variable B2 acts directly and indirectly on only one state, W2, the $\underline{\underline{B}}^T \underline{\underline{P}}\, \underline{\underline{B}}$ matrix in the recursive relations (Appendix A) became singular. There is also a physical anomaly to not weighting W2. If W2 is of no consequence and since B2 affects only W2 then B2 is of no consequence and should therefore be removed from the control vector.

A Liapunov quadratic weighting matrix was calculated [5] and used to weight the states. Table 6 shows the matrix and a comparison of the responses of the system designed from the Liapunov weighting and the chosen standard case. It is most interesting to note that although offsets were generally larger the normalized criteria were all less than unity. The Liapunov weighting was the only one used which removed the overshoot from the W2 response. The negative elements in $\underline{\underline{Q}}$ indicate a weighting in favour of certain interactions. These would be difficult to choose intuitively.

The observed effects which may be considered of general application in selecting a $\underline{\underline{Q}}$ diagonal may be summarized as follows.

TABLE 4

WEIGHTING W1 AND W2

Run	State Weighting					W1			W2			C2		
	W1	C1	H1	W2	C2	Gain	Offset†	ISE_N	Gain	Offset	ISE_N	Gain	Offset	ISE_N
11511	1	1	1	1	100	1.75	5.92	.986	15.8	.02	1.515	- 8.16	-.16	1.056
11510	10	1	1	10	100	3.95	3.13	.998	15.8	.02	1.464	-14.5	-.29	1.022
11520	100	1	1	100	100	5.22	2.45	1.001	15.8	.02	1.545	-17.2	-.36	1.014
11515*	.01	0	0	.01	100	4.75	2.35	1.002	15.8	.02	1.712	-29.2	-.045	1.271
11512	1	0	0	1	100	4.75	2.34	1.001	15.8	.02	1.657	-29.2	-.045	1.253
11810	10	0	0	10	100	4.75	2.34	1.001	15.8	.02	1.657	-29.2	-.045	1.253

*same as 11523 if $\underline{\underline{R}} = \underline{\underline{S}} = \underline{\underline{0}}$

†percentage of steady state

TABLE 5

WEIGHTING C1 AND H1

Run	State Weighting					W1			W2			C2		
	W1	C1	H1	W2	C2	Gain	Offset	ISE_N	Gain	Offset	ISE_N	Gain	Offset	ISE_N
11510	10	1	1	10	100	3.95	3.13	.998	15.8	.02	1.464	-14.5	-.29	1.022
11810	10	0	0	10	100	4.75	2.34	1.001	15.8	.02	1.657	-29.2	-.045	1.253

TABLE 6

LIAPUNOV STATE WEIGHTING

Run Number	State Weighting	W1			W2			C2		
		Gain	Offset	ISE_N	Gain	Offset	ISE_N	Gain	Offset	ISE_N
11510	10 1 1 10 100	3.95	3.13	.998	15.8	.02	1.464	-14.5	-.29	1.022
11628	Liapunov*	5.52	2.93	.998	15.8	.53	.999	- 6.8	-1.13	.977

$$^{*}\underline{\underline{Q}} = \begin{bmatrix} 5000 & -7 & -777 & 0 & 0 \\ -7 & 5 & 1 & 10 & 3 \\ -777 & 1 & 1030 & -6740 & -13 \\ 0 & 10 & -6740 & 50000 & 107 \\ 0 & 3 & -13 & 107 & 9 \end{bmatrix}$$

(a) If a control variable acts directly and indirectly on only one state then that state must be weighted.

(b) If a control variable acts principally through one state then weighting that state indirectly weights the control variable.

(c) Increasing the weighting on a state generally results in improved control for that state at the expense of the control of the rest.

(d) The useful range of weighting factors appears to be limited. If a diagonal term is relatively large it is the dominant term and increasing its relative size does not change the situation significantly.

(e) It appears from the models studied that the relative sizes of state weighting factors have little or no influence when the control variables are not weighted directly or indirectly.

(f) The weighting on "interior" states (those with zero columns in $\underline{\underline{C}}$ in Equation (7)) can be lighter especially in the cases where they are effectively constrained by a weighting on a related state e.g. C1 is effectively constrained by weighting C2 .

5.4. Control Weighting

Weighting control variables generally reduces the amount of control action (i.e. decreases the elements in the gain matrix) and effects one or more states depending on the process interactions. The responses of the states are generally more damped (less overshoot or slower) and have larger offsets.

Table 7 summarizes the effects of varying the control variable weighting on the evaporator. The first series show the trends mentioned with larger offsets and smaller values of the normalized criterion in all cases of control weighting. This has been confirmed by the experimental results in Figures 5 and 12. The second series, plotted in Figure 3, shows the gradation of the effect as control weighting on steam, S, is increased. The "saw-tooth" pattern evident in some of the curves is the result of the 64 second sampling interval.

5.5. Final State Weighting

The final state is the origin when loads on the system do not persist and it has been shown that when a load change does persist the optimal control actually drives the process to the offset so that as far as the control is concerned the final state is the "origin". For this reason the final state term does not contribute to the criterion in the infinite time case and hence the weighting matrix $\underline{\underline{S}}$ has no effect on the control matrices.

Simulation showed this effect with no changes in the control matrices even when the $\underline{\underline{S}}$ matrix was 10^6 times the $\underline{\underline{Q}}$ matrix. Control matrices derived using a finite number of time stages in the dynamic programming analysis do differ, often significantly, with and without final state weighting.

6. IMPLEMENTATION

The multivariable control scheme was implemented using an IBM 1800 digital control computer which is interfaced with the pilot plant evaporator. The process runs under Direct Digital Control (DDC)

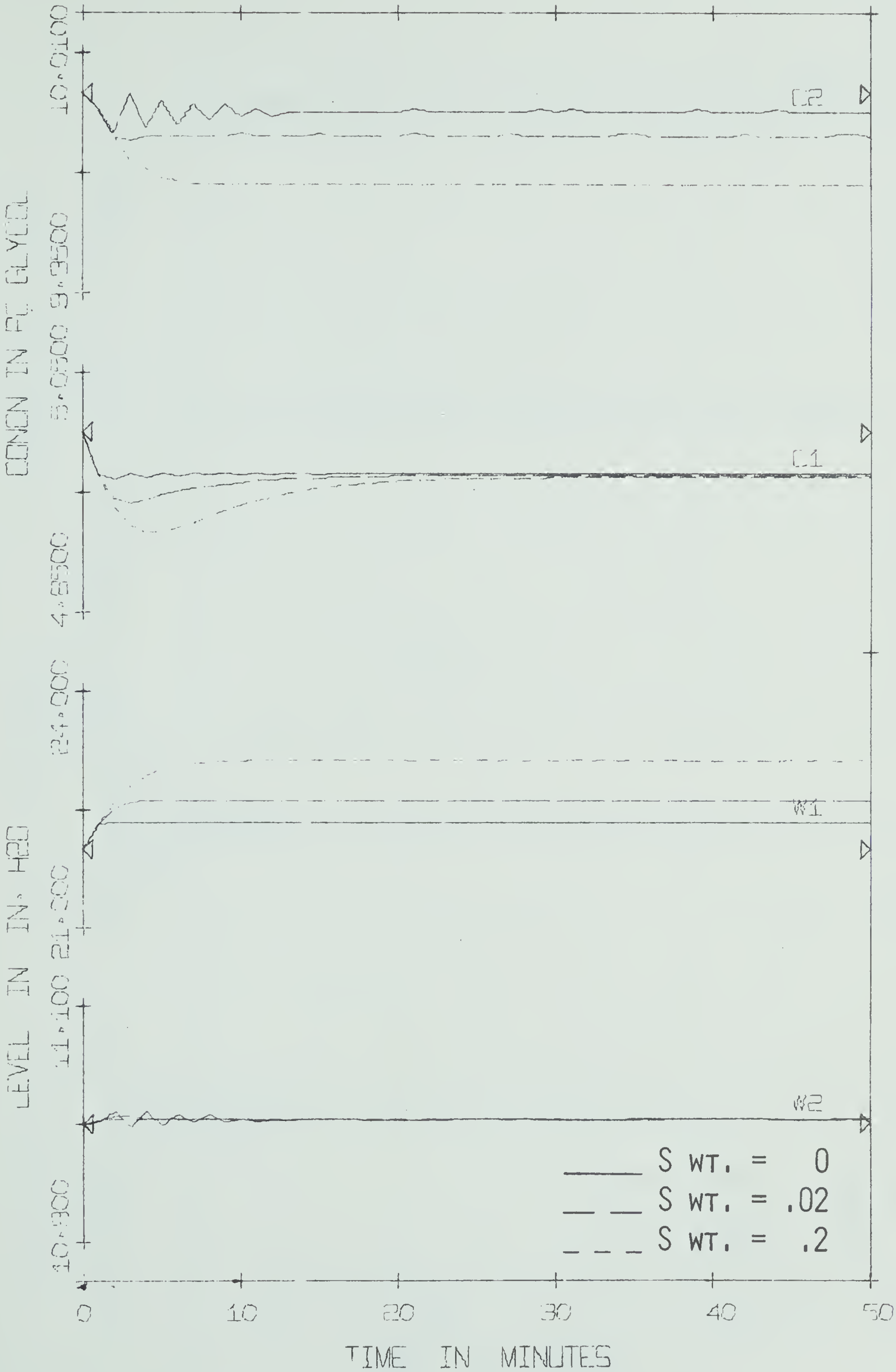


FIGURE 3a. SIMULATED EFFECT OF CONTROL WEIGHTING
(5L/+10%F/FB/Q1/R1, R3, R4/D1)

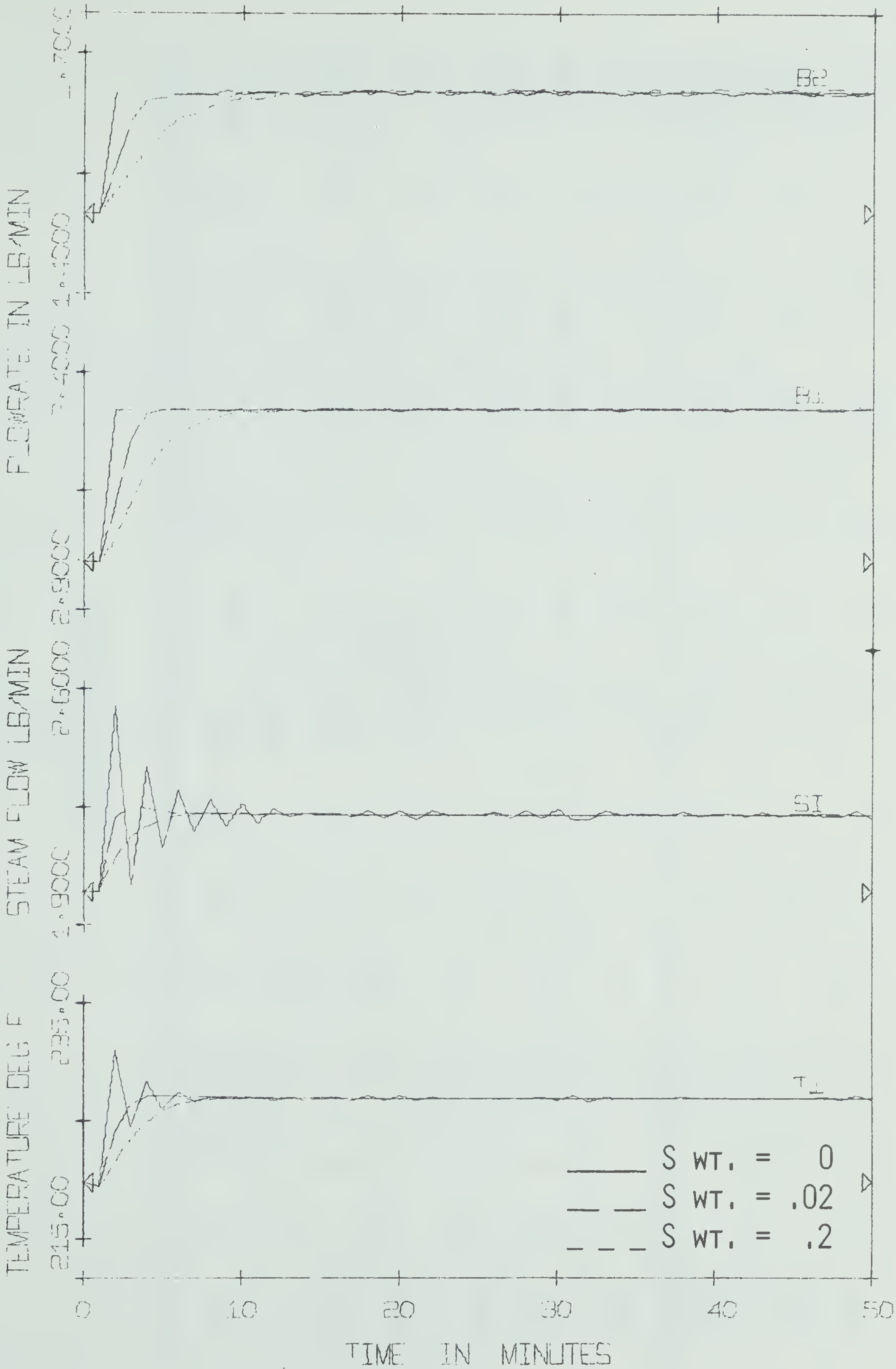


FIGURE 3b. SIMULATED EFFECT OF CONTROL WEIGHTING
(5L/+10%F/FB/Q1/R1, R3, R4/D1)

TABLE 7

CONTROL WEIGHTING

Run	Weighting					W1			W2			C2		
	W1	C1 S	H1 B1	W2 B2	C2	Gain	Offset	ISE_N	Gain	Offset	ISE_N	Gain	Offset	ISE_N
11810	10	0 0	0 0	10 0	100	4.75	2.34	1.001	15.8	.02	1.657	-29.2	-.045	1.255
11811	10	0 1	0 0	10 0	100	3.52	5.16	.988	15.8	.02	1.442	-4.89	-.97	.988
11819	10	0 0	0 1	10 0	100	2.19	4.69	.989	15.8	.02	1.560	-32.8	+.23	1.023
11820	10	0 0	0 0	10 1	100	4.25	3.23	.982	2.47	2.50	.947	-28.3	+.34	.969
11812	10	0 1	0 1	10 1	100	1.73	7.52	.977	2.56	2.24	.941	- 5.07	-.33	1.085
11512	1	0 0	0 0	1 0	100	4.75	2.34	1.001	15.8	.02	1.657	-29.2	-.045	1.253
11617	1	0 .02	0 0	1 0	100	2.18	4.22	.995	15.8	.02	1.446	-11.3	-.10	1.084
11614	1	0 .2	0 0	1 0	100	1.29	7.81	.983	15.8	.02	1.476	- 4.83	-.23	1.019

using a time-sharing executive system which permits simultaneous execution of off-line jobs such as the plotting of the figures for this paper.

The multivariable control algorithm was programmed in FORTRAN and executed each control interval as a process coreload. Design factors and principles are discussed in some detail in Chapter 9 . System time for the coreload varied from two to five seconds in every control interval, usually 64 seconds. This time was mainly concerned with disk input and output. A core resident algorithm would take about five times the execution time of the equivalent DDC configuration (15 multiplications compared to 3). The program obtained state variable measurements from DDC data acquisition loops and made control variable changes through the setpoints of DDC flow control loops.

There were two basic problems to be faced when working with real processes. These were noisy measurement signals and state variables that were not measured.

Measurements were conditioned by standard digital exponential filters within the DDC data acquisition loops. However, filtering was light so as not to introduce undue phase lags.

The linear process model was used to predict the state of the process and was supplied with measured values of the load and control variables and the last estimate of the state.

$$\underline{x}^{n}_{calc} = \underline{\underline{A}}\,\underline{x}^{n-1}_{est} + \underline{\underline{B}}\,\underline{u}^{n-1}_{meas} + \underline{\underline{D}}\,\underline{d}^{n-1}_{meas} \quad (12)$$

This predicted state and the available measured states were "exponentially combined" [13] to provide the state estimate both for the control algorithm and the next model calculation.

$$\underline{x}^n_{est} = \underline{\underline{\alpha}}\, \underline{x}^n_{meas} + (\underline{\underline{I}} - \underline{\underline{\alpha}})\underline{x}^n_{calc} \qquad (13)$$

$\underline{\underline{\alpha}}$ is a diagonal matrix of "filter constants" which can vary between zero, indicating a calculated value of the state, and unity, indicating a measured value. While the control law is optimal with respect to the model and criterion, this state estimation procedure results in an overall suboptimal control system. Work is also under way on using a Kalman optimal filter to predict the values of the state variables. A Kalman filter would give an optimal control system subject to the additional conditions of the Separation Theorem.

At first the model gave inaccurate estimates of the liquid levels since the integrating nature of these two states accumulated the measurement noise and calibration errors in the feed and bottoms flowrates. When the bottoms flowrates were eliminated from the model using two rows of the appropriate control matrices much improved model estimates were obtained. Model predictions were good since the multivariable control kept the model close to its point of linearization.

7. EXPERIMENTAL RESULTS

A number of experimental runs have been performed on the equipment to illustrate the effects of the control parameters on the response of the evaporator to load changes. All responses were to 20 percent steps up and down (at times indicated in the diagrams) in feed

flowrate F which open loop studies showed to be the most severe of all input disturbances. This approaches the limit of validity of the linearized model.

Evaporator responses with control at intervals of 16, 64, 256, and 448 seconds are shown in Figures 4 to 7. The increasing offset with increasing control interval is particularly noticeable in first effect level W1 and to a lesser extent in the product concentration C2. The effect of high control gains and process noise was evident in Figures 4a and 4b, particularly in the control variables (Figure 4b) while the feed flowrate, F, was noisy. The "sawtooth pattern" in T1 on all high gain runs is due to venting vapour to control P1.

The effects of the state weighting matrix $\underline{\underline{Q}}$ are illustrated in Figures 8 to 12. Figures 8 and 9 show the evaporator responses to feed flowrate changes with increasing weighting on product concentration. The improved control of C2 and larger offsets in levels, particularly W1, can be seen. Increasing the weighting on the levels reduces the level offsets and increases the offset in product concentration. These effects, particularly the first, are shown in Figures 10 and 11. The effects of increased gains resulting from increased weighting and process noise are evident in Figure 11.

Direct weighting of the control variables showed a general deterioration of control quality as might be predicted. This is shown in Figure 12 which is comparable to Figure 5 which has no direct control weighting.

The control configuration illustrated in Figure 5 with a control interval of 64 seconds and state weighting

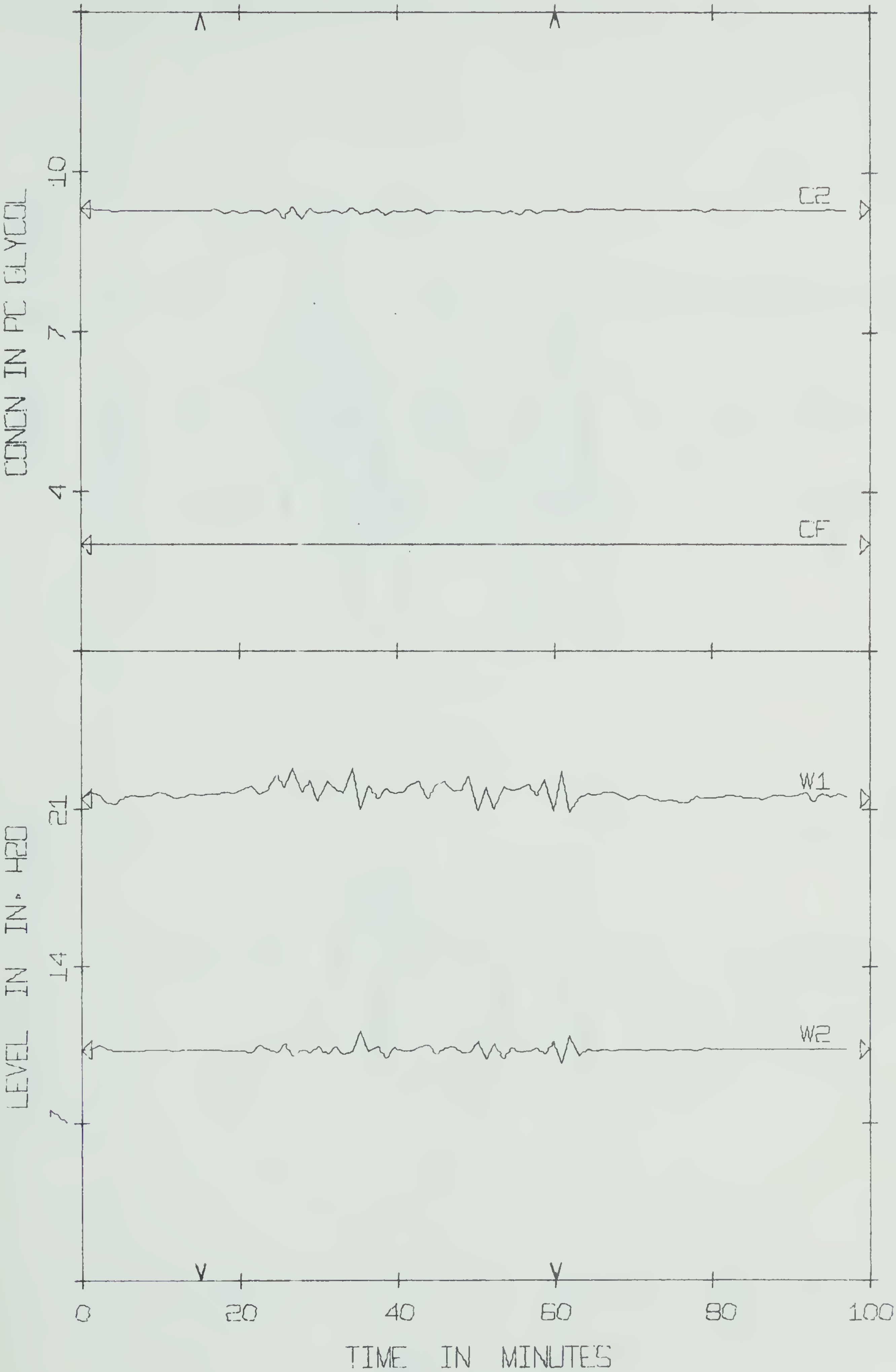


FIGURE 4a. EXPERIMENTAL RESPONSE WITH 16 SEC. CONTROL
(EXP/20%F/FB/Q1/R1/D2/A1/MVC12)

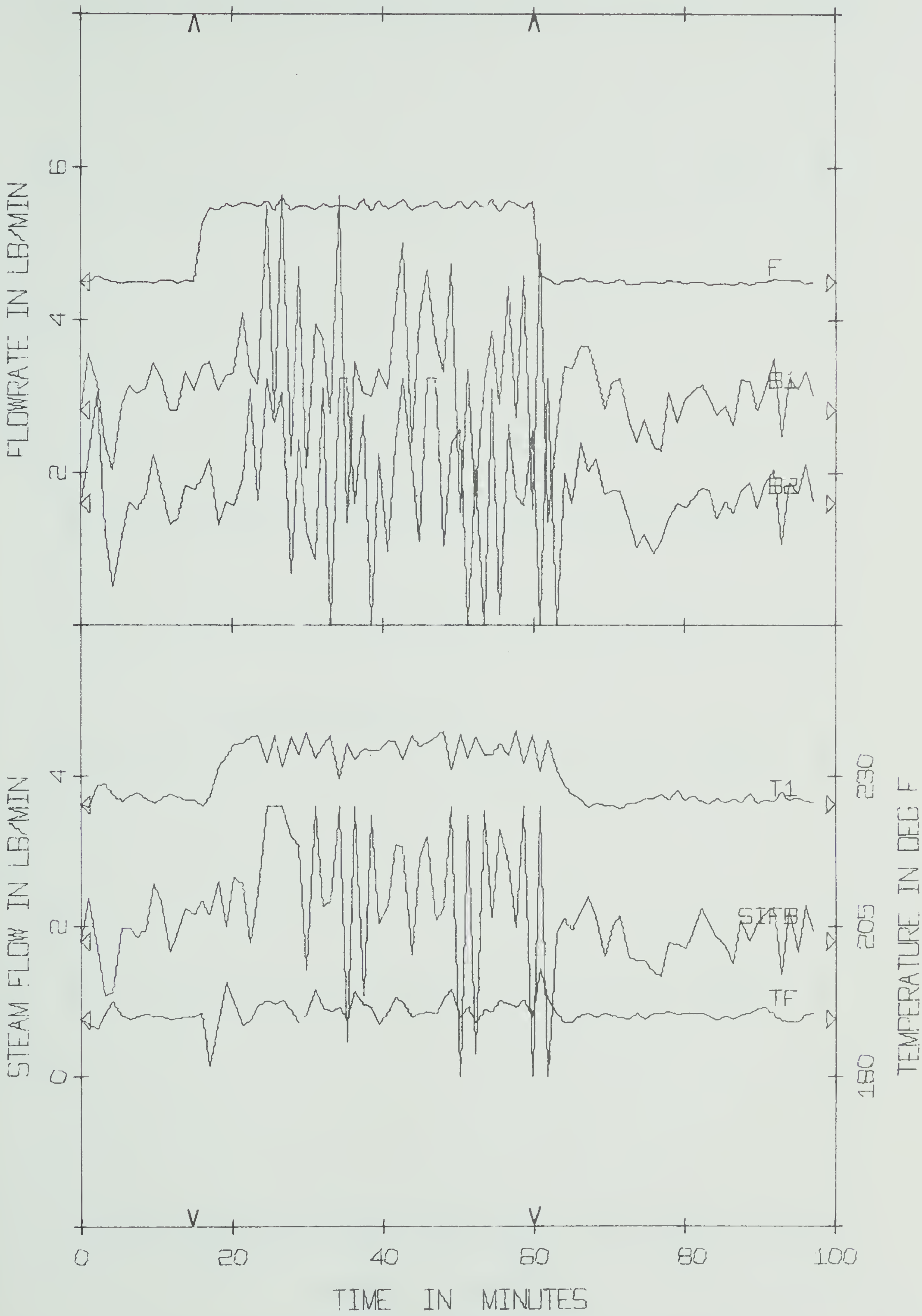


FIGURE 4b. EXPERIMENTAL RESPONSE WITH 16 SEC. CONTROL (EXP/20%F/FB/Q1/R1/D2/A1/MVC12)

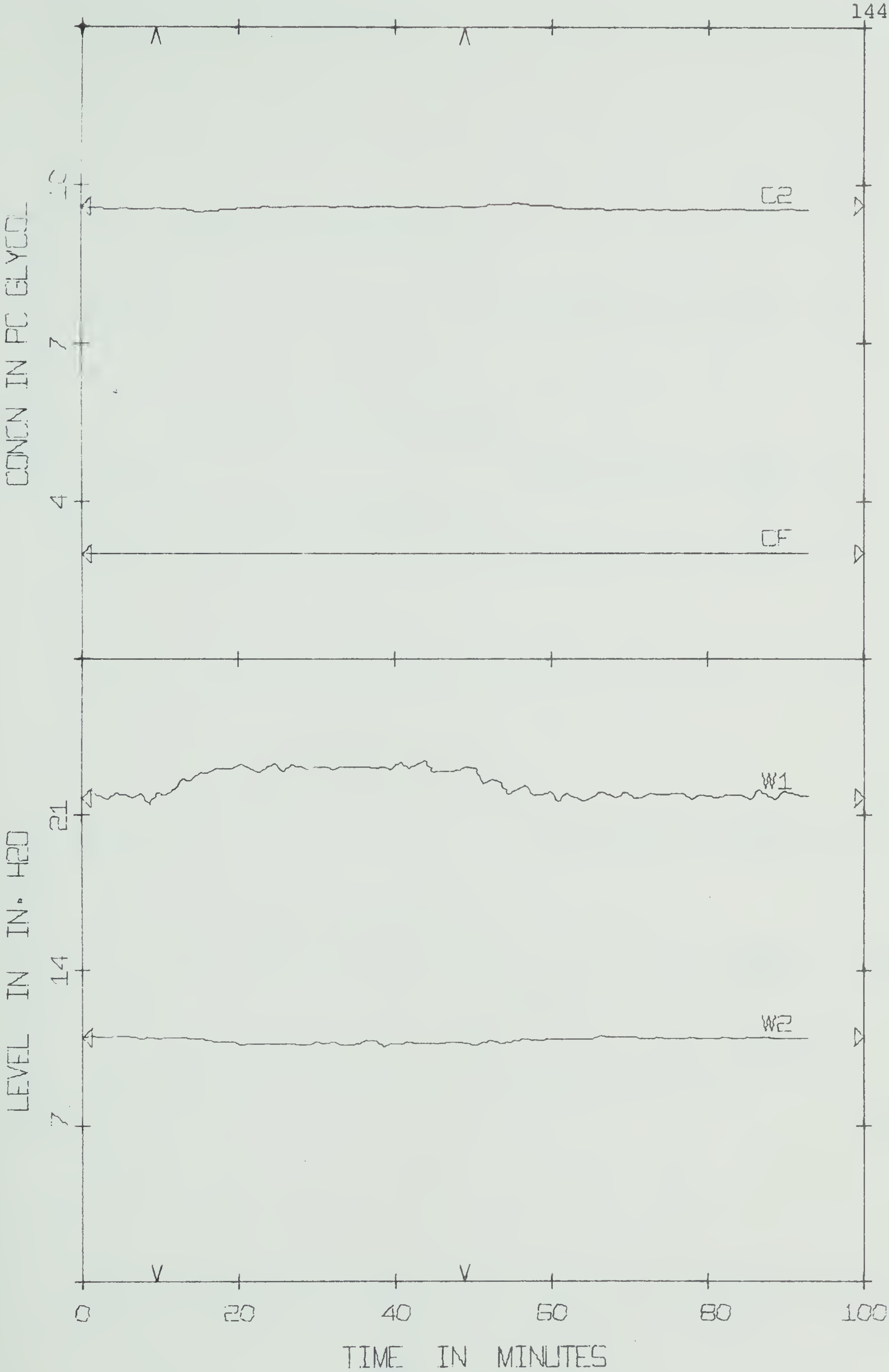


FIGURE 5a. EXPERIMENTAL RESPONSE WITH 64 SEC. CONTROL
(EXP/20%F/FB/Q1/R1/D1/A1/MVC41)

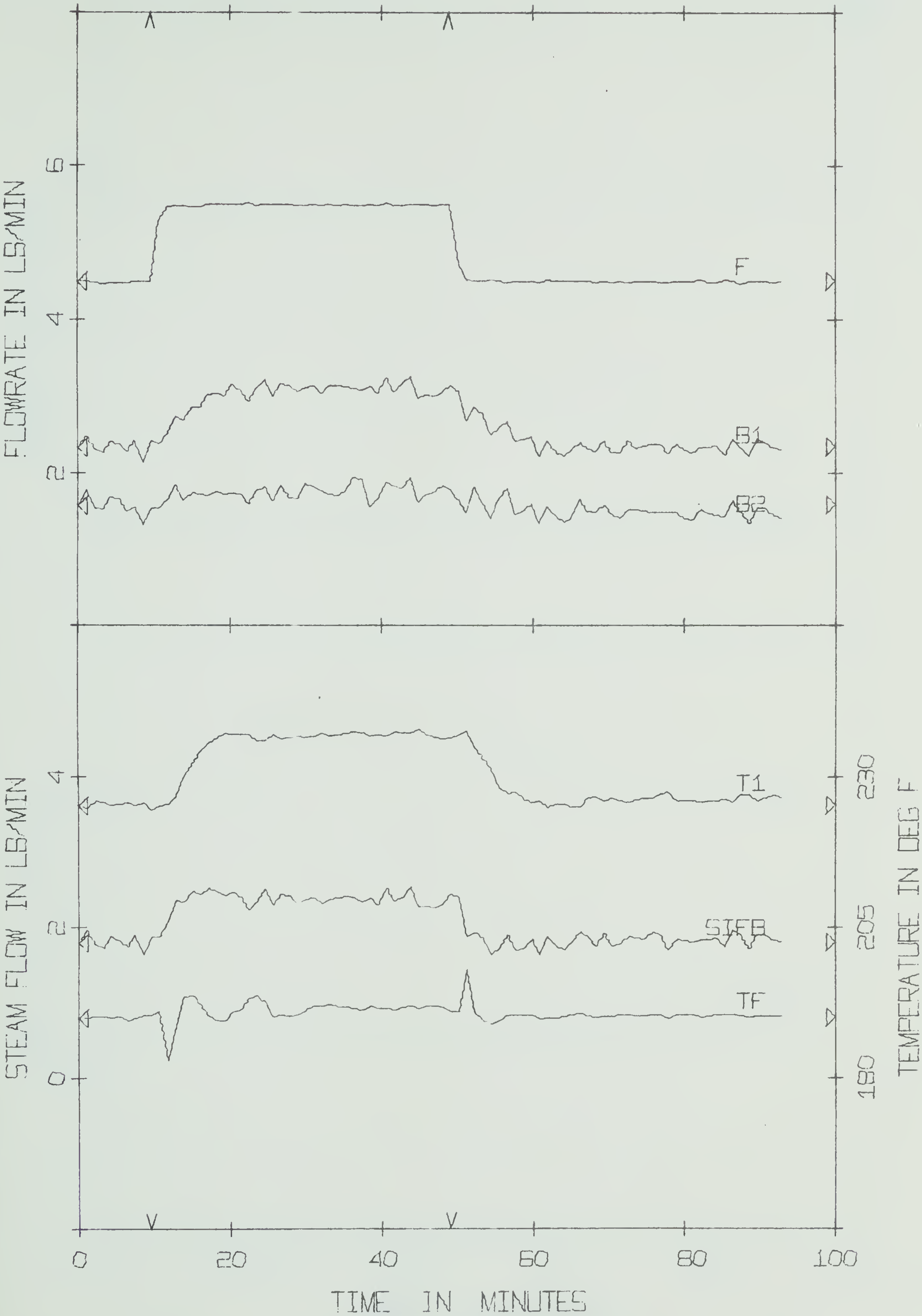


FIGURE 5b. EXPERIMENTAL RESPONSE WITH 64 SEC. CONTROL (EXP/20%F/FB/Q1/R1/D1/A1/MVC41)

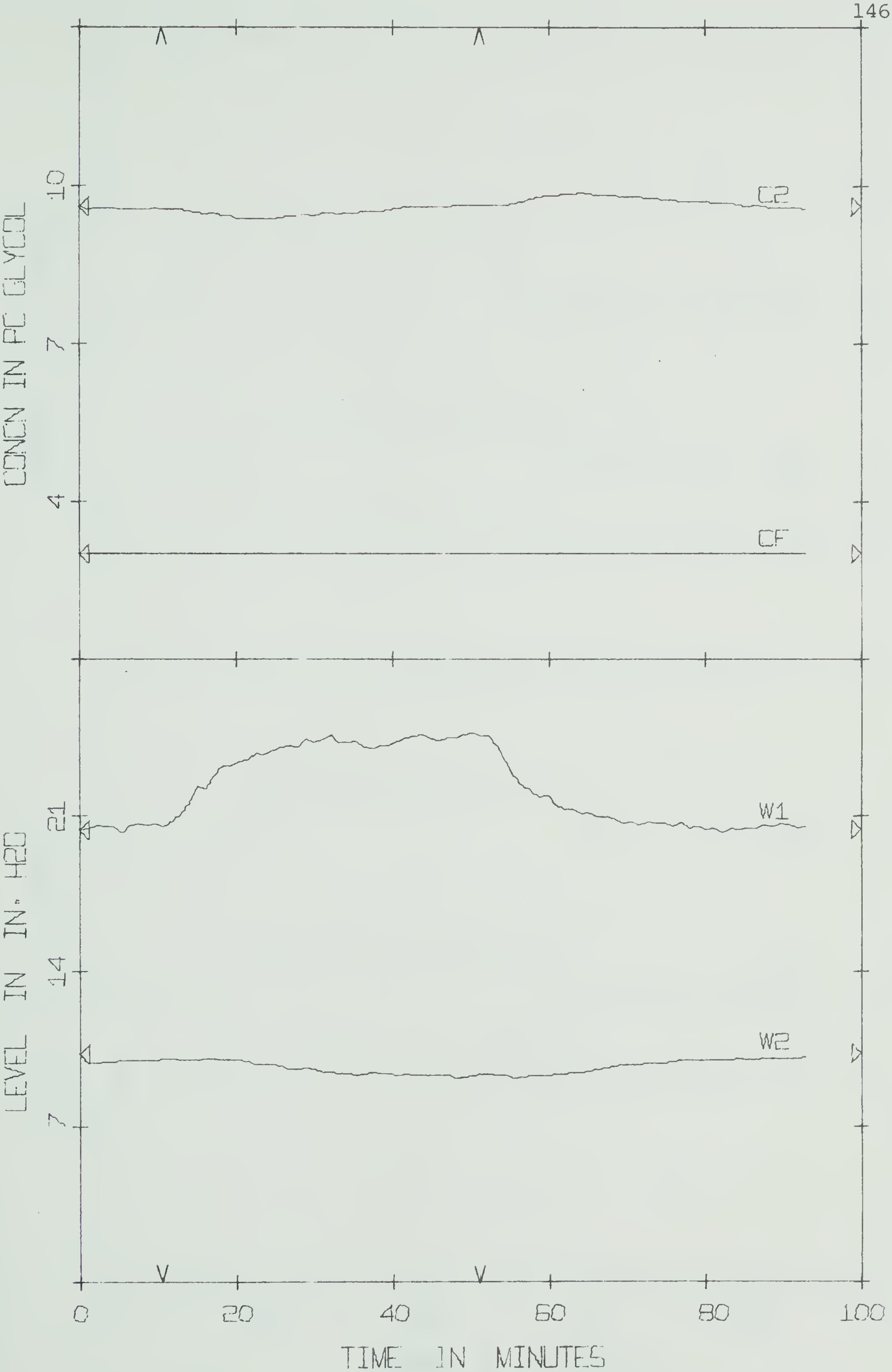


FIGURE 6a. EXPERIMENTAL RESPONSE WITH 256 SEC. CONTROL (EXP/20%F/FB/Q1/R1/D4/A1/MVC42)

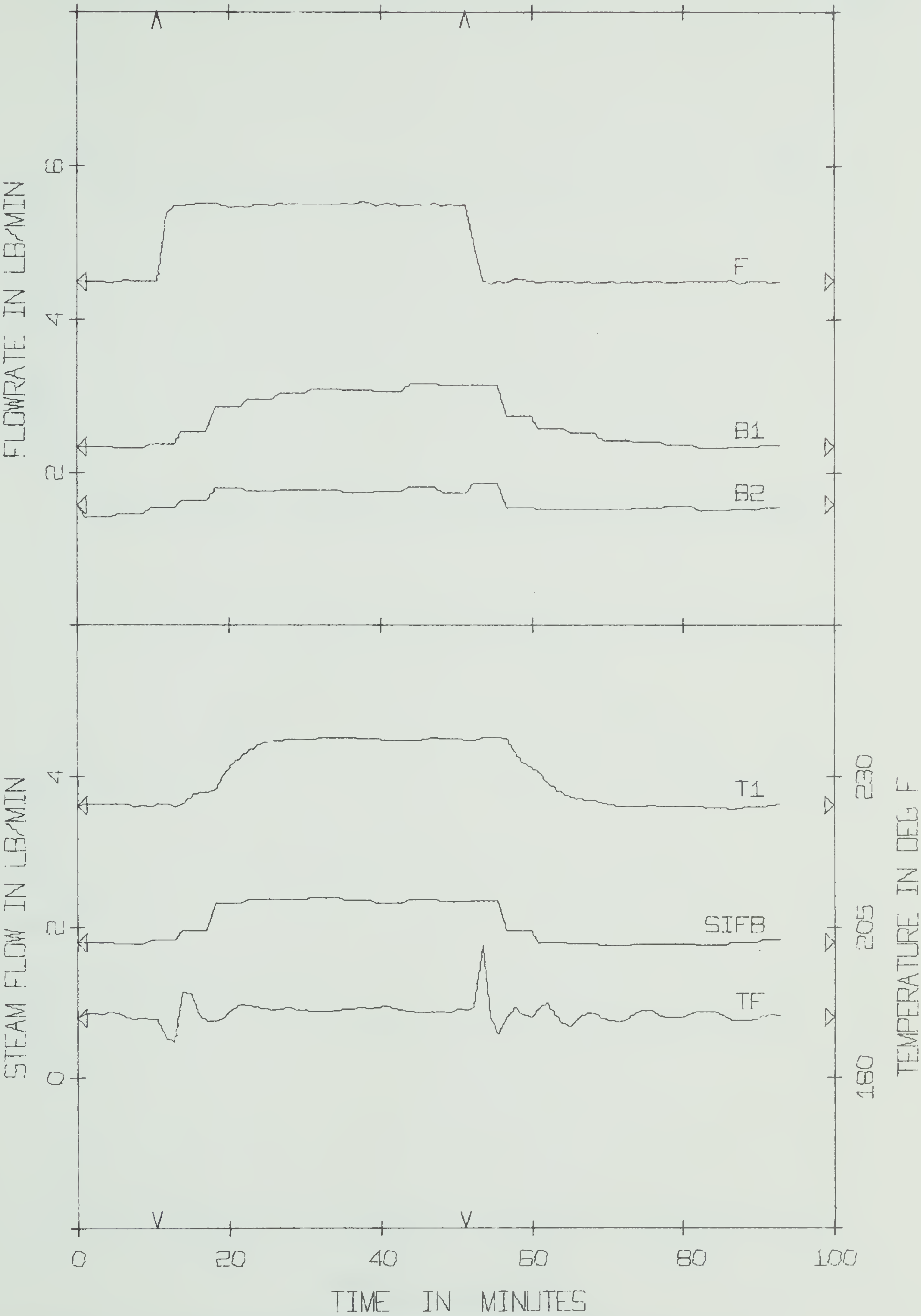


FIGURE 6b. EXPERIMENTAL RESPONSE WITH 256 SEC. CONTROL (EXP/20%F/FB/Q1/R1/D4/A1/MVC42)

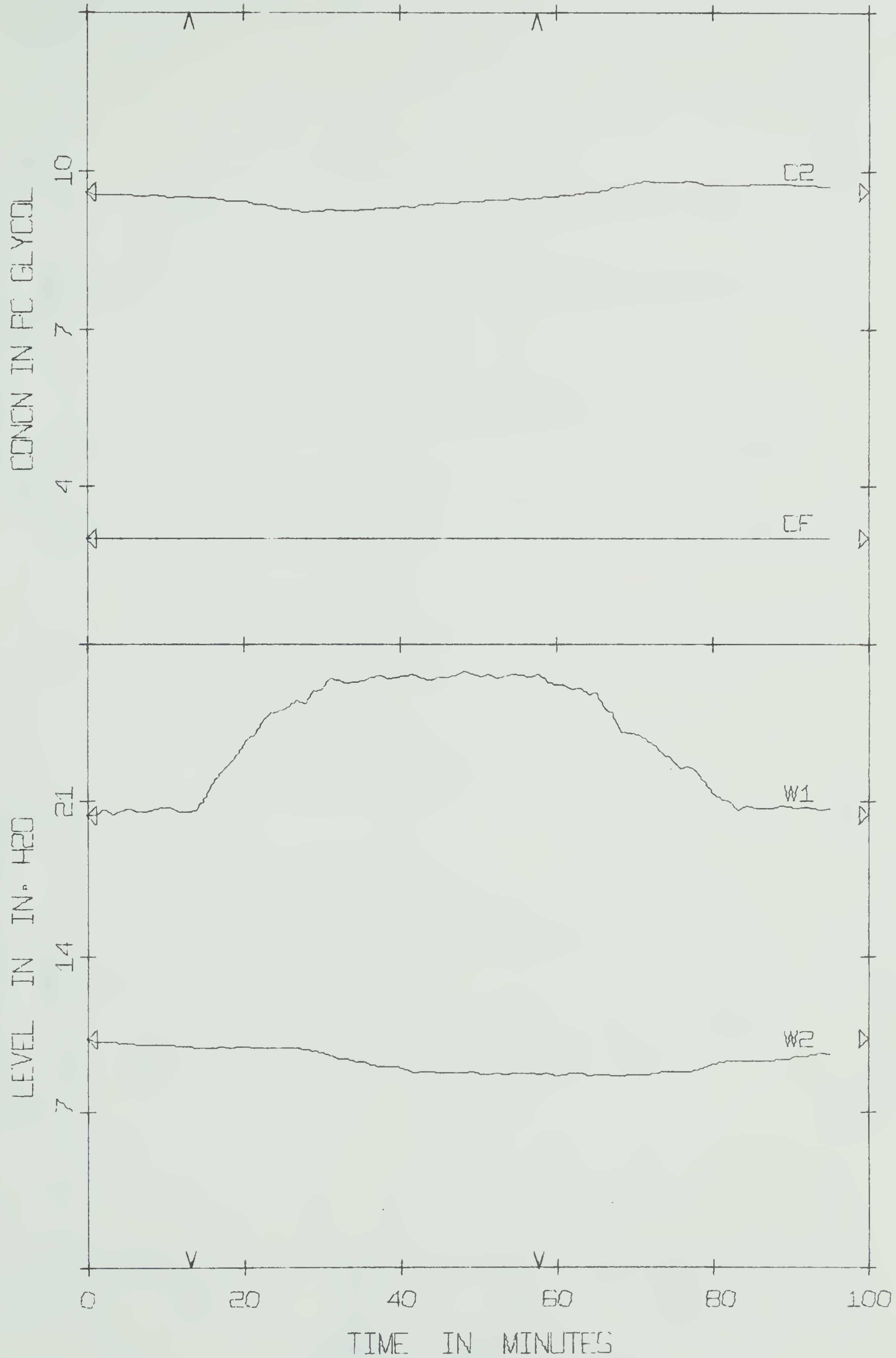


FIGURE 7a. EXPERIMENTAL RESPONSE WITH 448 SEC. CONTROL (EXP/20%F/FB/Q1/R1/D3/A1/MVC15)

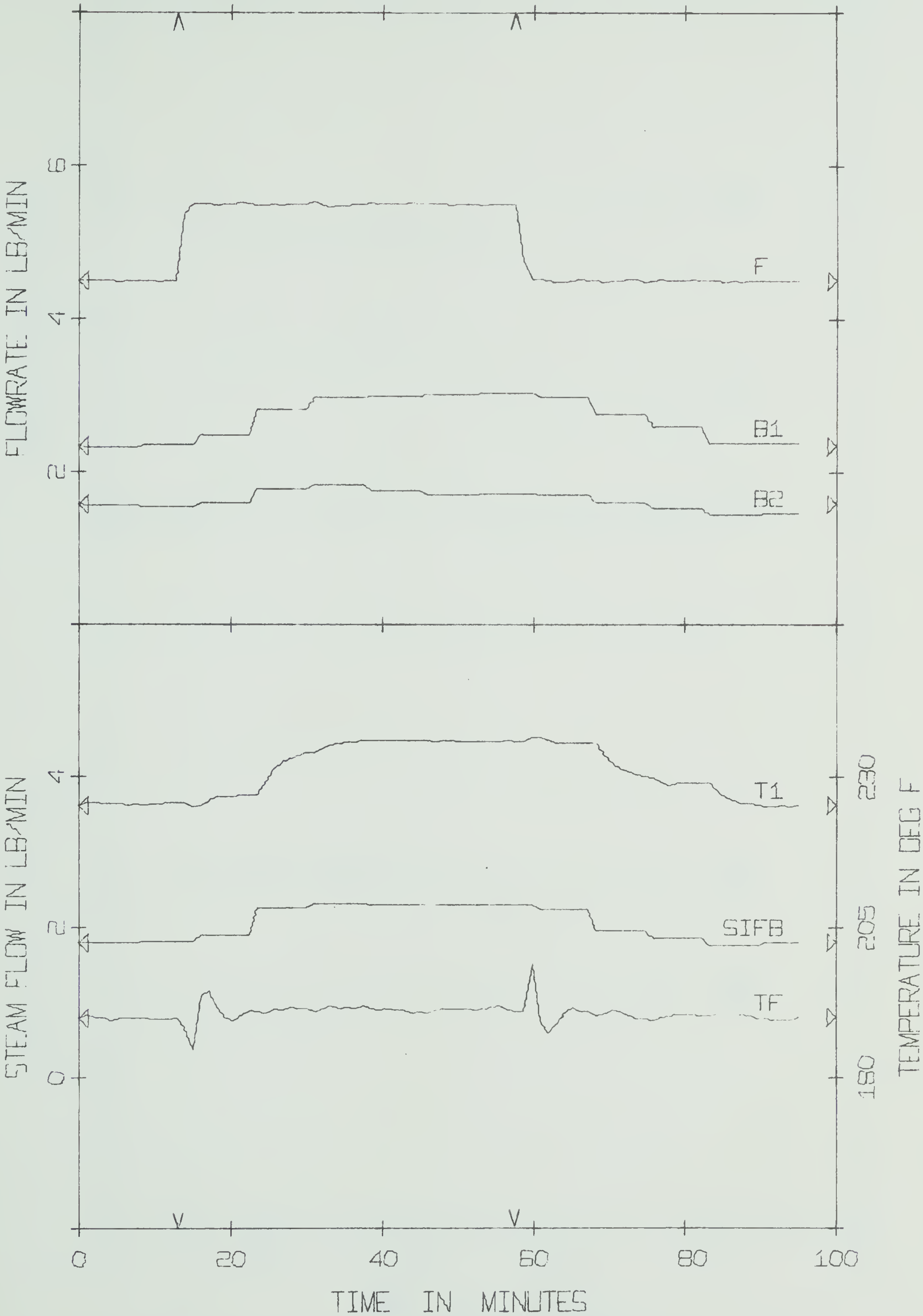


FIGURE 7b. EXPERIMENTAL RESPONSE WITH 448 SEC. CONTROL (EXP/20%F/FB/Q1/R1/D3/A1/MVC15)

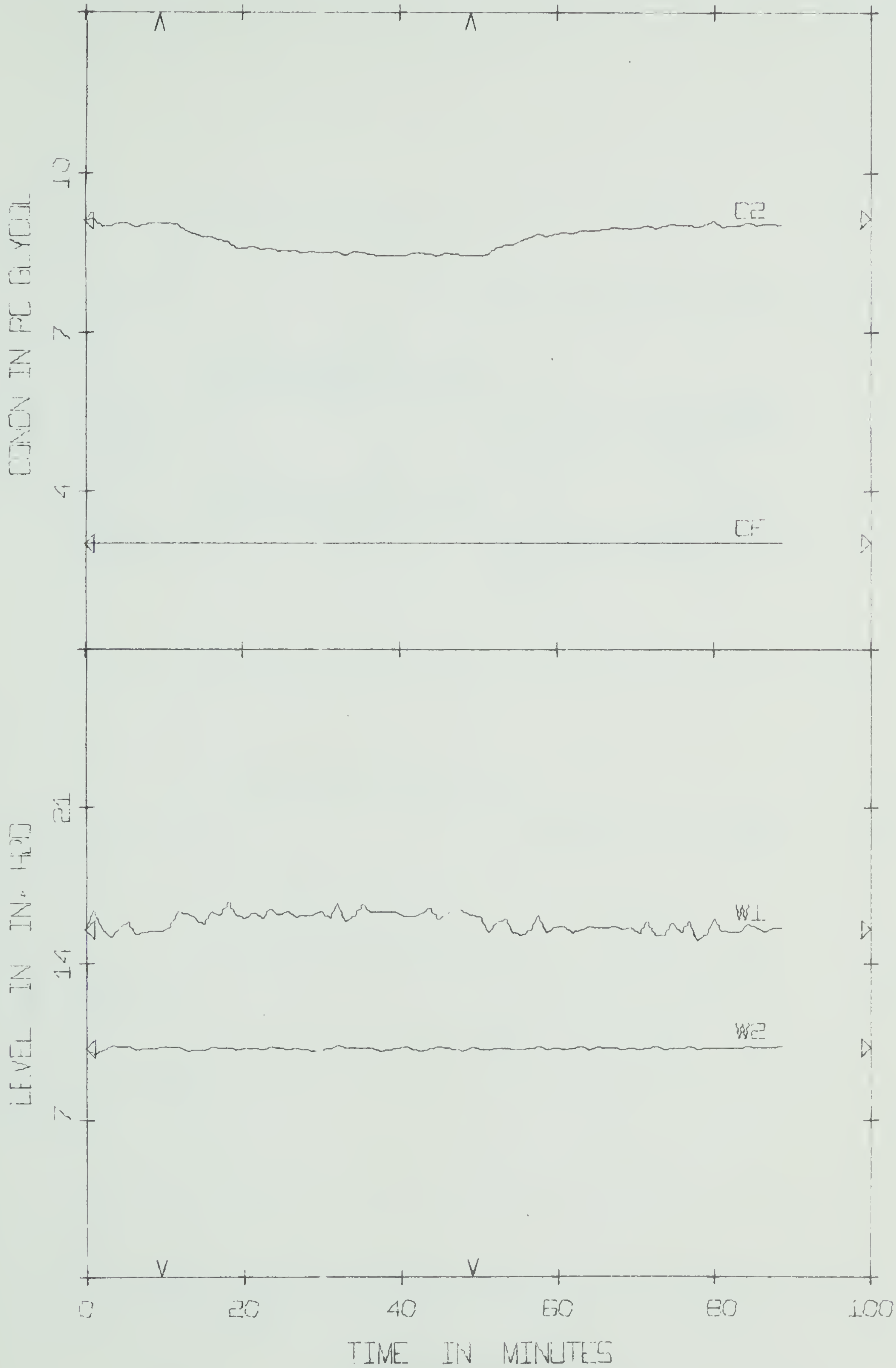


FIGURE 8a. EXPERIMENTAL EFFECT OF WEIGHTING C2 I
(EXP/20%F/FB/Q2/R1/D1/A1/MVC55)

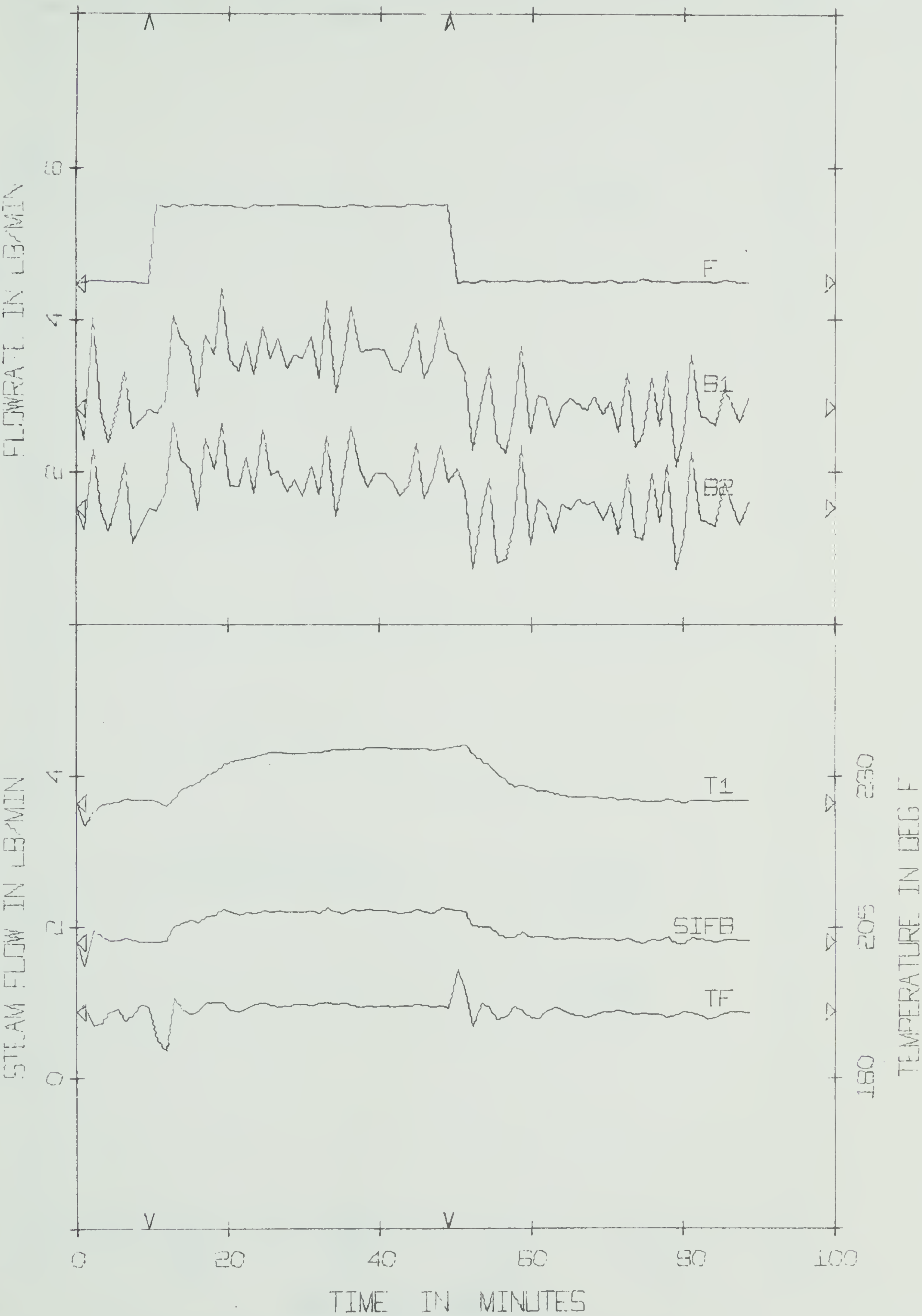


FIGURE 8b. EXPERIMENTAL EFFECT OF WEIGHTING C2 I
(EXP/20%F/FB/Q2/R1/D1/A1/MVC55)

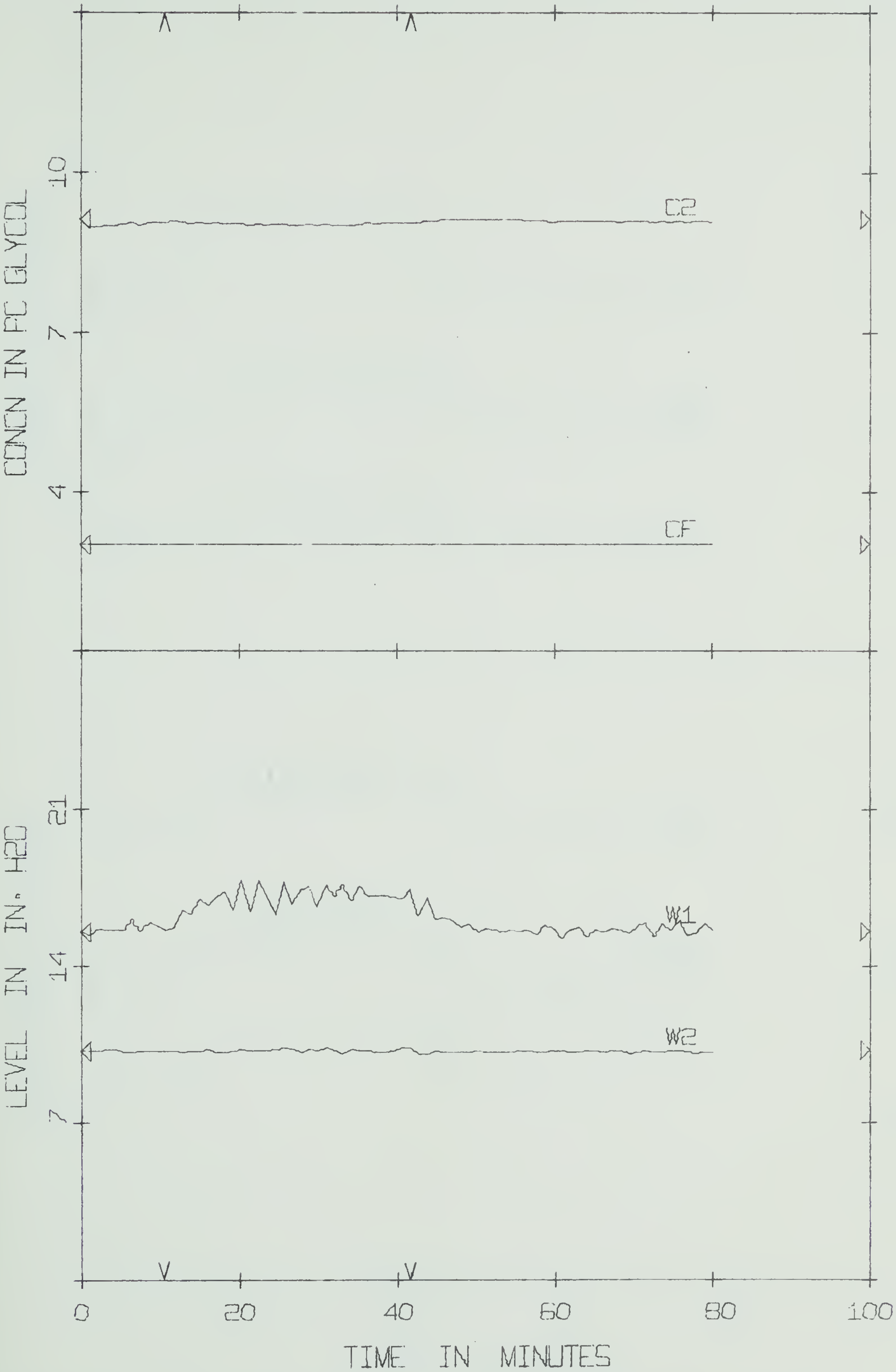


FIGURE 9a. EXPERIMENTAL EFFECT OF WEIGHTING C2 II
(EXP/20%F/FB/Q3/R1/D1/A1/MVC56)

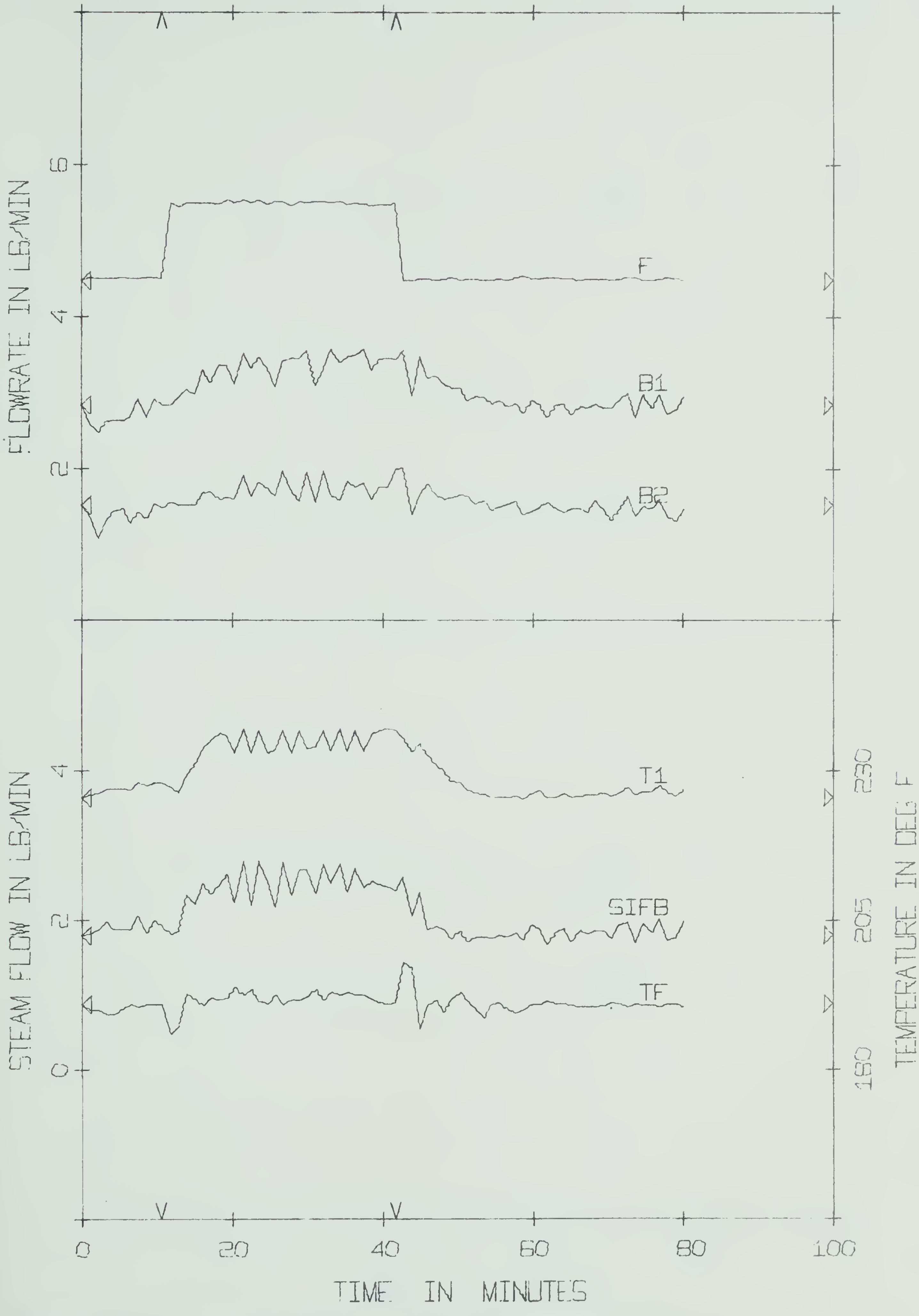


FIGURE 9b. EXPERIMENTAL EFFECT OF WEIGHTING C2 II
(EXP/20%F/FB/Q3/R1/D1/A1/MVC56)

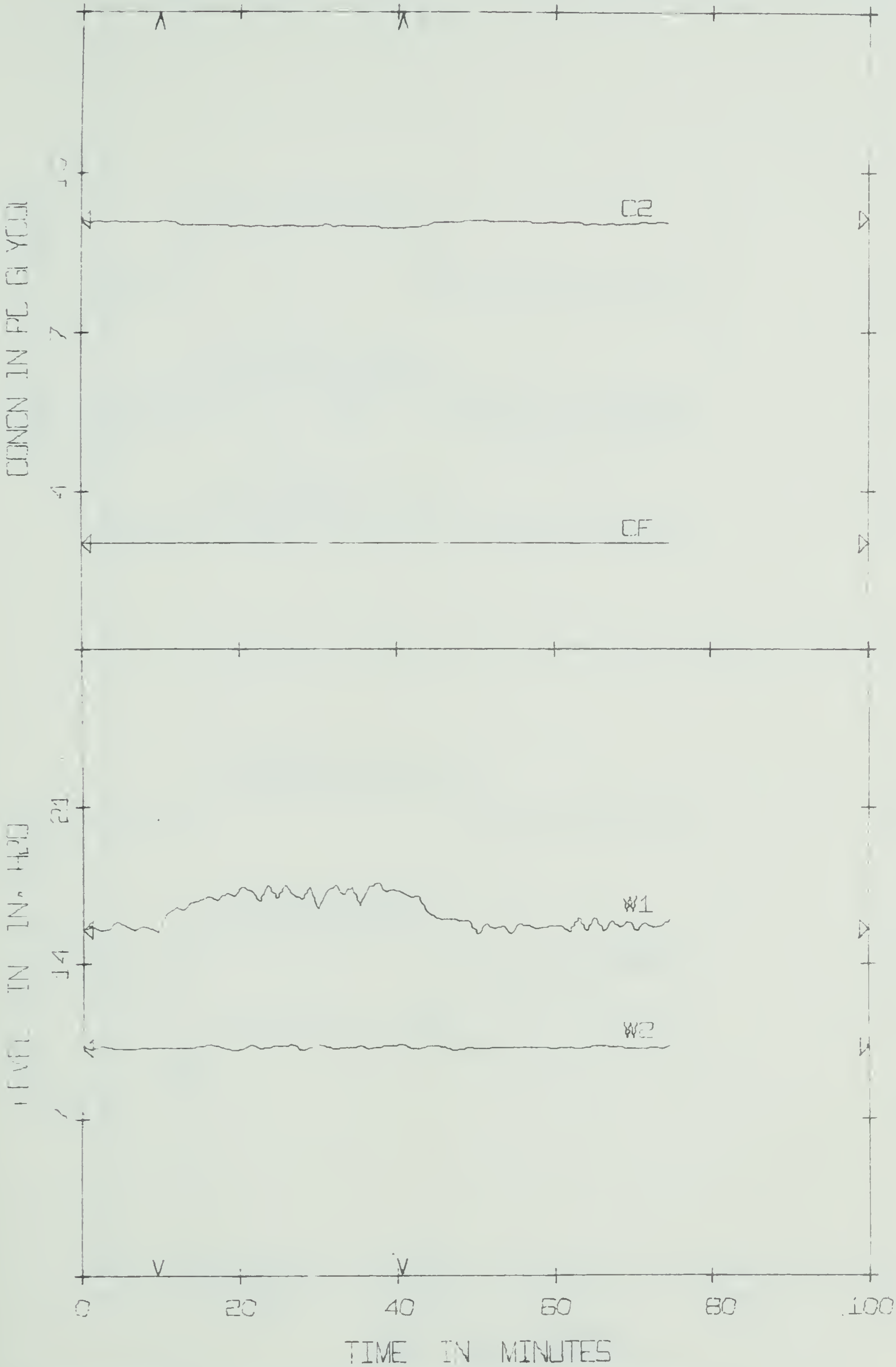


FIGURE 10a. EXPERIMENTAL EFFECT OF WEIGHTING LEVELS I
(EXP/20%F/FB/Q4/R1/D1/A1/MVC57)

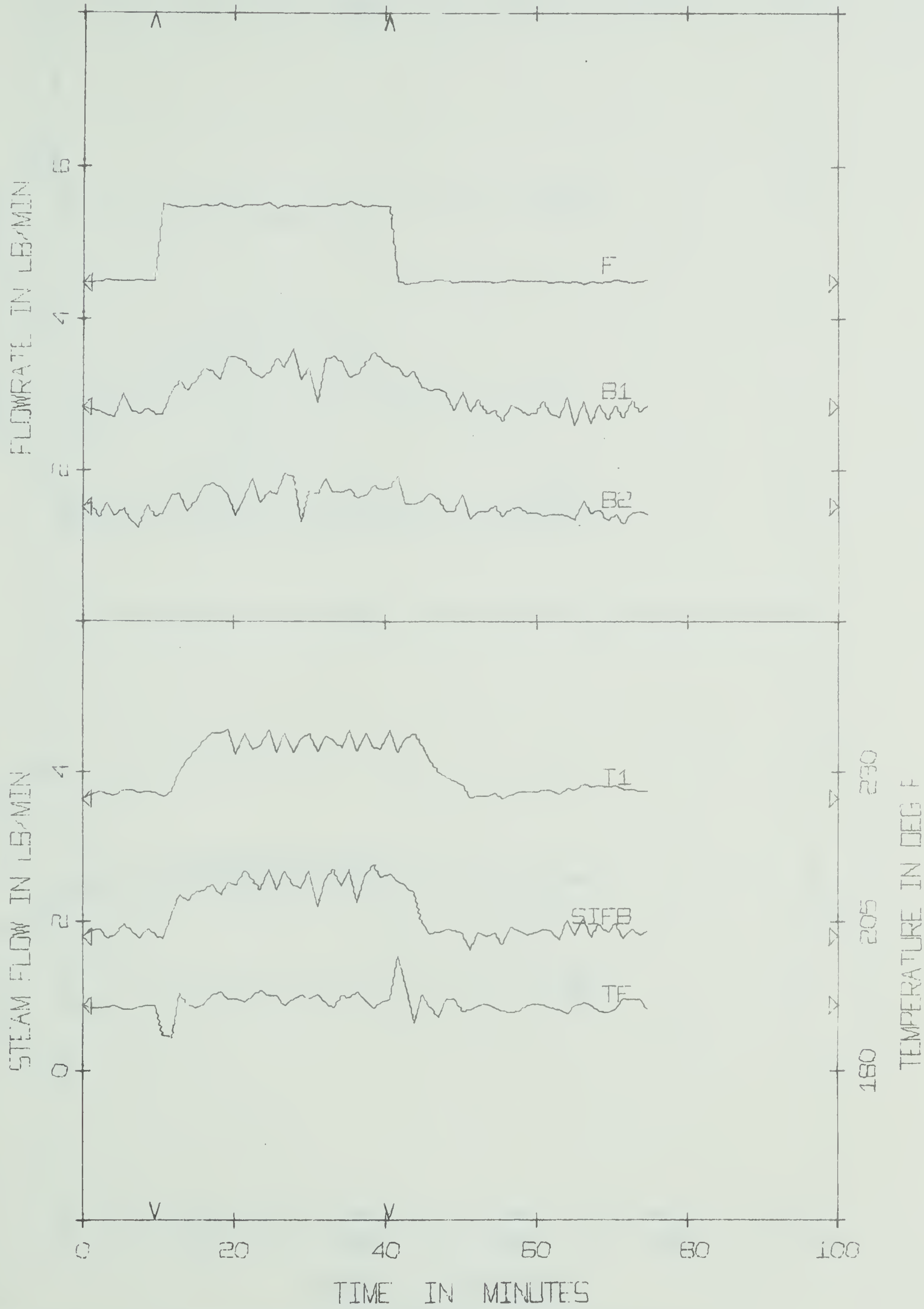


FIGURE 10b. EXPERIMENTAL EFFECT OF WEIGHTING LEVELS I
(EXP/20%F/FB/Q4/R1/D1/A1/MVC57)

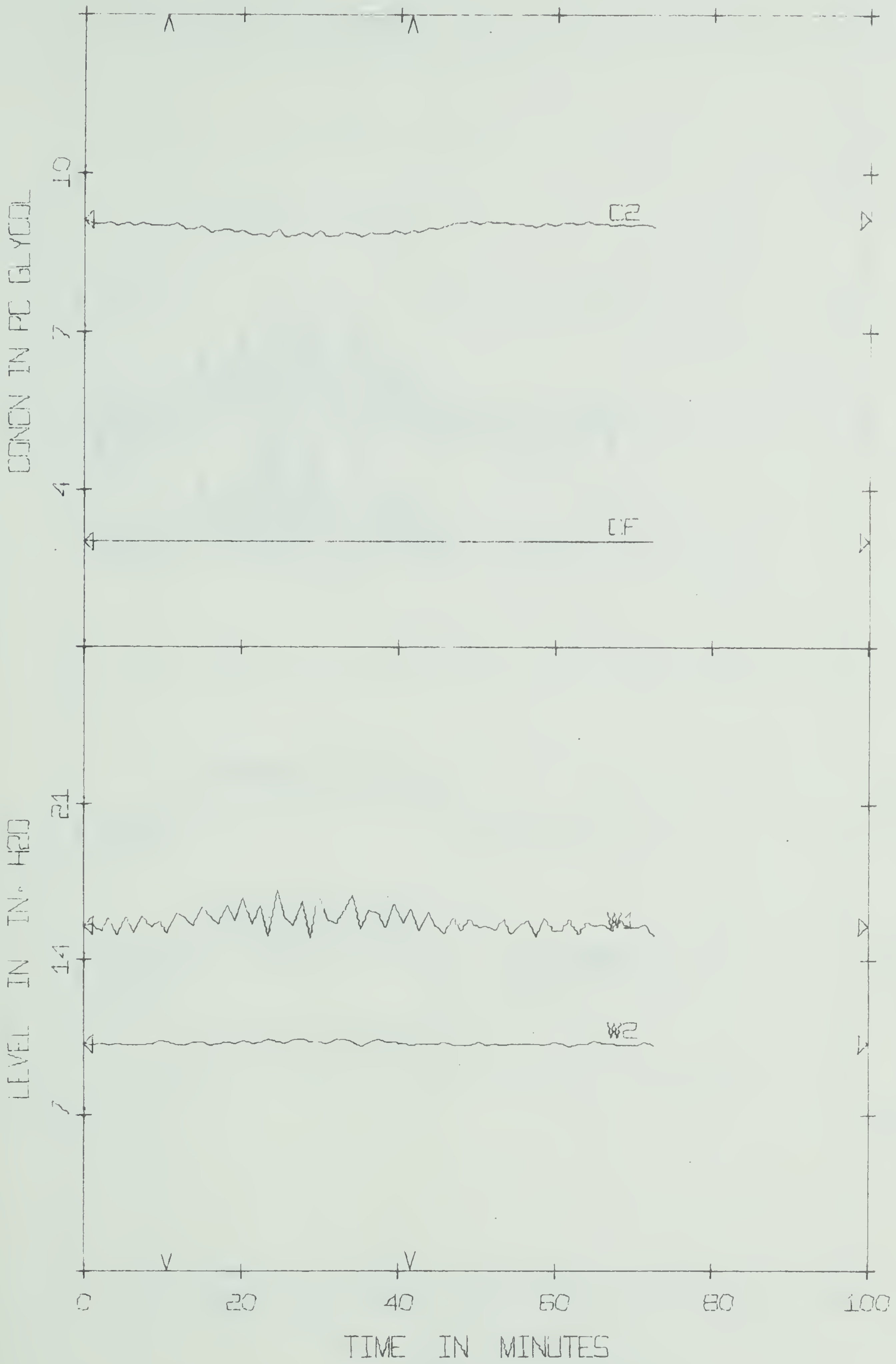


FIGURE 11a. EXPERIMENTAL EFFECT OF WEIGHTING LEVELS II
(EXP/20%F/FB/Q5/R1/D1/A1/MVC58)

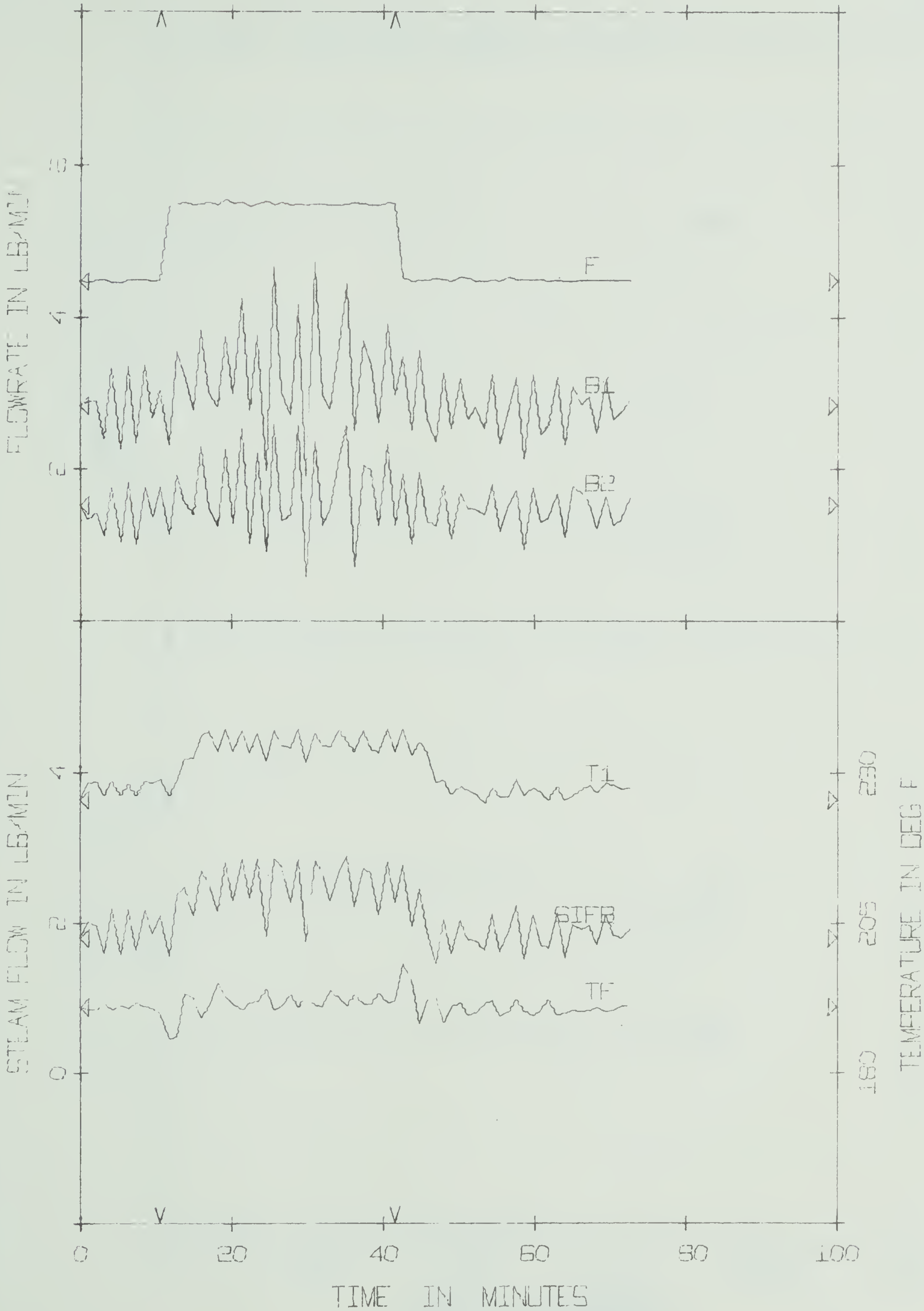


FIGURE 11b. EXPERIMENTAL EFFECT OF WEIGHTING LEVELS II
(EXP/20%F/FB/Q5/R1/D1/A1/MVC58)

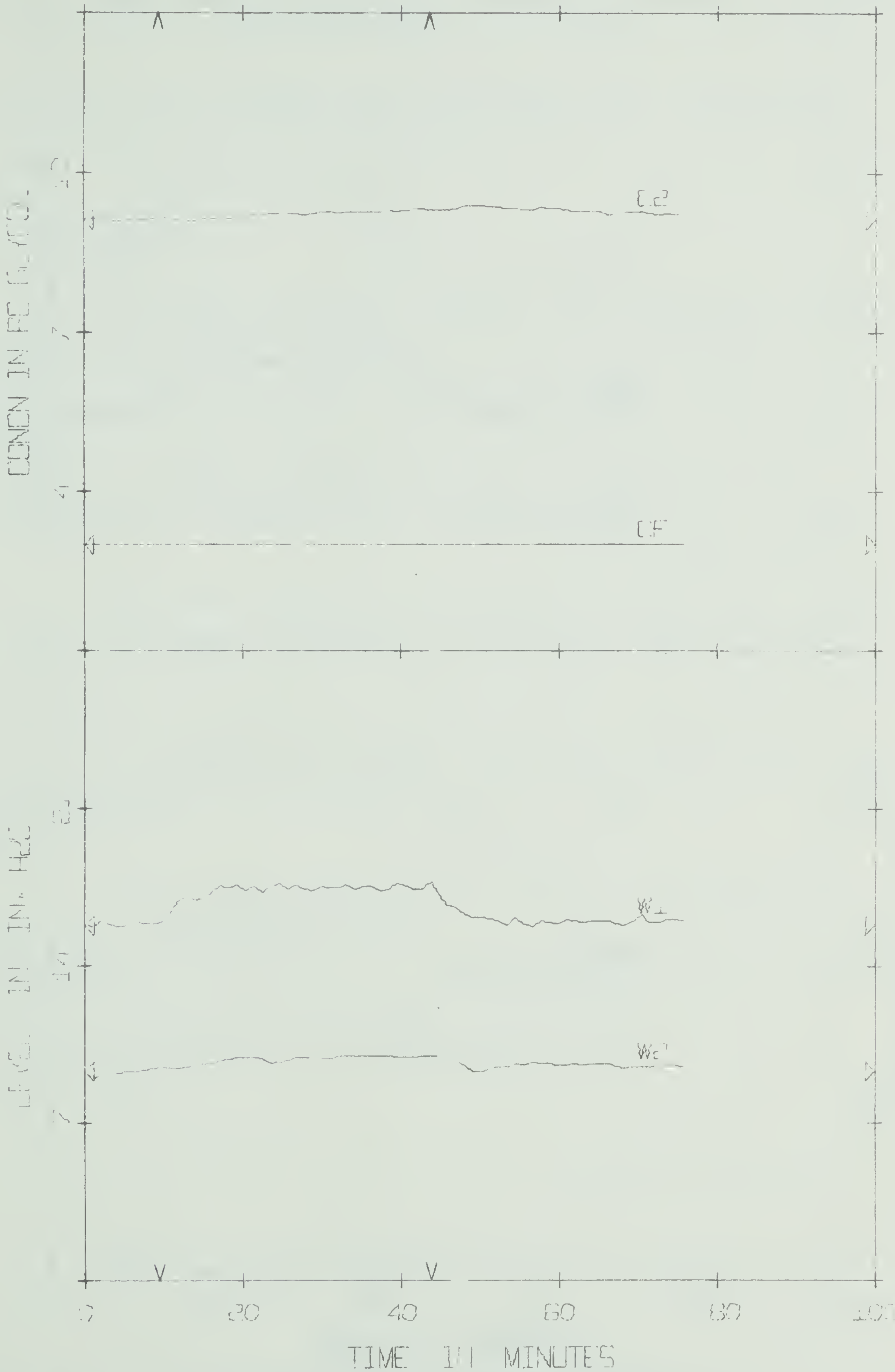


FIGURE 12a. EXPERIMENTAL EFFECT OF CONTROL WEIGHTING
(EXP/20%F/FB/Q1/R2/D1/A1/MVC59)

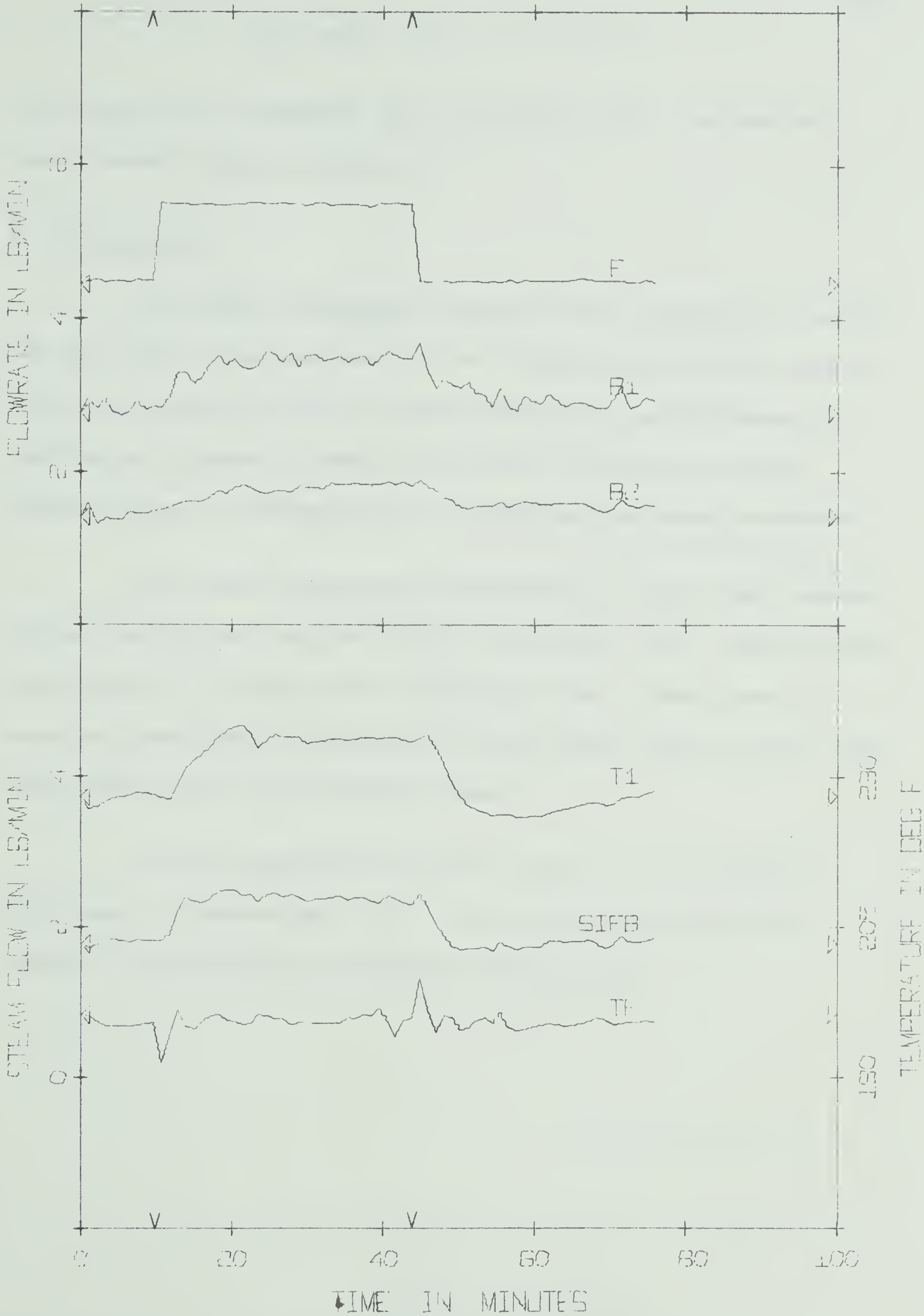


FIGURE 12b. EXPERIMENTAL EFFECT OF CONTROL WEIGHTING
(EXP/20%F/FB/Q1/R2/D1/A1/MVC59)

$$\underline{\underline{Q}} = \text{diag.}\ (10,\ 1,\ 1,\ 10,\ 100)$$

was chosen as the "standard" case for further work. The feedback control matrix appears in Table 8.

8. CONCLUSIONS

The dynamic programming approach gave a proportional control law with none of the realizeability or computational problems common when other methods are used. Design parameters could be chosen by considering a number of general guidelines and practical points. Simulation was an invaluable help in selecting the "best" parameters.

The control scheme was implemented on a pilot plant process and gave much better response than a conventional DDC control system (see Figure 13). Good control was obtained over a large range of operating conditions even though the linear model did not predict open loop behavior well in the complete range.

The work showed multivariable control to be a practical improvement on conventional DDC. The required modelling could probably be justified on the improved control alone.

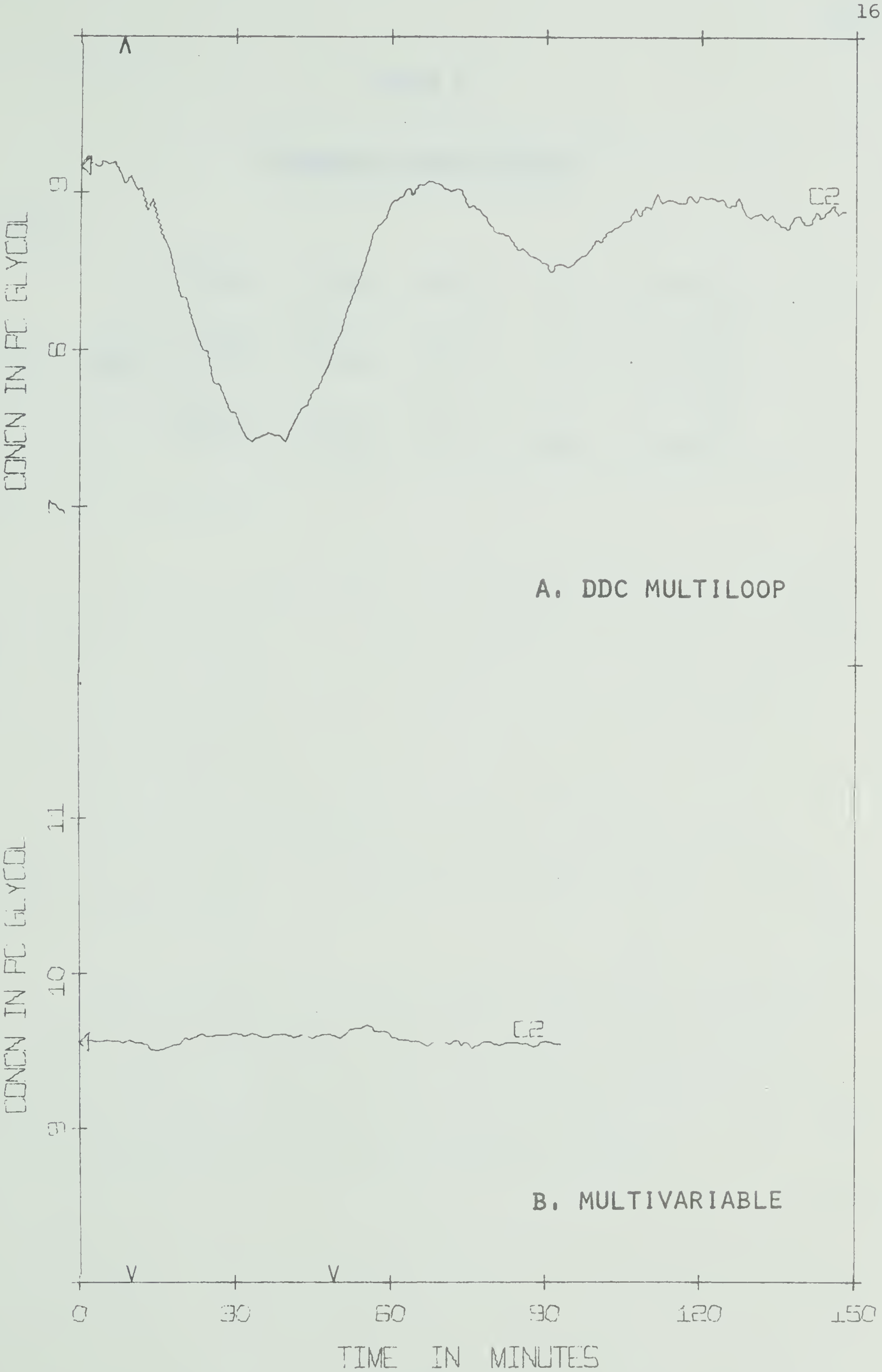


FIGURE 13. COMPARISON OF MULTIVARIABLE AND MULTILOOP
(EXP/20%F/FB, DDC/Q1/R1/D1/A1/T1/MVC41, DDC14)

TABLE 8

FEEDBACK CONTROL MATRIX

$$\underset{=}{K}_{FB} = \begin{bmatrix} 5.09 & -1.48 & -2.68 & 0 & -14.6 \\ 3.95 & 0.36 & 0.21 & 0 & 7.39 \\ 5.31 & 1.19 & -0.11 & 15.8 & 18.8 \end{bmatrix}$$

CHAPTER SIX

MULTIVARIABLE FEEDFORWARD CONTROL

ABSTRACT

A generalized formulation of the multivariable feedforward control problem is presented to clarify the relationships between different design approaches. Four multivariable feedforward schemes were developed from this formulation and tested experimentally on a pilot plant evaporator.

The four control schemes are designed using three criteria, zero offsets in some states, a summed quadratic index, and a quadratic function of the offsets. Design methods for the second and third criteria are developed.

Simulated results from an evaporator model are used to compare the four schemes. All four cases produce similiar control matrices and responses.

Multivariable feedforward-feedback control is implemented on a double effect evaporator using an IBM 1800 control computer. The experimental data demonstrate the improved control when feedforward is used and confirm the similarity of the four schemes.

1. INTRODUCTION

The aim of the work was to examine multivariable feedforward control with the goal of developing a control algorithm suitable for implementation on computer controlled industrial processes.

Earlier work [1] has shown that an optimal proportional feedback control system, in the presence of constant loads, will drive the process to a state different from the desired state. This difference in states will be termed offset. It is therefore of interest to augment the feedback controller in order to minimize or remove this offset. Conventional feedforward control is designed to compensate completely for loads, and feedback control is added only to allow for modelling errors and other disturbances. However the multivariable problem is generally formulated so that feedforward action reduces or removes offsets caused by proportional feedback action in the presence of constant loads.

A generalized formulation is presented for the multivariable feedforward control problem. Two existing design approaches presented in the literature [2,3] are shown to solve special cases of this generalized formulation and two additional design approaches are derived. One minimizes a summed quadratic criterion by discrete dynamic programming and the other uses differential calculus to minimize a quadratic function of the final offsets produced by proportional feedback control. Both approaches lead to a proportional feedforward control law.

A simulated comparison of four formulations and their

solutions is presented. This comparison is backed by experimental results obtained by implementing feedforward control on a pilot plant evaporator.

2. LITERATURE SURVEY

The design theory of single variable feedforward controllers is well known although implementation is still based mainly on empirical knowledge or "tuning" [[4] Chapter 8]. On the other hand multivariable feedforward control has not received very wide attention. As with single variable feedforward control, multivariable control can be either static or dynamic.

2.1. Dynamic Compensation

Bollinger and Lamb [5] have presented a dynamic design approach based upon a model expressed as a matrix of transfer functions. However, there are a number of limitations. The number of outputs must equal the number of control variables and problems of physically realizing the calculated controllers are often severe, particularly if the disturbance acts upon the process faster than the control variables. An alternative approach is to select a physically realizable dynamic controller and optimize its parameters. This alternative is generally unsuitable for multivariable systems where the number of parameters to optimize becomes relatively large. Corlis and Luus [6] used the approach of identifying a dynamic model of the process and using this as a predictor for feedforward compensation. A two-input shell and tube heat exchanger model was used to simulate their multi-input single-output design technique. Johnson [7] and Sobral and Stefanek [8] transform the control problem and "eliminate" the effects of disturbances by

controls which generate an equal and opposite effect in the state variables to that produced by the disturbances. These disturbances need not be measured but must be known to satisfy a given linear differential equation. In the special case of a constant disturbance the solution simplifies to a form of integral control. A disadvantage of the technique is a rather stringent collinearity condition.

2.2. Static Compensation

Multivariable static or proportional feedforward control is easier to design and implement. Anderson [2] presented a design technique based on an "error coordinate" transformation. Assuming a constant disturbance the system transformed to the standard regulator problem without disturbances which could then be solved by existing optimal techniques. The resulting control law, when transformed out of "error coordinates", included feedforward action which essentially ensured that some of the outputs were at their desired values when the process reached steady state. The number of outputs which can be controlled in this manner is dependent on the degrees of freedom for driving states or outputs to specific values [1].

Alternatively a criterion can be optimized with respect to the control variables. The solution gives both the form of the control law and its parameter values. The form and parameter values depend on the criterion chosen. When a quadratic index of state and control variables is used a proportional control law results which includes both the optimal feedback and feedforward modes. This problem has been solved by Solheim and Saethre [3] using calculus of variations for the continuous and discrete cases and by West and McGuire [9] using

continuous dynamic programming.

3. GENERALIZED FORMULATION

This section will present a generalized formulation of the static feedforward control problem.

Consider the state space model of the following form

$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} + \underline{\underline{D}}\,\underline{d} \tag{1}$$

where $\underline{x}$ is the n dimensional state vector,

$\underline{u}$ is the m dimensional control vector,

$\underline{d}$ is the p dimensional load vector whose elements are all measurable,

and $\underline{\underline{A}}$, $\underline{\underline{B}}$, and $\underline{\underline{D}}$ are appropriately dimensioned constant coefficient matrices.

In general it is desirable to eliminate offsets in the state variables completely. However since this is impossible in some problems the designer may choose to minimize some function of a subset of the offsets, to make another subset of them zero, and to ignore the offsets in the remaining state variables.

The degrees of freedom for driving states or outputs to specific values [1] is the number of control variables, m. Therefore it is possible to have zero offsets in only q states where q is less than or equal to m. If $q < m$ then it is possible to use the remaining degrees of freedom to minimize a function of the offsets in some or all of the remaining $(n-q)$ states.

The state and control vectors may be partitioned with the partitions $\underline{x}_1$ and $\underline{u}_1$ of dimension $q(0 \leq q \leq m)$, zero offset being desired in these q states.

The resulting control problem formulation can then be stated as follows:

$$\dot{\underline{x}} = [\underline{\underline{A}}_1 \; \underline{\underline{A}}_2]\begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + [\underline{\underline{B}}_1 \; \underline{\underline{B}}_2]\begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \end{bmatrix} + \underline{\underline{D}}\,\underline{d} \tag{2}$$

where it is desired that:

(a) when $\dot{\underline{x}} = 0$, $\underline{x}_1 = \underline{x}_{1d}$

(b) the criterion $J = J(\underline{x}_2, \underline{u}_2)$ is minimized with respect to $\underline{u}_2$.

Since the final offsets are a function of the feedback control matrix, the design of the feedback control matrix must be carried out before designing for zero offsets. This results in two possible design approaches.

(a) The order of design is:

 (i) design the feedback control matrix and replace the $\underline{\underline{A}}$ matrix in the feedforward problem statement (Equation (2)) by the closed loop form.

 (ii) design the $\underline{u}_1$ feedforward control matrix for zero offsets and substitute for $\underline{u}_1$ in Equation (2).

 (iii) minimize the criterion J using a design method giving only feedforward.

(b) The order of design is:

(i) design the feedback and $\underline{u}_2$ feedforward matrices based on a common criterion J and using a feedback/feedforward design technique. Substitute for $\underline{u}_2$ in Equation (2).

(ii) design the $\underline{u}_1$ feedforward control matrix for zero offsets.

The latter approach would result in feedback action from $\underline{u}_2$ only which would be generally undesirable.

4. EXISTING DESIGN TECHNIQUES

Common design techniques are discussed relative to the generalized formulation.

4.1. Design for Zero Offset

The usual design approach would be as follows. Substitute the conditions $\dot{\underline{x}} = 0$ and $\underline{x}_1 = \underline{x}_{1d}$ into the partitioned problem (Equation (2))

$$0 = [\underline{\underline{A}}_1 \ \underline{\underline{A}}_2]\begin{bmatrix} \underline{x}_{1d} \\ \underline{x}_2 \end{bmatrix} + [\underline{\underline{B}}_1 \ \underline{\underline{B}}_2]\begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \end{bmatrix} + \underline{\underline{D}}\ \underline{d} \tag{3}$$

This equation can be rearranged to the expression

$$[\underline{\underline{B}}_1 \ \underline{\underline{A}}_2]\begin{bmatrix} \underline{u}_1 \\ \underline{x}_2 \end{bmatrix} = - \underline{\underline{B}}_2\ \underline{u}_2 - \underline{\underline{A}}_1\ \underline{x}_{1d} - \underline{\underline{D}}\ \underline{d} \tag{4}$$

From which a control law of the following form can be partitioned

$$\underline{u}_1 = \underline{\underline{K}}_1 \underline{u}_2 + \underline{\underline{K}}_2 \underline{x}_{1d} + \underline{\underline{K}}_3 \underline{d} \tag{5}$$

The "error coordinate" approach of Anderson [2] can also be used to evaluate this control law. It is shown in Appendix A that the "error coordinate" approach is exactly equivalent to the above simple analysis where $\underline{\underline{A}}$ is the closed loop matrix incorporating the optimal feedback matrix evaluated by dynamic programming . Anderson solved the special case where $q = m$.

If the dimension of $\underline{u}_2$ is non-zero then Equation (5) can be substituted into the original problem (Equation (2)) to produce the remaining problem,

$$\dot{\underline{x}} = [\underline{\underline{A}}_1 \ \underline{\underline{A}}_2]\begin{bmatrix} \underline{\underline{x}}_1 \\ \underline{\underline{x}}_2 \end{bmatrix} + (\underline{\underline{B}}_1 \underline{\underline{K}}_1 + \underline{\underline{B}}_2)\underline{u}_2 + (\underline{\underline{B}}_1 \underline{\underline{K}}_3 + \underline{\underline{D}})\underline{d} + \underline{\underline{B}}_1 \underline{\underline{K}}_2 \underline{x}_{1d} \tag{6}$$

where $\underline{u}_2$ is evaluated to minimize the criterion $J(\underline{x}_2, \underline{u}_2)$. If the minimization had been carried out first $\underline{u}_2$ would be substituted for in Equation (3) and would not appear in the zero offset design.

4.2. Minimization of Criteria

Solheim and Saethre [3] and West and McGuire [9] presented design techniques for the minimization of a quadratic criterion in state and control variables. Both methods resulted in both feedback and feedforward proportional control matrices which would necessitate the second design approach for the generalized formulation. The problems solved in their papers were special cases where the first condition on zero offsets was not included.

5. ALTERNATIVE DESIGN METHODS

This section presents two other design techniques which produce proportional feedforward control matrices. In both cases the feedforward action augments basic feedback control and reduces the offsets resulting from proportional feedback action in the presence of constant loads.

5.1. Minimization of Offsets

The criterion for the feedforward control is to minimize a weighted sum of the squared offsets resulting from a load change.

$$J = \underline{x}_s^T \underline{\underline{Q}}\, \underline{x}_s \tag{7}$$

Consider a process described by the discrete state equation

$$\underline{x}_{n+1} = \underline{\underline{A}}\, \underline{x}_n + \underline{\underline{B}}\, \underline{u}_n + \underline{\underline{D}}\, \underline{d} \tag{8}$$

where $\underline{d}$ is the measureable load change (a constant vector for $t > 0$).

Given a proportional feedback control matrix, a control law of the following form is considered.

$$\underline{u}_n = \underline{\underline{K}}_{FB}\, \underline{x}_n + \underline{u}_{FF} \tag{9}$$

At steady state $(t \to \infty)$ $\underline{x}_{n+1} = \underline{x}_n = \underline{x}_s$ so that

$$\underline{x}_s = -(\underline{\underline{I}} - \underline{\underline{A}} - \underline{\underline{B}}\, \underline{\underline{K}}_{FB})^{-1}(\underline{\underline{B}}\, \underline{u}_{FF} + \underline{\underline{D}}\, \underline{d}) \tag{10}$$

Differentiating the criterion with respect to the feedforward control vector

$$\frac{1}{2}\frac{\partial J}{\partial \underline{u}_{FF}} = \frac{\partial \underline{x}_s}{\partial \underline{u}_{FF}} \underline{\underline{Q}}\, \underline{x}_s = 0 \tag{11}$$

from which it follows that

$$\underline{u}_{FF} = -(\underline{\underline{B}}^T \underline{\underline{A}}^{*T} \underline{\underline{Q}}\, \underline{\underline{A}}^* \underline{\underline{B}})^{-1} \underline{\underline{B}}^T \underline{\underline{A}}^{*T} \underline{\underline{Q}}\, \underline{\underline{A}}^* \underline{\underline{D}}\, \underline{d} \tag{12}$$

where

$$\underline{\underline{A}}^* = -(\underline{\underline{I}} - \underline{\underline{A}} - \underline{\underline{B}}\, \underline{\underline{K}}_{FB})^{-1} \tag{13}$$

(Note that if the system is "square", i.e. the dimensions of the state and control vectors are the same, there is exact compensation with

$$\underline{u}_{FF} = -\underline{\underline{B}}^{-1} \underline{\underline{D}}\, \underline{d}$$

The feedforward control action at time t is minimizing the offset that would result at $t = \infty$ from proportional feedback and the load at time t.

5.2. Dynamic Programming Design

This design technique makes use of the discrete dynamic programming approach to minimize the following general quadratic performance index.

$$J = \underline{x}_N^T \underline{\underline{S}}\, \underline{x}_N + \sum_{i=1}^{N} (\underline{x}_i^T \underline{\underline{Q}}\, \underline{x}_i + \underline{u}_{i-1}^T \underline{\underline{R}}\, \underline{u}_{i-1}) \tag{14}$$

Considering the process represented by the discrete state equation (Equation 8) the following relations define the optimal feedback

and feedforward control matrices

$$\underline{\underline{K}}_{FB}^{N-i} = -(\underline{\underline{B}}^T \underline{\underline{P}}_{i-1} \underline{\underline{B}} + \underline{\underline{R}})^{-1} \underline{\underline{B}}^T \underline{\underline{P}}_{i-1} \underline{\underline{A}} \tag{15}$$

where

$$\underline{\underline{P}}_{i-1} = \underline{\underline{T}}_{N-i+1}^T \underline{\underline{P}}_{i-2} \underline{\underline{T}}_{N-i+1} + \underline{\underline{K}}_{FB}^{N-i+1T} \underline{\underline{R}} \underline{\underline{K}}_{FB}^{N-i+1} + \underline{\underline{Q}} \tag{16}$$

initialized by $\underline{\underline{P}}_o = \underline{\underline{Q}} + \underline{\underline{S}}$

$$\underline{\underline{K}}_{FF}^{N-i} = -(\underline{\underline{B}}^T \underline{\underline{P}}_{i-1} \underline{\underline{B}} + \underline{\underline{R}})^{-1} (\underline{\underline{B}}^T \underline{\underline{P}}_{i-1} \underline{\underline{D}} + \underline{\underline{B}}^T \underline{\underline{O}}_{i-1}) \tag{17}$$

where

$$\underline{\underline{O}}_{i-1} = \underline{\underline{T}}_{N-i+1}^T \underline{\underline{O}}_{i-2} + \underline{\underline{T}}_{N-i+1}^T \underline{\underline{P}}_{i-1} (\underline{\underline{B}} \underline{\underline{K}}_{FF}^{N-i+1} + \underline{\underline{D}})$$
$$+ \underline{\underline{K}}_{FB}^{N-i+1T} \underline{\underline{R}} \underline{\underline{K}}_{FF}^{N-i+1} \tag{18}$$

initialized by $\underline{\underline{O}}_o = 0$ and where

$$\underline{\underline{T}}_{N-i+1} = \underline{\underline{A}} + \underline{\underline{B}} \underline{\underline{K}}_{FB}^{N-i+1} \tag{19}$$

These recursive relations converge to give constant control matrices $\underline{\underline{K}}_{FB}$ and $\underline{\underline{K}}_{FF}$ for a control law of the form

$$\underline{u}_n = \underline{\underline{K}}_{FB} \underline{x}_n + \underline{\underline{K}}_{FF} \underline{d}_n \tag{20}$$

It is assumed in the derivation that the load change is of constant magnitude.

It should be noted that this design technique generates both the optimal feedback and feedforward control matrices. The feedback matrix $\underline{\underline{K}}_{FB}$ is independent of $\underline{\underline{K}}_{FF}$ and is optimal for the formulation used when applied alone. However $\underline{\underline{K}}_{FF}$ is not independent of $\underline{\underline{K}}_{FB}$ and cannot be used alone.

Since this approach generates both control matrices it is suitable for the second design approach to the generalized formulation or the special case where no zero offsets are desired.

6. SIMULATED COMPARISON OF DESIGNS

A simulation study was carried out using a model of a pilot plant double effect evaporator in the Department of Chemical and Petroleum Engineering at the University of Alberta.

6.1. Double Effect Evaporator and Model

A schematic flow diagram of the process connected for operation in a double effect "forward feed" mode of operation is shown in Figure 1. The first effect is a calandria type unit with an eight inch diameter tube bundle. It operates with a nominal feedrate of 5 lb./min. of 3 percent aqueous triethylene glycol. The second effect is a long tube vertical unit with three 1" × 6' tubes and is operated with externally forced circulation. The second effect is operated under vacuum and utilizes the vapour from the first effect as heating medium. The product is about 10 percent glycol when the steam to the first effect is at its nominal flowrate of 2 lb./min.

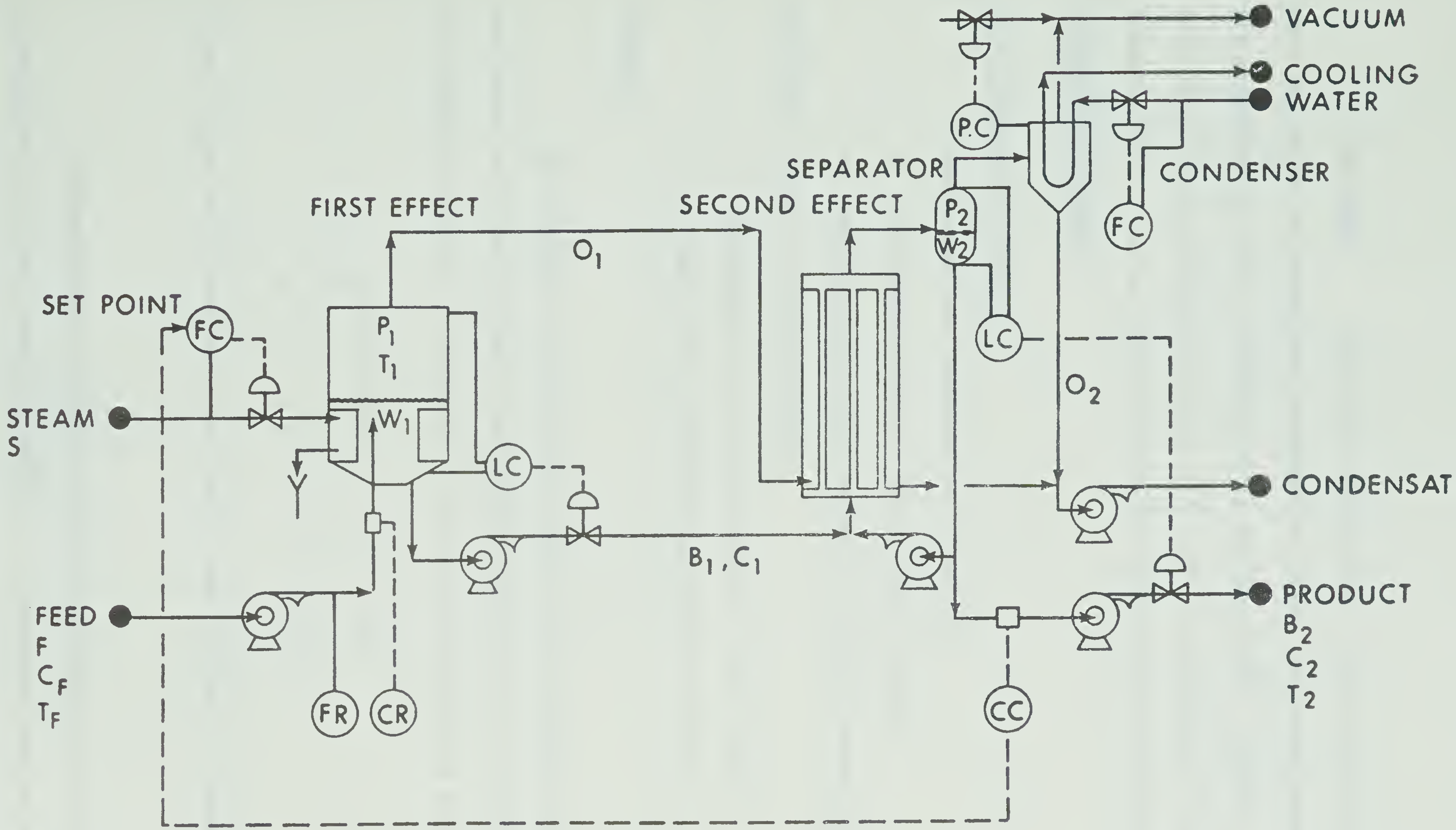


FIGURE 1. PILOT PLANT DOUBLE EFFECT EVAPORATOR

The model is a five dimensional linear state equation with variables in normalized perturbation form. The model equations are presented in full in Equation (21) and the state, control, and load vectors are defined in terms of the process variables in the Nomenclature. The two liquid levels and the product concentration are the defined outputs.

6.2. Simulation Results

Four cases were examined in the study:

(a) designed for zero offsets in all outputs.

(b) designed for a zero offset in product concentration and minimized offsets in liquid levels.

(c) designed for minimized offsets in all outputs.

(d) designed for a minimized quadratic criterion in all outputs.

The feedback control matrix used in all four cases was evaluated by discrete dynamic programming (Equation (15)) based upon the quadratic criterion (Equation (14)) with the following parameters.

$$\underline{\underline{Q}} = \text{diag}\ (10,\ 1,\ 1,\ 10,\ 100)$$

$$\underline{\underline{R}} = 0, \quad \underline{\underline{S}} = 0, \quad \Delta t = 64 \text{ secs.}$$

These values were found to be the "best" combination in Chapter 5.

The feedforward control matrices evaluated in the four cases were found to be very similiar (Table 1). Simulated results are exemplified by the plots in Figures 2a and 2b where multivariable

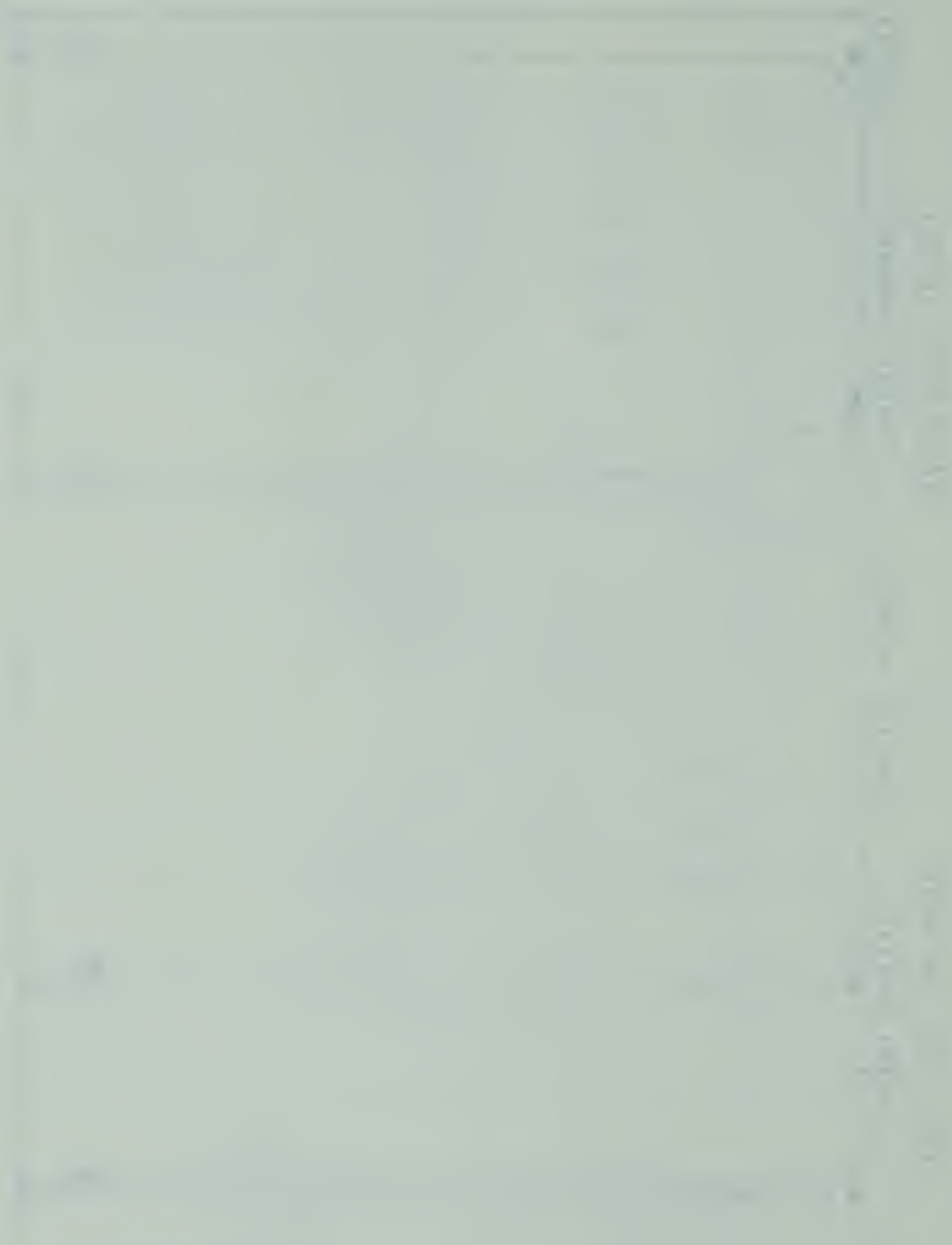
$$\begin{bmatrix} \dot{W1} \\ C1 \\ H1 \\ W2 \\ C2 \end{bmatrix} = \begin{bmatrix} 0 & -.00156 & -.1711 & 0 & 0 \\ 0 & -.1419 & .1711 & 0 & 0 \\ 0 & -.00875 & -1.102 & 0 & 0 \\ 0 & -.00128 & -.1489 & 0 & .00013 \\ 0 & .0605 & .1489 & 0 & -.0591 \end{bmatrix} \begin{bmatrix} W1 \\ C1 \\ H1 \\ W2 \\ C2 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & -.143 & 0 \\ 0 & 0 & 0 \\ .392 & 0 & 0 \\ 0 & .108 & -.0592 \\ 0 & -.0486 & 0 \end{bmatrix} \begin{bmatrix} S \\ B1 \\ B2 \end{bmatrix}$$

$$+ \begin{bmatrix} .2174 & 0 & 0 \\ -.074 & .1434 & 0 \\ -.036 & 0 & .1814 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F \\ CF \\ HF \end{bmatrix}$$

$$\begin{bmatrix} W1 \\ W2 \\ C2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} W1 \\ C1 \\ H1 \\ W2 \\ C2 \end{bmatrix}$$

Equation (21)



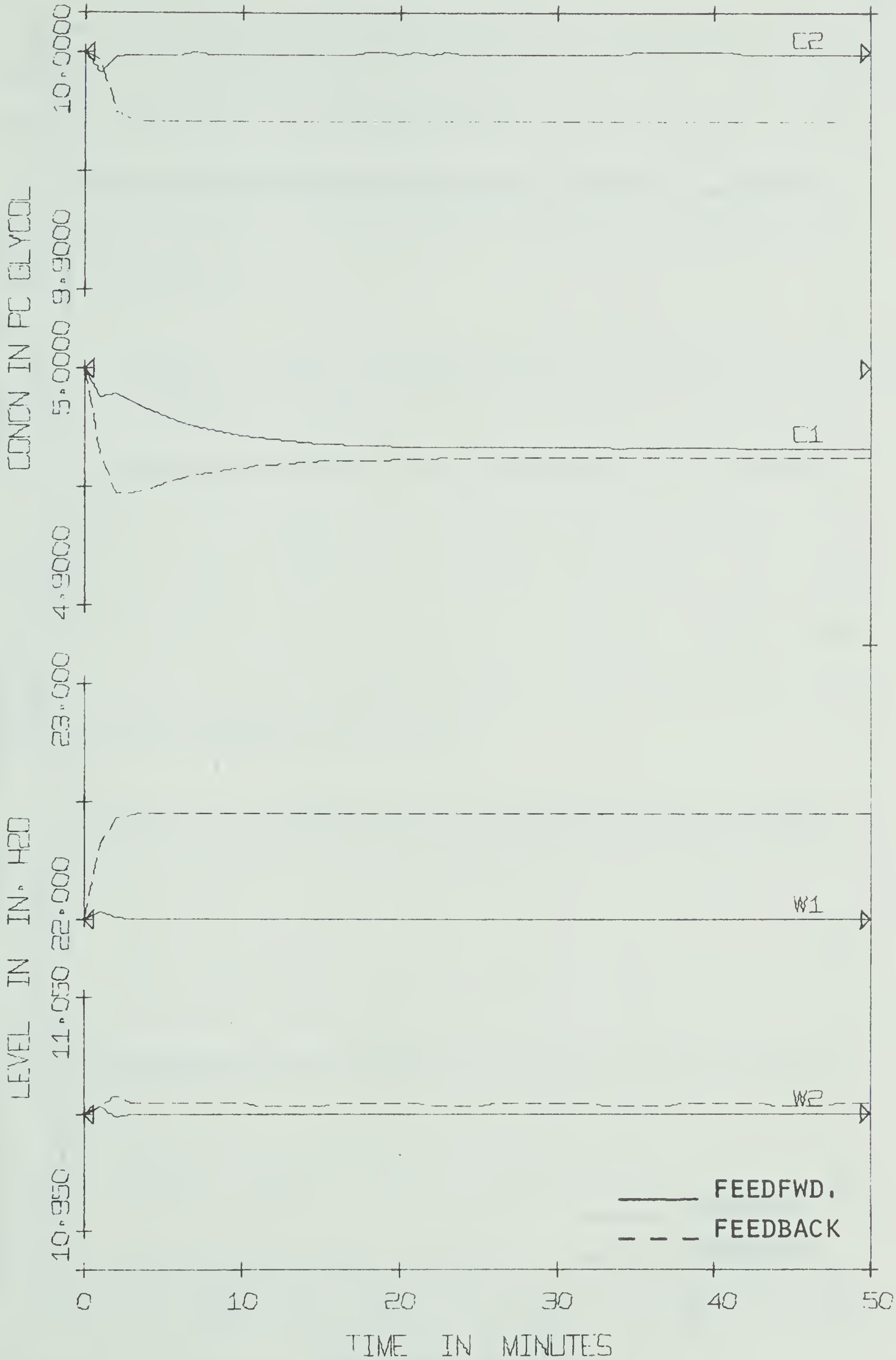


FIGURE 2a. SIMULATED FEEDFORWARD VS FEEDBACK CONTROL (5L/+10%F/FB, FF/Q1/R1/D1)

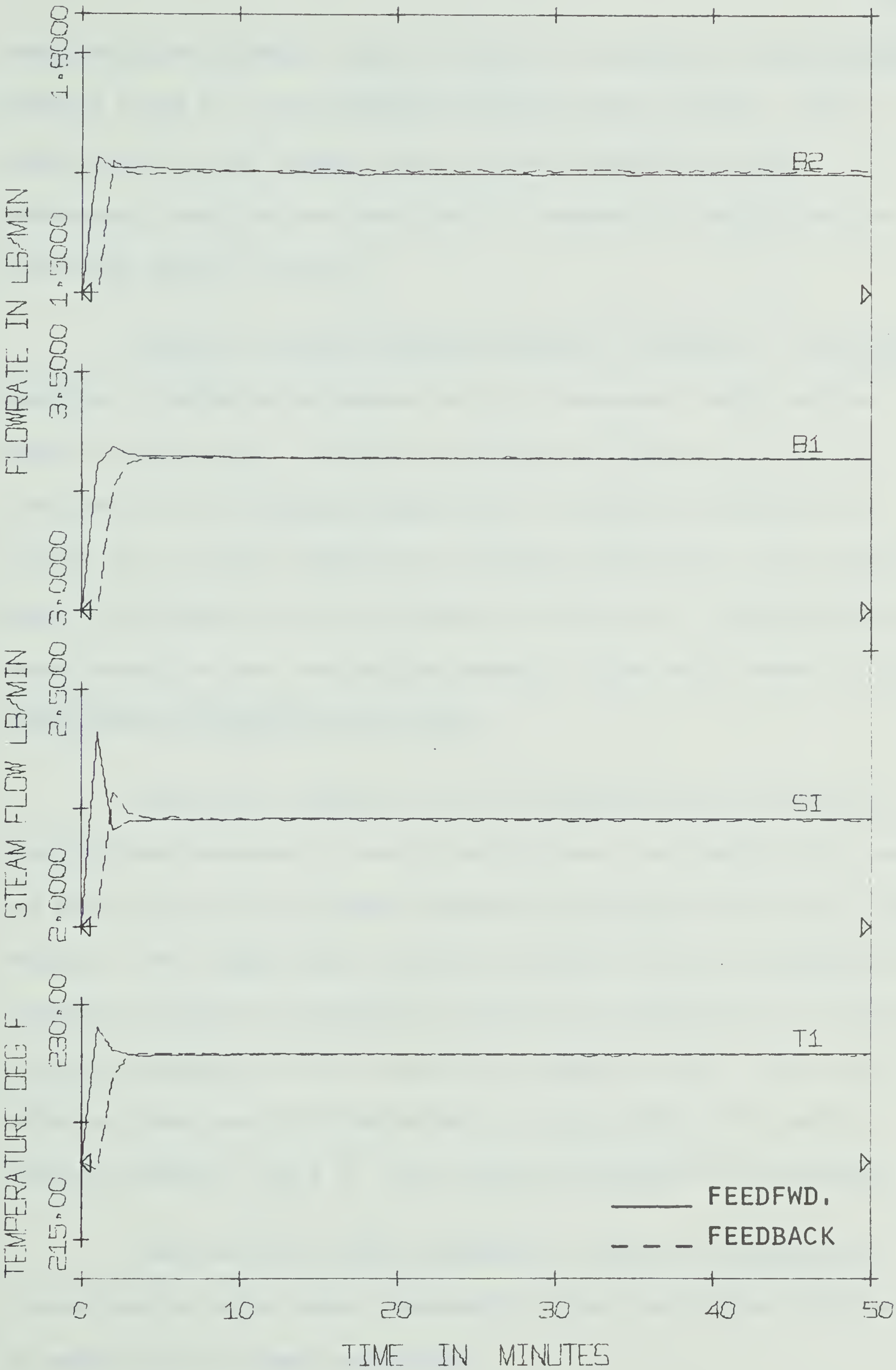


FIGURE 2b. SIMULATED FEEDFORWARD VS FEEDBACK CONTROL
(5L/+10%F/FB, FF/Q1/R1/D1)

feedforward and feedback (case d) control is compared to multivariable feedback alone for a ten percent increase in feed flowrate. The oscillations in W2 result from simulated roundoff in DDC measurement loops and the large scale. Comparative results for the four cases appear in Table 2.

In all cases the response exhibited "overshoot" in the initial transient. Reduction or removal of this "overshoot" would require dynamic feedforward. But with the overshoots being small for the fifty percent step changes imposed here, it would be difficult to justify the increased complexity of dynamic design for a real system where disturbances of such magnitude are not usual. The steady state design technique gave the smallest average offset while those for the minimization techniques were larger.

Practically speaking all the feedforward cases resulted in a significant increase in control quality over the feedback only case and there was little to choose between on the basis of the four cases examined. The steady state technique involved the least computation although the dynamic programming design can be carried out in conjunction with the evaluation of the proportional feedback matrix. The latter technique does have the disadvantage of being affected by control variable weighting ($\underline{\underline{R}} \neq 0$) while the other methods do not involve $\underline{\underline{R}}$.

The simulation study presented in Chapter 10 which uses a distillation column model developed by Rafal and Stevens [10] led to essentially the same conclusions.

TABLE 1

CONTROL MATRICES

$$\underline{\underline{K}}_{FB} = \begin{bmatrix} 5.095 & -1.475 & -2.68 & 0 & -14.56 \\ 3.95 & .36 & .21 & 0 & 7.39 \\ 5.31 & 1.19 & -\ .11 & 15.83 & 18.81 \end{bmatrix}$$

Case (a):

$$\underline{\underline{K}}_{FF} = \begin{bmatrix} 2.055 & -.090 & -.463 \\ 1.014 & .032 & 0 \\ 1.122 & .091 & 0 \end{bmatrix}$$

Case (b):

$$\underline{\underline{K}}_{FF} = \begin{bmatrix} 2.050 & -.088 & -.463 \\ 1.014 & .031 & 0 \\ 1.122 & .089 & 0 \end{bmatrix}$$

Case (c):

$$\underline{\underline{K}}_{FF} = \begin{bmatrix} 2.045 & -.114 & -.463 \\ 1.018 & .043 & 0 \\ 1.134 & .119 & 0 \end{bmatrix}$$

Case (d):

$$\underline{\underline{K}}_{FF} = \begin{bmatrix} 2.047 & -.136 & -.463 \\ 1.019 & .037 & 0 \\ 1.135 & .116 & 0 \end{bmatrix}$$

TABLE 2

COMPARISON OF FEEDFORWARD RESULTS

(50% Step in Feed Flowrate)

Case Run Number	W1 Offset	$W1_{max}$	ISE_N	W2 Offset	$W2_{max}$	ISE_N	C2 Offset	$C2_{max}$	ISE_N
Feedback 11510	15.5	15.5	1.0	.11	.25	1.4	-1.45	-1.45	.99
(a) 22010	.0002	1.22	10^6	.0009	.002	1.8	- .0002	- .38	10^5
(b) 24010	.015	1.22	283.	.0069	.014	1.8	- .0093	- .38	62.
(c) 23010	.00003	1.19	10^8	.00001	.00002	1.8	- .031	- .40	8.8
(d) 21010	.014	1.19	280.	10^{-5}	10^{-5}	1.8	- .028	- .40	10.

Notes 1. Offsets and maximum values are in percent of steady state.

2. ISE_N is an approximate measure of overshoot past the offset.

7. IMPLEMENTATION

The multivariable control system was implemented with an IBM 1800 digital control computer which is interfaced with the pilot plant double effect evaporator. The process runs under Direct Digital Control (DDC) under a time-sharing executive system which permits simultaneous execution of off-line jobs such as the plotting of the figures for this paper.

Multivariable control calculations are carried out by a queued process coreload written in Fortran which executes every control interval. System time for the coreload varies from two to five seconds in every 64 second control interval, depending on the disk operations. Actual CPU time would be considerably less but was not readily available. The computation time for the multivariable control algorithm is essentially fifteen multiplications compared with three for conventional DDC. In addition there are some extra calculations required for state estimation and model calculations. The program obtains state variable measurements from DDC data acquisiton loops and makes control variable changes by adjusting the setpoints of DDC flow control loops.

Details of the implementation and some of the design considerations are available in Chapter 9 and a program users manual [11].

8. EXPERIMENTAL RESULTS

The offsets which result from load changes when multivariable proportional feedback control is implemented are exemplified by the first effect level (W1) in Figure 3. The two twenty percent feed flowrate changes, an increase and then a decrease, also left offsets in product concentration (C2) and second effect level (W2) although in this case they were not significant.

The addition of feedforward control was found to eliminate significant offsets due to load changes in feed flowrate (Figure 4), feed concentration (Figure 5), and feed temperature (Figure 6). The feedforward control matrix was evaluated by the dynamic programming algorithm. The increased noise in first effect level and hence the control variables in Figures 4 to 8 was a result of a noisy differential pressure transmitter. This noise produced a band on the recorder of about two percent of the span (about 0.6 inches of level). Although due to an instrument fault the results are a good illustration of the effect on the manipulated variables of high gain control applied to noisy measurements.

Feedforward control matrices calculated by the steady state technique and by minimization of offsets were also evaluated for a twenty percent feed flowrate change (Figures 7 and 8 respectively). No significant difference in the performance of the three control matrices was apparent in the experimental results.

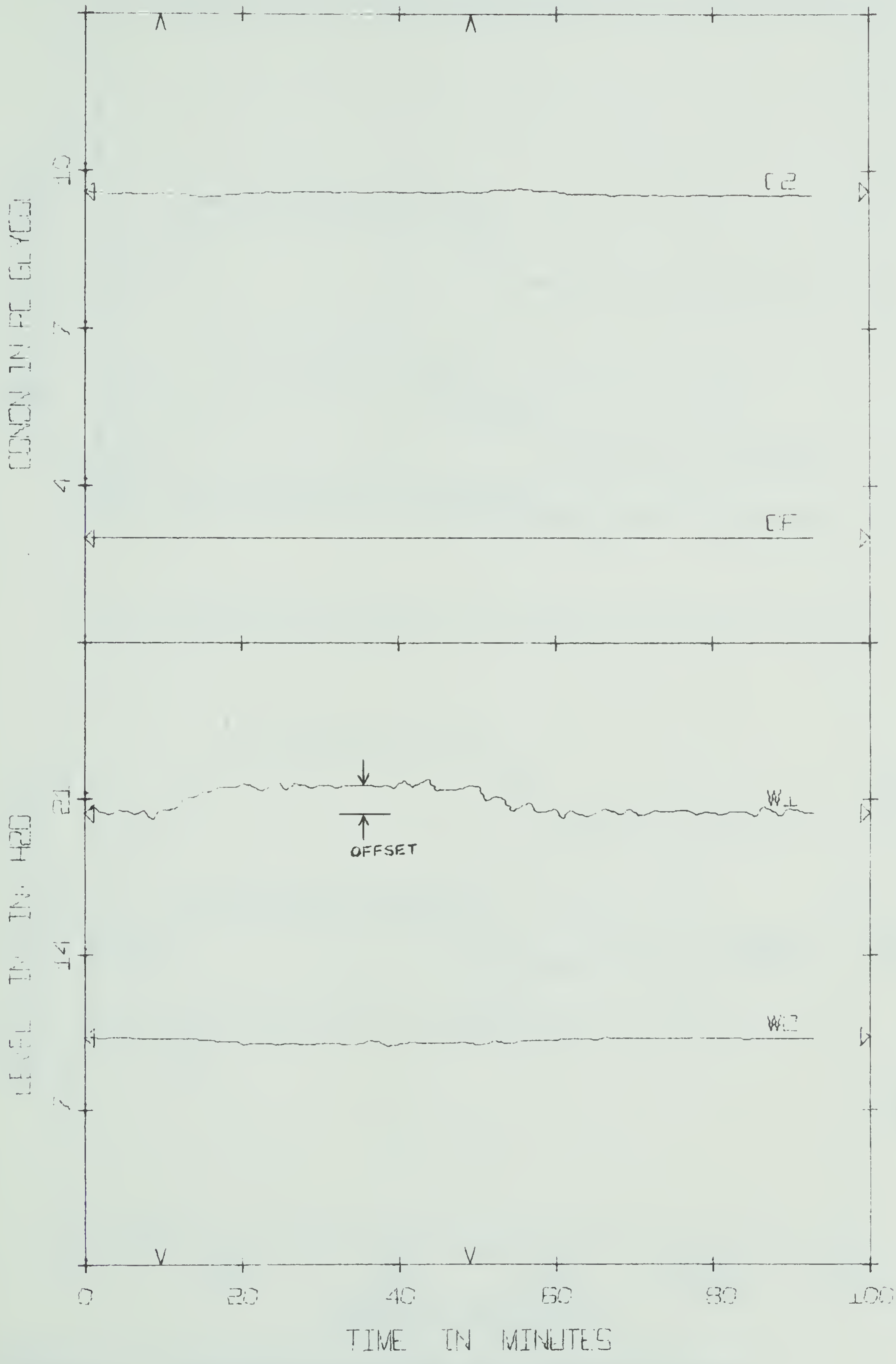


FIGURE 3a. EXPERIMENTAL MULTIVARIABLE FEEDBACK CONTROL
(5L/20%F/FB/Q1/R1/D1/A1/MVC41)

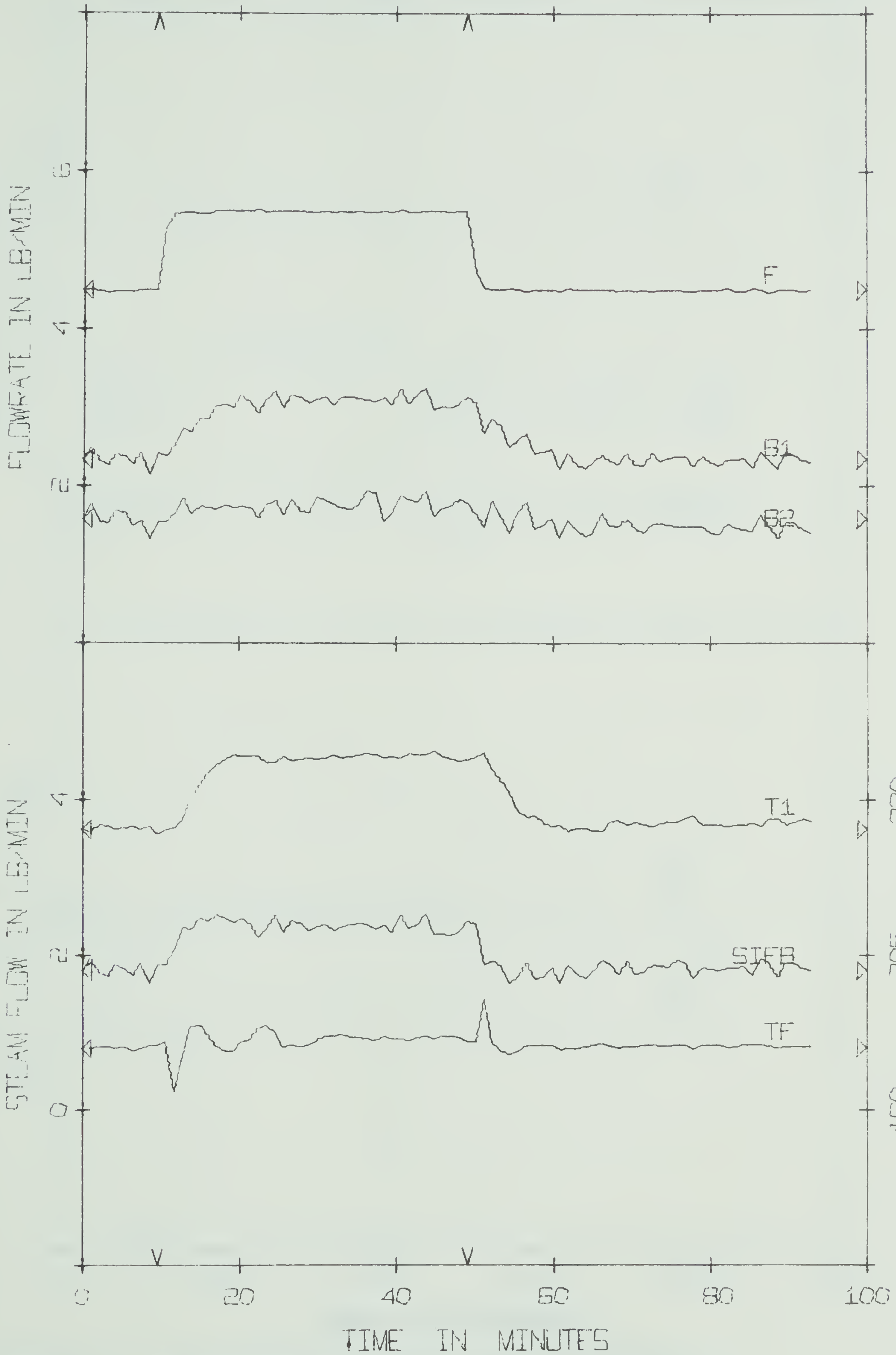


FIGURE 3b. EXPERIMENTAL MULTIVARIABLE FEEDBACK CONTROL
(5L/20%F/FB/Q1/R1/D1/A1/MVC41)

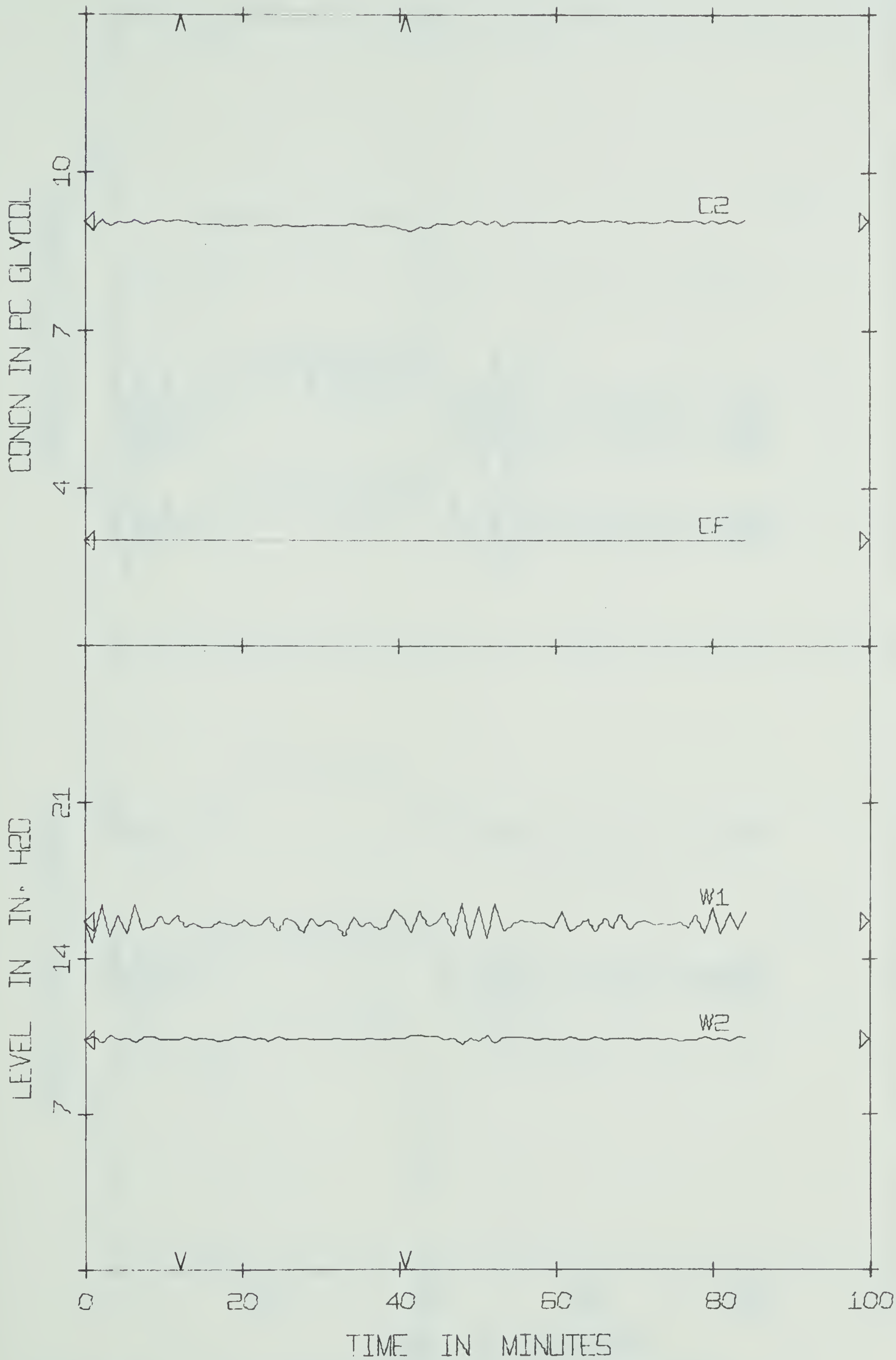


FIGURE 4a. EXPERIMENTAL FEEDFORWARD-QUADRATIC INDEX- I (5L/20%F/FF-QI/Q1/R1/D1/A1/MVC61)

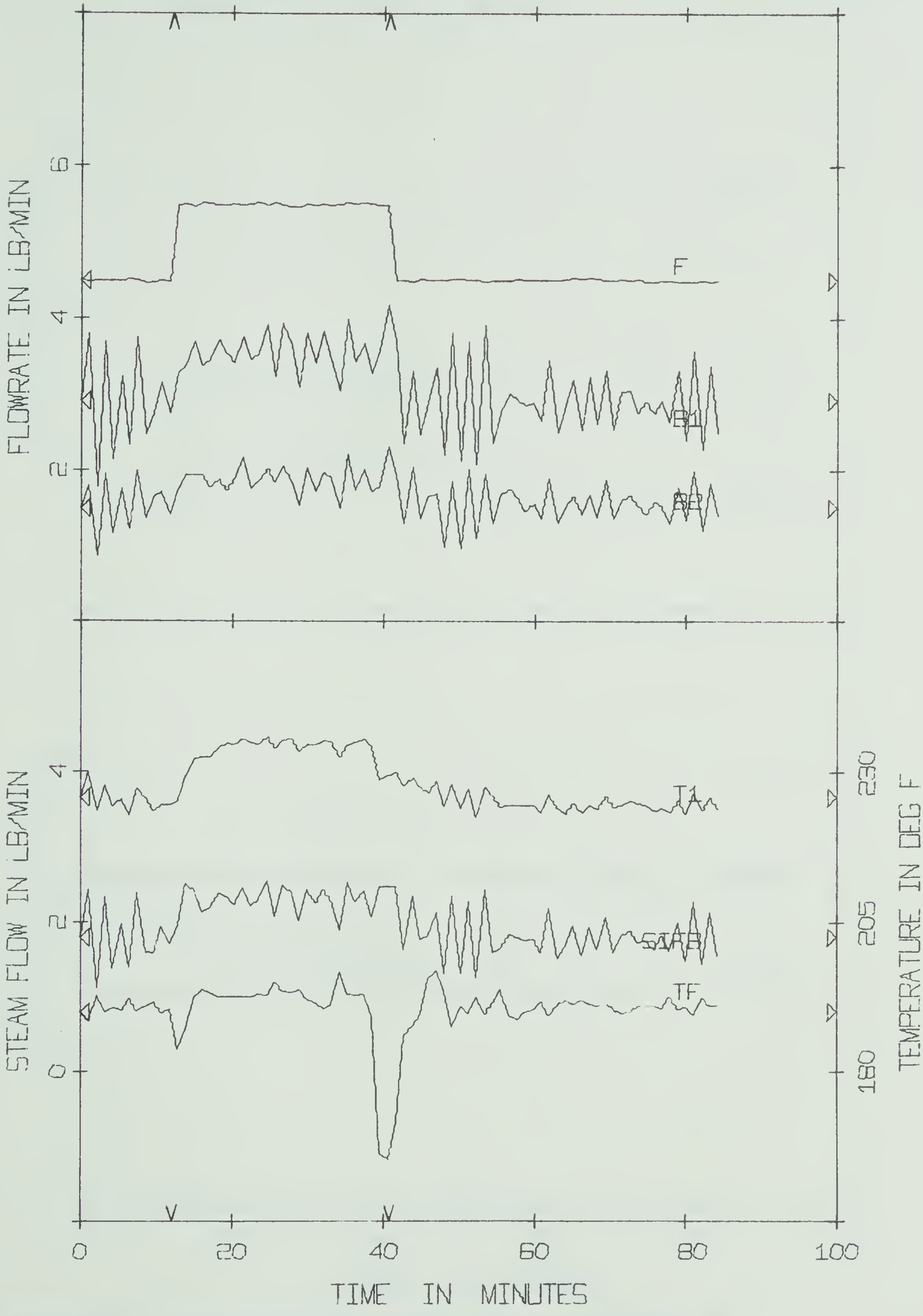


FIGURE 4b. EXPERIMENTAL FEEDFORWARD-QUADRATIC INDEX- I (5L/20%F/FF-QI/Q1/R1/D1/A1/MVC61)

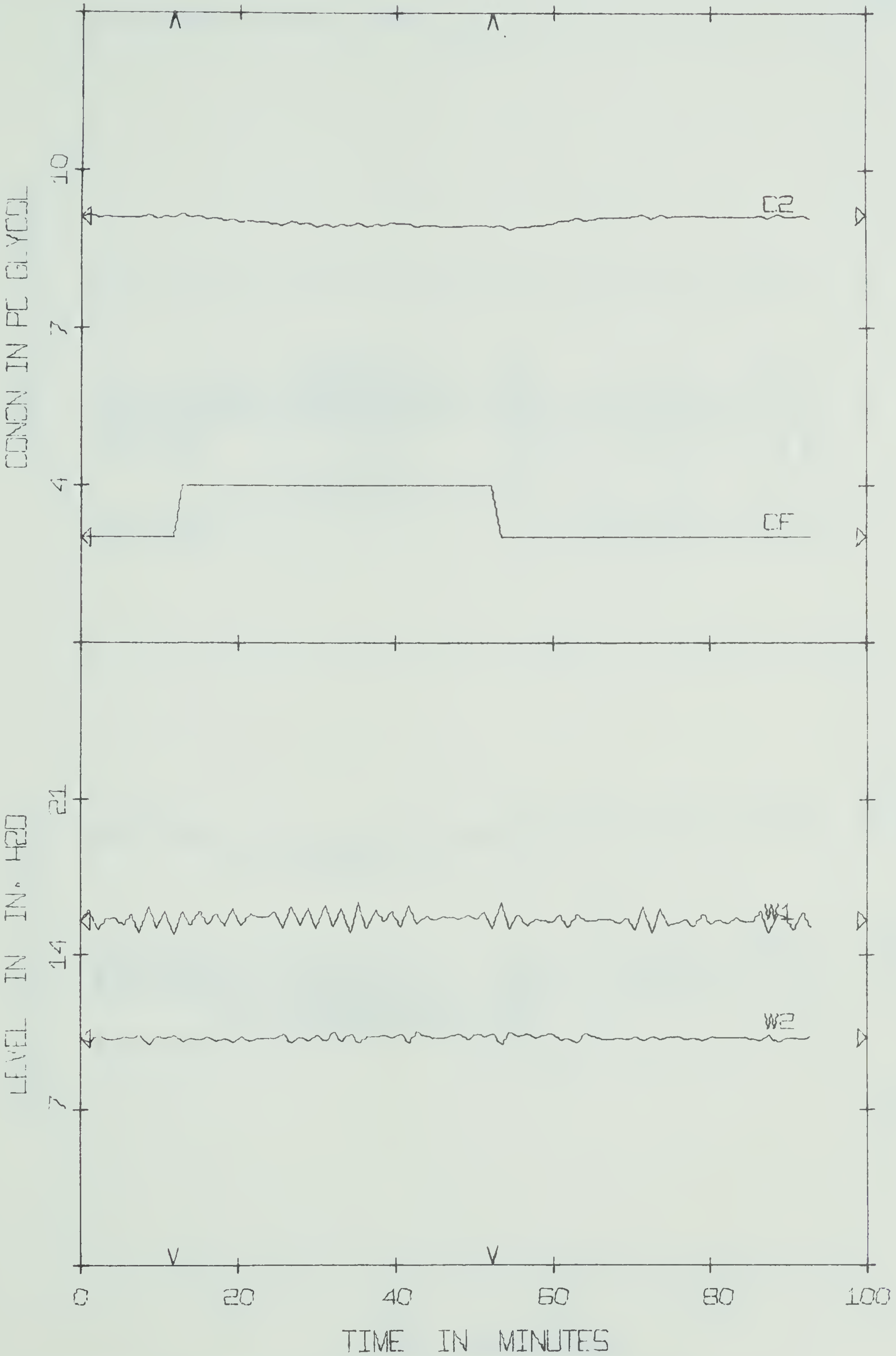


FIGURE 5a. EXPERIMENTAL FEEDFORWARD-QUADRATIC INDEX- II (5L/29%CF/FF-QI/Q1/R1/D1/A1/MVC64)

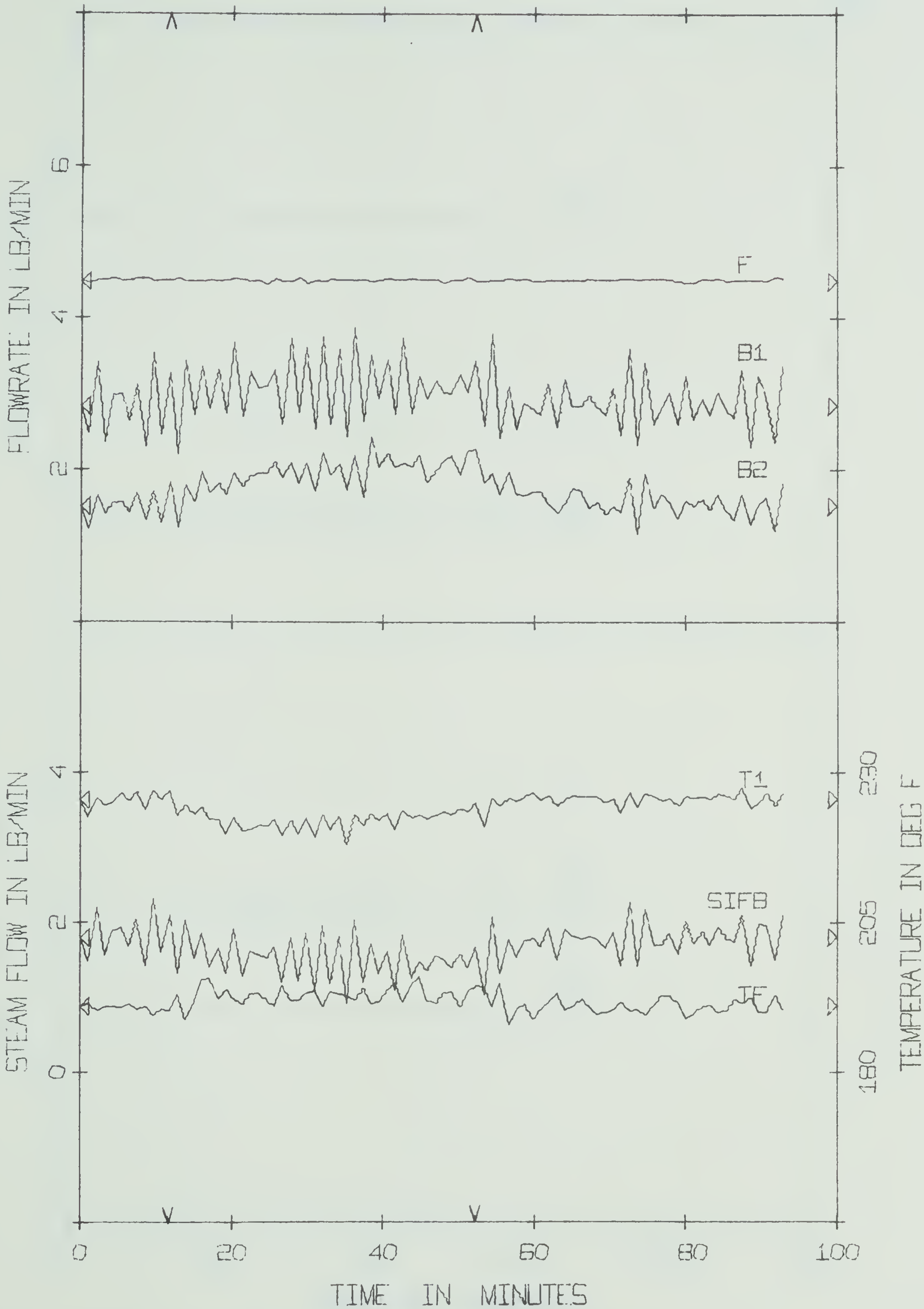


FIGURE 5b. EXPERIMENTAL FEEDFORWARD-QUADRATIC INDEX- II
(5L/29%CF/FF-QI/Q1/R1/D1/A1/MVC64)

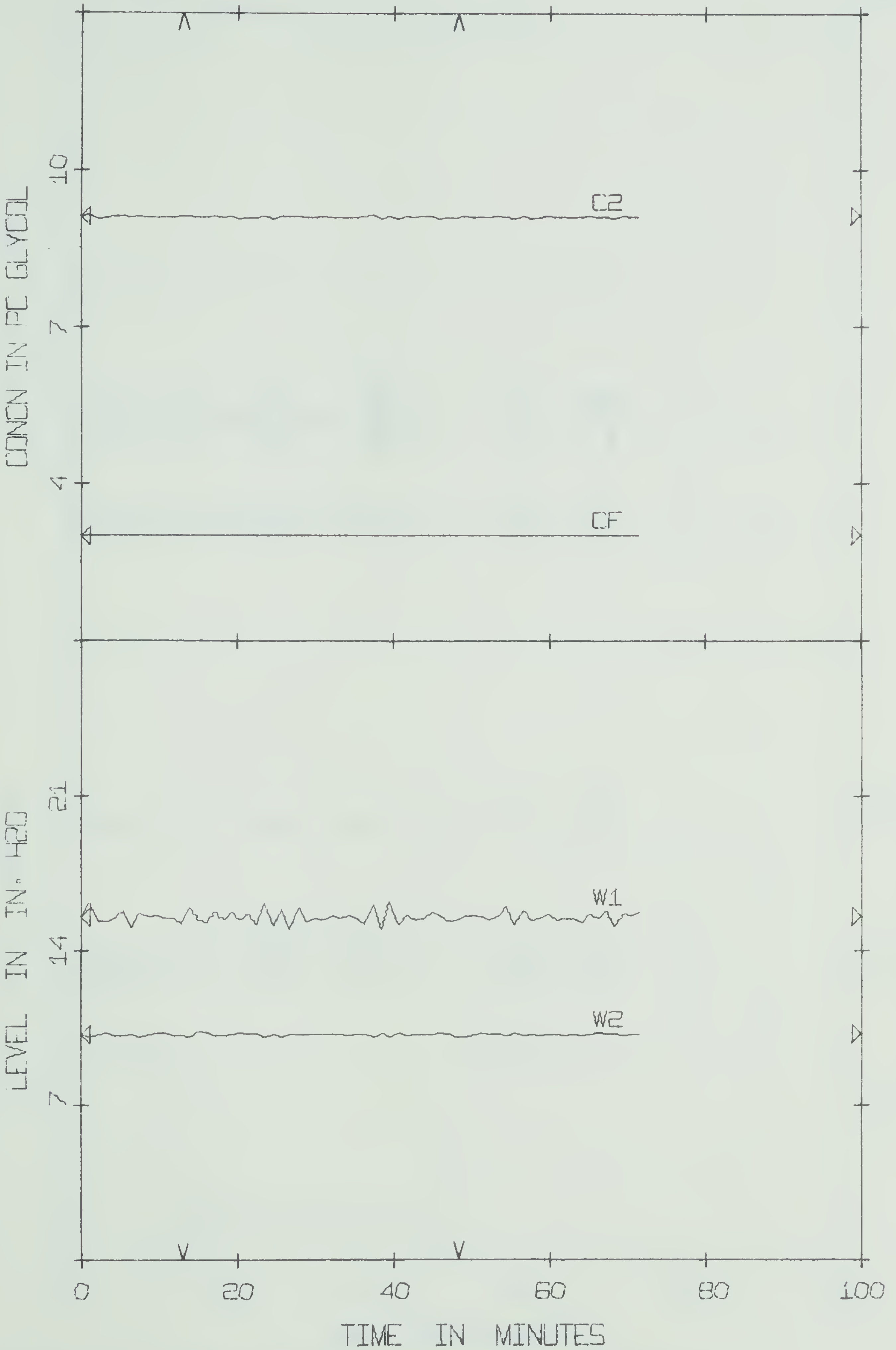


FIGURE 6a. EXPERIMENTAL FEEDFORWARD-QUADRATIC INDEX- III
(5L/16%TF/FF-QI/Q1/R1/D1/A1/MVC65)

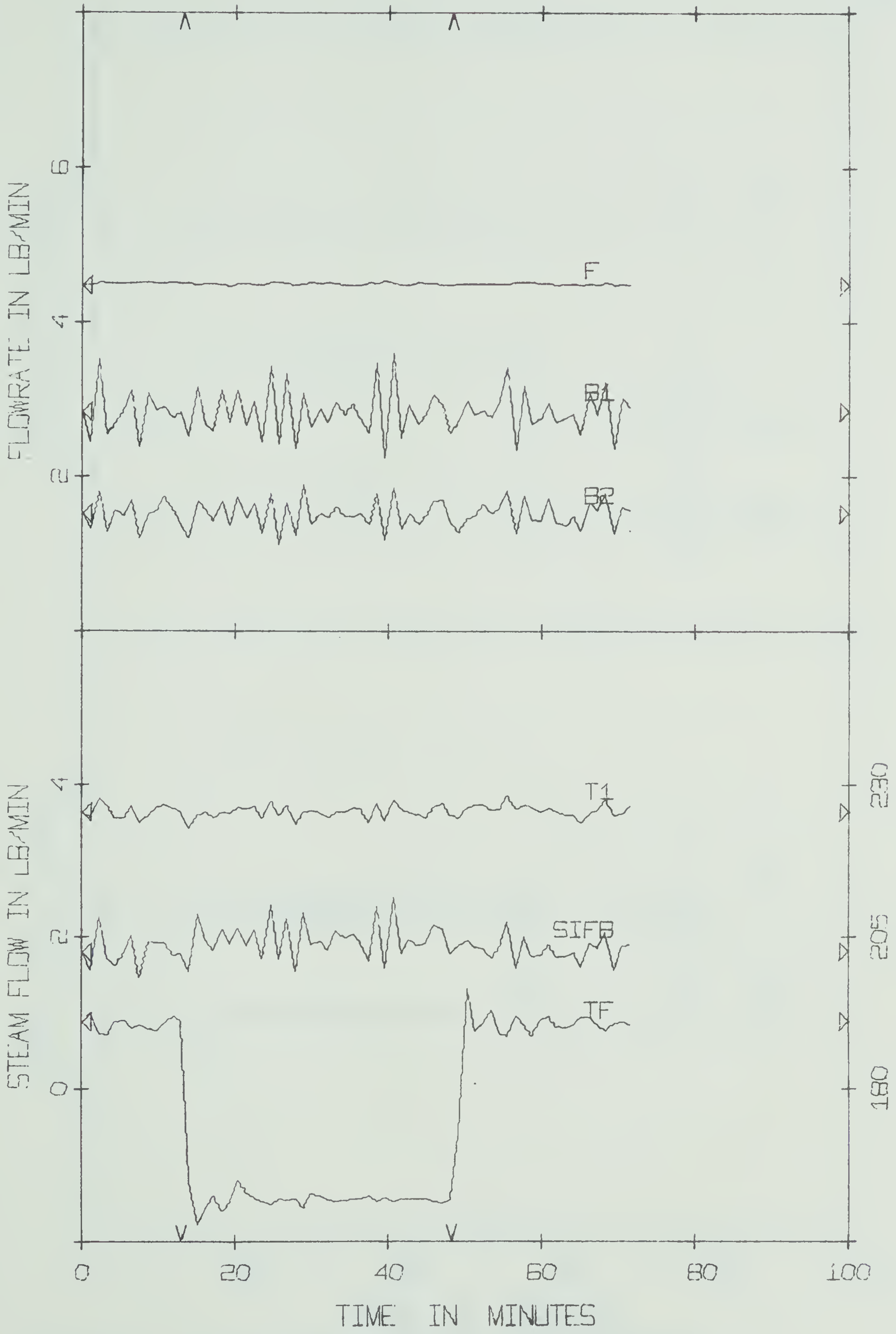


FIGURE 6b. EXPERIMENTAL FEEDFORWARD-QUADRATIC INDEX- III
(5L/16%TF/FF-QI/Q1/R1/D1/A1/MVC65)

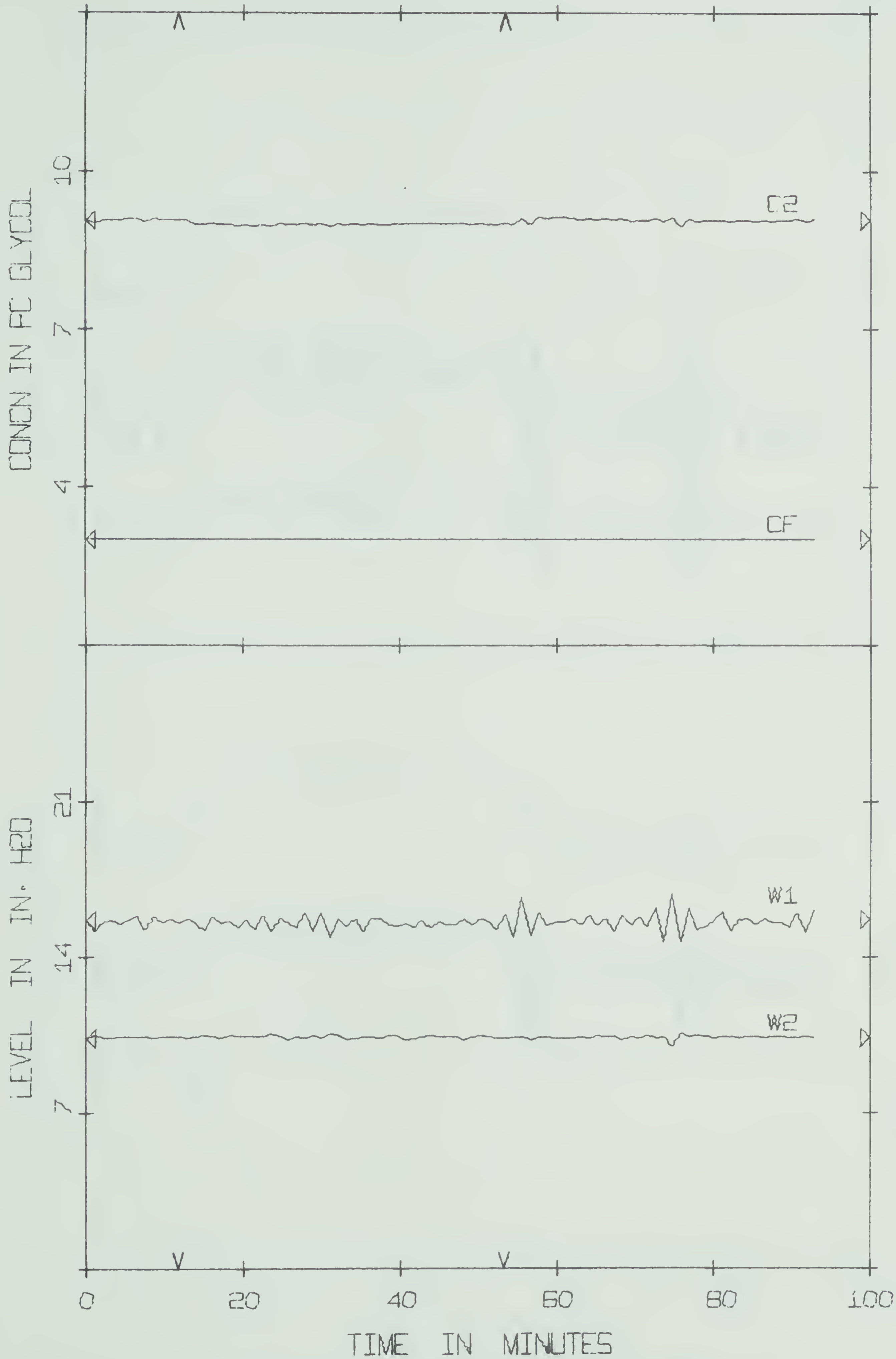
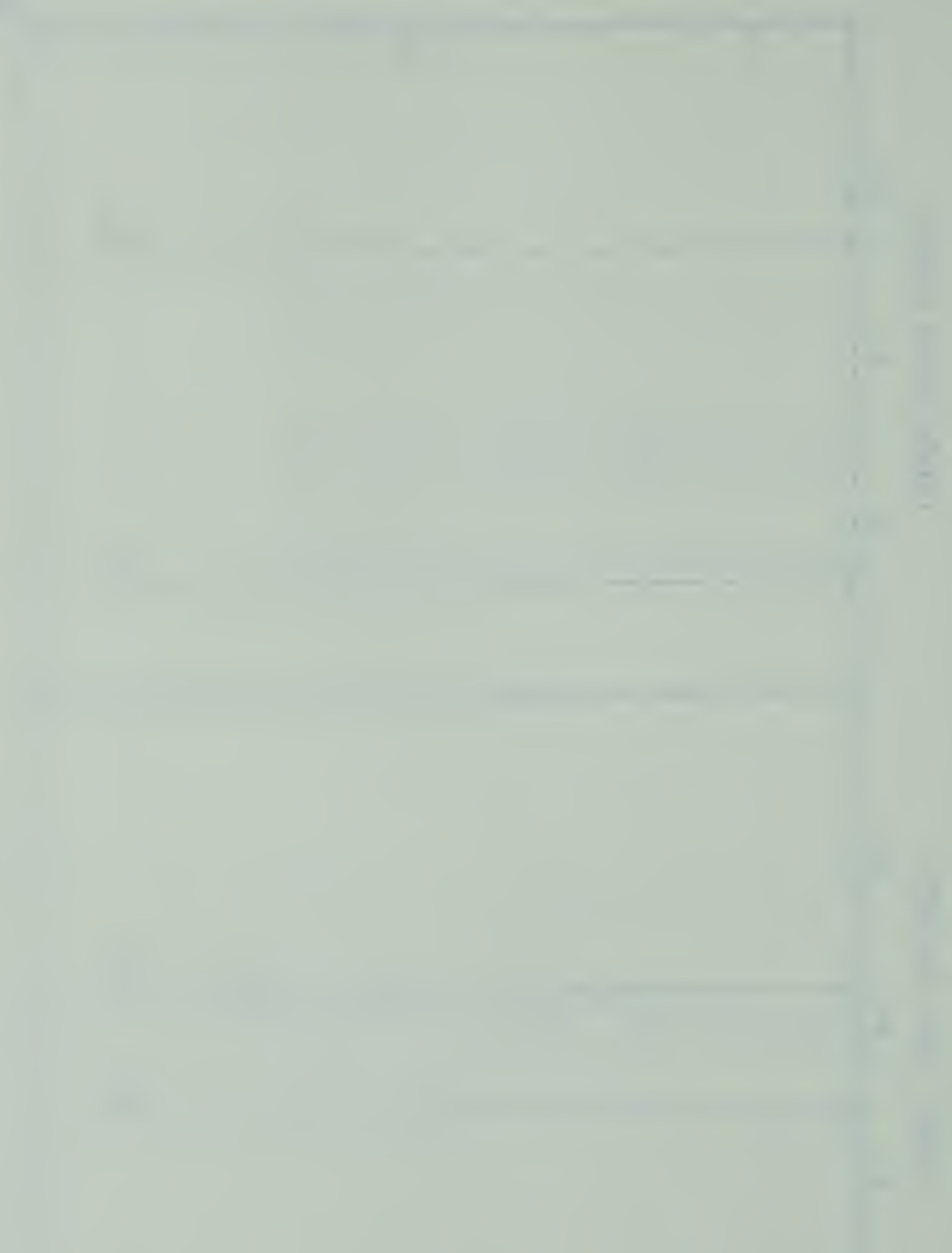


FIGURE 7a. EXPERIMENTAL FEEDFORWARD TO ZERO OFFSETS (5L/20%F/FF-ZO/Q1/R1/D1/A1/MVC62)



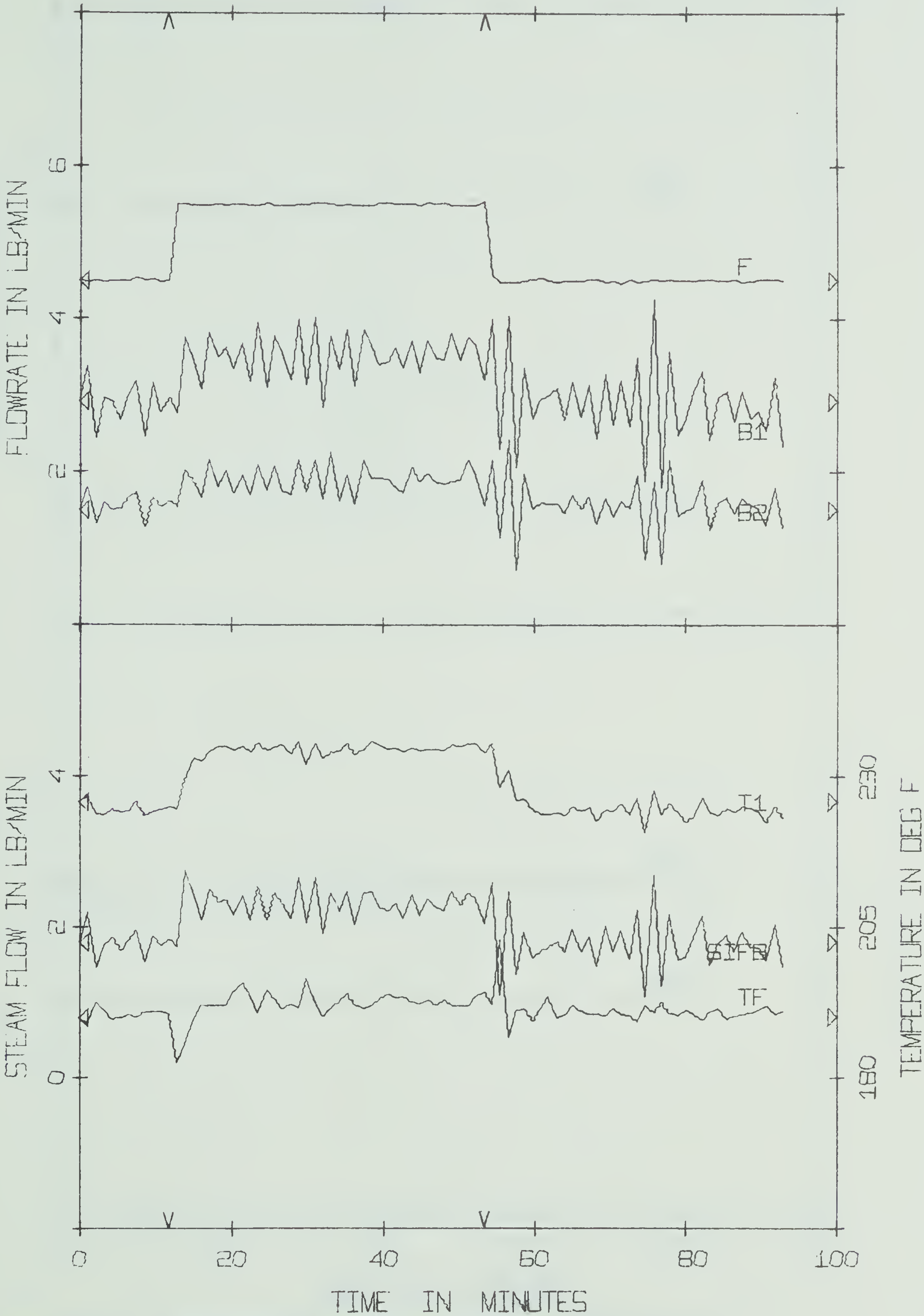
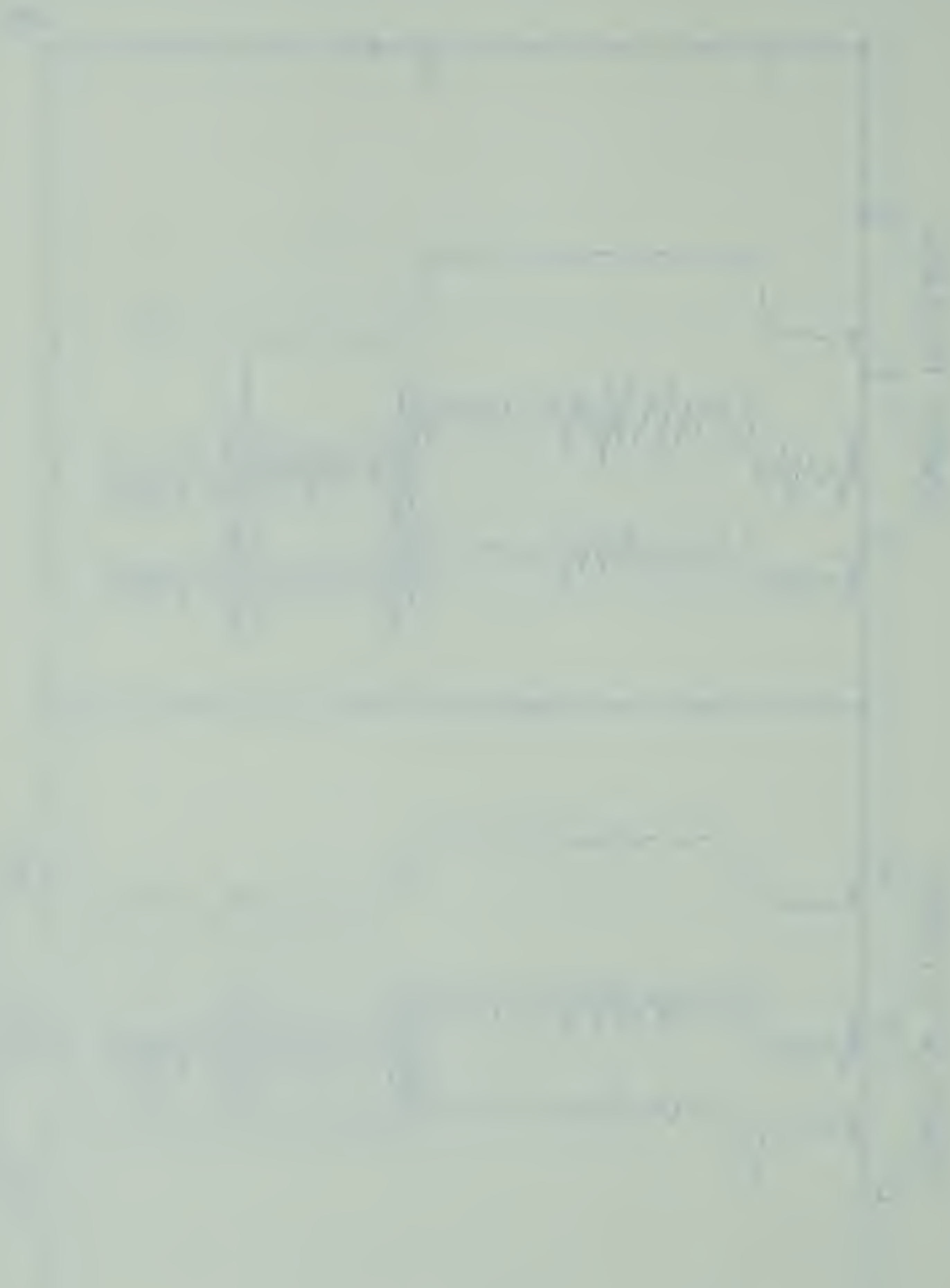


FIGURE 7b. EXPERIMENTAL FEEDFORWARD TO ZERO OFFSETS (5L/20%F/FF-ZO/Q1/R1/D1/A1/MVC62)



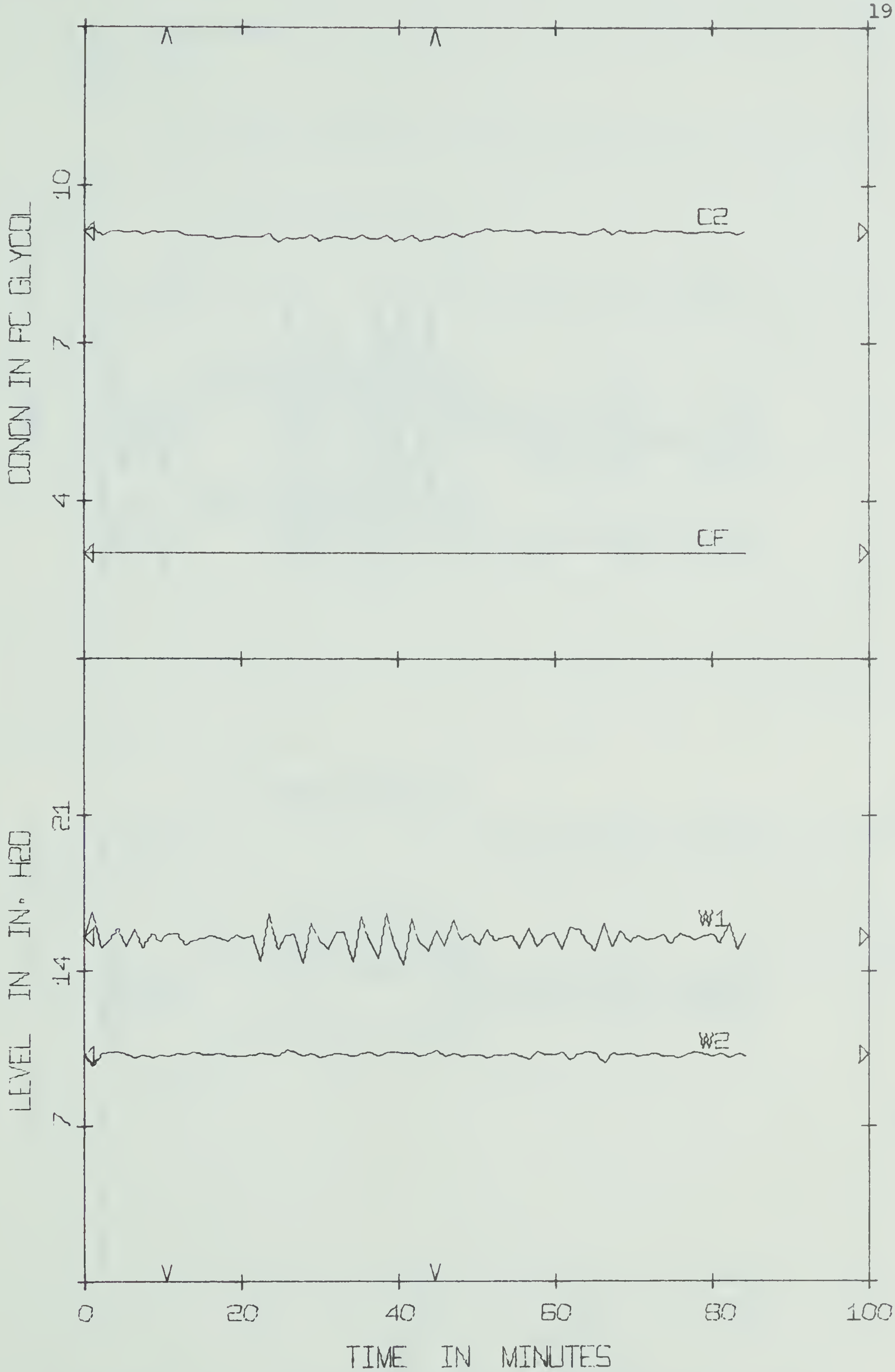


FIGURE 8a. EXPERIMENTAL FEEDFORWARD TO MINIMIZE OFFSETS
(5L/20%F/FF-MO/Q1/R1/D1/A1/MVC63)

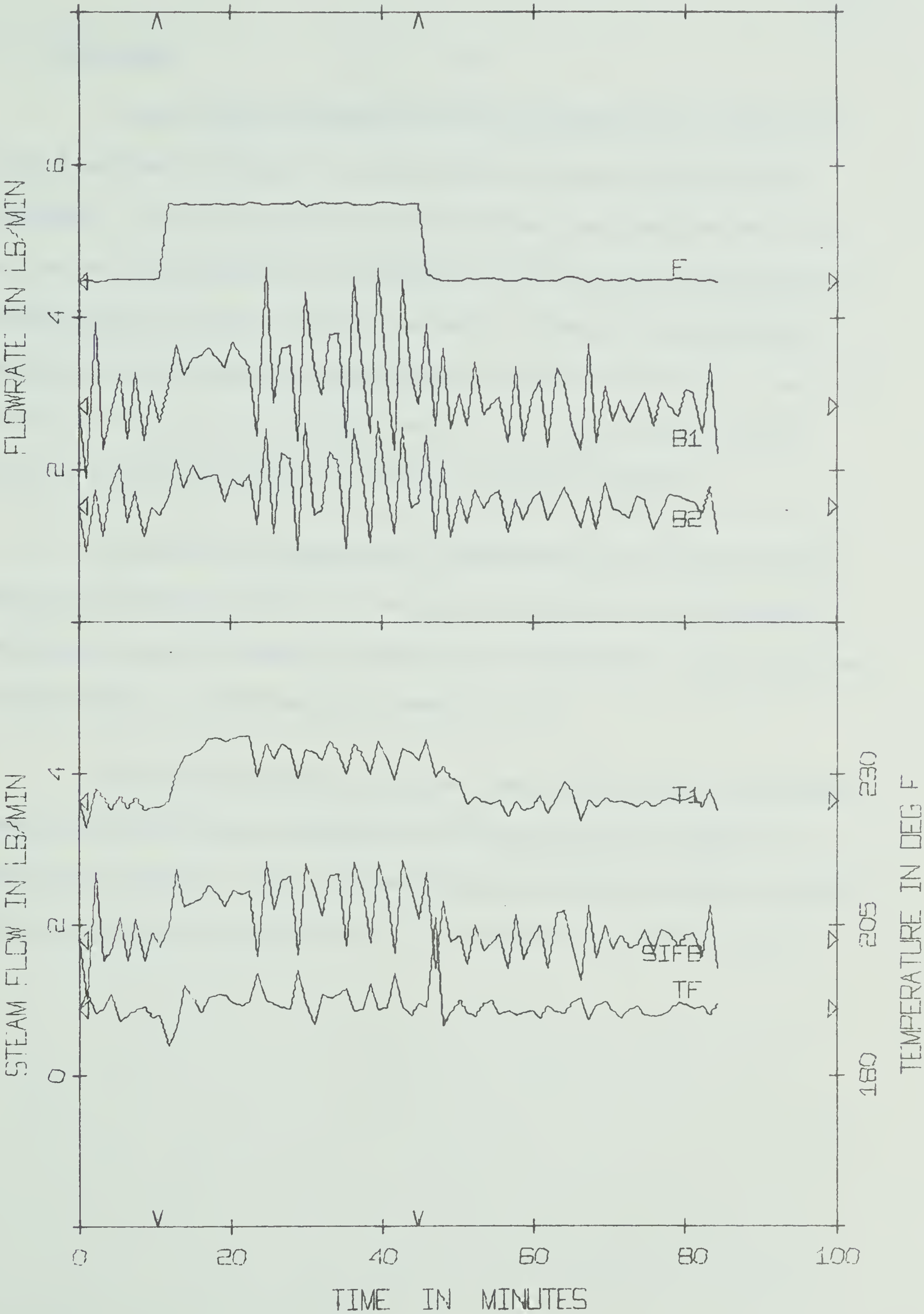


FIGURE 8b. EXPERIMENTAL FEEDFORWARD TO MINIMIZE OFFSETS
(5L/20%F/FF-MO/Q1/R1/D1/A1/MVC63)

9. CONCLUSIONS

A generalized formulation of the multivariable feedforward control problem clarified the relationship between several design approaches. Deviations from the steady state were smallest when the problem was formulated for the removal of offsets and solved using the steady state or error coordinate approach. However the deviations resulting from formulations minimizing the offsets or using a quadratic criterion were not significantly larger. The latter design method could be adversely affected by control variable weighting.

Simulated and experimental comparisons of four formulations using the different methods of designing proportional multivariable feedforward control showed no significant differences. All results were an improvement on multivariable feedback alone.

Experimental tests also showed that multivariable feedforward control could be easily implemented by digital control computer in practical situations. The implementation successfully handled problems such as process noise with little or no decrease in control quality.

CHAPTER SEVEN

MULTIVARIABLE INTEGRAL CONTROL

ABSTRACT

An optimal, multivariable control system was implemented on a computer controlled pilot plant evaporator and produced significantly better results than conventional methods.

A general form of the integral quadratic index was used as a measure of performance and the optimization problem was formulated such that dynamic programming techniques generated a proportional-plus-integral control law. The dynamic behavior of the process was represented by a linear, time-invariant, fifth-order state variable model.

Simulated control runs were carried out to demonstrate the effect of weighting the "integral states" and to illustrate the results when more integral control states are added than are permitted by the allowable degrees of freedom.

Experimental results were obtained under both the multivariable system and a typical industrial multiloop system. The response of the pilot plant evaporator to sizeable load changes under multivariable control showed a significant improvement. It was of particular interest that excellent control results were obtained on the basis of a process model that was only moderately successful in predicting open loop transients.

1. INTRODUCTION

This paper considers the design and implementation of an optimal multivariable control system. The optimization is based on a quadratic performance index and formulated such that the application of dynamic programming techniques generates a proportional-plus-integral control law. The process is assumed to be adequately represented by a linear, time-invariant state variable model.

The algebraic recursive relationships developed from the dynamic programming formulation were used to generate the optimal control law for a double effect evaporator represented by a fifth-order model. The results from several digital simulation runs are included to illustrate the effect of parameters such as the weighting matrix in the quadratic criterion and the control interval.

Experimental data from the pilot plant evaporator unit are used to show the improved performance produced by the optimal control law over a conventional multiloop DDC system.

2. LITERATURE SURVEY

Several design methods for multivariable regulatory control exist [1,2] but many result in complex control laws and require the real-time solution of systems of nonlinear differential equations. Dynamic programming has been found to generate proportional state feedback control [1] when the discrete time optimization problem is formulated with a summed quadratic error criterion. This configuration is well suited to practical implemen-

tation. Both Nicholson [3] and Anderson [4] used this approach to determine the optimal regulatory control of simulated steam boilers. Nicholson presents simulated comparisons with the "single step" optimal control formulation and with conventional Direct Digital Control (DDC). He found the control from the results of the multi-step formulation superior but did not consider that the additional off-line computation was justified by the small improvement over control resulting from a "single step" formulation. Noton and Choquette [5] reported on a project underway in which a dynamic programming control law will be applied to an industrial system, basically for state driving and reactor startup. A Kalman filter was used to identify parameters for the model as well as for state estimation.

Multivariable proportional-plus-integral control has not received wide attention in the literature although the integral action would remove some offsets with little increase in the complexity of the implementation. Smith and Murrill [6] transformed the model equations and defined an integral controller through a solution of the Riccati equation. They presented no results and their transformation necessitated the assumption of step load changes. Moore [7] used a formulation necessitating a term in the criterion involving the control vector derivative. He proposed the use of a least squares technique for systems with more states than control variables. This would jeopardize the removal of offset. Shih [8] formulated optimal integral control for a single control variable and output. This paper extends the formulation

to the multivariable case.

The weighting matrices in the quadratic criterion are an important design consideration and have been considered by several authors for the optimal proportional control formulation. Nicholson mentioned his trials but drew no general conclusions. Tyler and Tuteur [9] defined a complex procedure for determining the system poles from the elements of diagonal weighting matrices which would enable a trial and error design for specified root loci. Chant and Luus [10] used a "brute force" optimization technique requiring iterative designs and simulations. However there do not appear to be any *a priori* guides other than "experience". The criterion for comparing different weighting matrices opens a whole new area.

3. DESIGN PROCEDURE

3.1. Problem Formulation

The process plant model is assumed to be of the form,

$$\underline{\dot{x}} = \underline{\underline{A}}_c \underline{x} + \underline{\underline{B}}_c \underline{u} + \underline{\underline{D}}_c \underline{d} \tag{1}$$

where $\underline{x}$, $\underline{u}$, and $\underline{d}$ are the state (dimension n), control (dimension m), and load (dimension p) vectors respectively. In this work it is assumed that all the variables are in a normalized perturbation form. $\underline{\underline{A}}_c$, $\underline{\underline{B}}_c$, and $\underline{\underline{D}}_c$ are the constant coefficient matrices for a linear time-invariant process.

An "integral" state vector is defined by

$$\underline{z} = \int \underline{y} dt \tag{2}$$

where

$$\underline{y} = \underline{\underline{C}}\,\underline{x} \tag{3}$$

with $\underline{y}$ an output vector of dimension q .

By augmenting the process model (Equation (1)) with the differentiated form of Equation (2) the following relation is obtained (which is identical in form to the original process model).

$$\begin{bmatrix} \dot{\underline{x}} \\ \dot{\underline{z}} \end{bmatrix} = \begin{bmatrix} \underline{\underline{A}}_c & \underline{\underline{0}} \\ \underline{\underline{C}} & \underline{\underline{0}} \end{bmatrix} \begin{bmatrix} \underline{x} \\ \underline{z} \end{bmatrix} + \begin{bmatrix} \underline{\underline{B}}_c \\ \underline{\underline{0}} \end{bmatrix} \underline{u} + \begin{bmatrix} \underline{\underline{D}}_c \\ \underline{\underline{0}} \end{bmatrix} \underline{d} \tag{4}$$

The solution to this augmented system, neglecting the disturbance term can be derived in the following discrete form:

$$\underline{x}'_{k+1} = \underline{\underline{A}}\,\underline{x}'_k + \underline{\underline{B}}\,\underline{u}_k \tag{5}$$

where k indicates the control interval, $\underline{x}'^T = [\underline{x}^T, \underline{z}^T]$, and $\underline{\underline{A}}$ and $\underline{\underline{B}}$ are constant coefficient matrices.

3.2. Design Algorithm

The optimal control problem has been formulated with a discrete process model (Equation (5)) and the general discrete quadratic criterion.

$$\begin{aligned} J = {} & \beta^N \underline{x}'^T_N \underline{\underline{S}}\,\underline{X}'_N \\ & + \sum_{k=1}^{N} \beta^k [\underline{x}'^T_K \underline{\underline{Q}}\,\underline{X}'_k \\ & \quad + \underline{u}^T_{k-1} \underline{\underline{R}}\,\underline{u}_{k-1}] \end{aligned} \tag{6}$$

where $\underline{\underline{S}}$, $\underline{\underline{Q}}$, and $\underline{\underline{R}}$ are weighting matrices of appropriate order and are usually positive definite and often diagonal. The scalar β is a time weighting parameter, greater than or equal to unity, which increases the weighting on state and control deviations with time.

Application of dynamic programming to the problem produces a solution of the form

$$\underline{u} = \underline{\underline{K}}\,\underline{x}' \tag{7}$$

which transforms, using equation (2), to

$$\underline{u} = \underline{\underline{K}}_{FB}\underline{x} + \underline{\underline{K}}_{I} \int \underline{y}dt \ . \tag{8}$$

The control matrix, $\underline{\underline{K}} = [\underline{\underline{K}}_{FB}, \underline{\underline{K}}_{I}]$, results from the convergence of the following recurrence relations.

$$\underline{\underline{P}}_{o} = \underline{\underline{Q}} + \underline{\underline{S}} \tag{9}$$

and

$$\underline{\underline{K}}_{N-i} = -(\underline{\underline{B}}^{T}\underline{\underline{P}}_{i}\underline{\underline{B}} + \underline{\underline{R}})^{-1}\underline{\underline{B}}^{T}\underline{\underline{P}}_{i}\underline{\underline{A}} \tag{10}$$

$$\underline{\underline{S}}'_{N-i} = \underline{\underline{A}} + \underline{\underline{B}}\,\underline{\underline{K}}_{N-i} \tag{11}$$

$$\underline{\underline{P}}_{i+1} = \beta(\underline{\underline{S}}'^{T}_{N-i}\underline{\underline{P}}_{i}\underline{\underline{S}}'_{N-i} + \underline{\underline{K}}^{T}_{N-i}\underline{\underline{R}}\,\underline{\underline{K}}_{N-i}) + \underline{\underline{Q}} \tag{12}$$

for $i = 0,1,2,\ldots$.

3.3 "Integral" States

If the design were carried out with more "integral" states then control variables, i.e., $q > m$, it would be found

that offsets of the states would vary but not be removed. The error in this approach can be shown by the following analysis.

If q "integral" states are introduced it is expected that q of the n states will be zero at steady state, that is from equation (1) there are n equations of the form

$$\dot{x}_i = \underline{a}_i^T \underline{x} + \underline{b}_i^T \underline{u} + \underline{d}_i^T \underline{d} = 0 \tag{13}$$

where q elements of $\underline{x}$ are zero, the remaining $(n - q)$ elements of $\underline{x}$ and m elements of $\underline{u}$ are unknowns, and the p elements of $\underline{d}$ are constants. Hence there are n equations in $(n - q + m)$ unknowns. For a unique solution q must be equal to m. If $m > q$ then $(m - q)$ elements of $\underline{u}$ can be specified and a unique solution follows. If $m < q$ there is no unique solution.

Hence the dimension of the control vector defines the "degrees of freedom" or the number of states, or functions of the state, defined by equation (3) that can be driven to zero at steady state.

4. SIMULATION STUDY

4.1. Process and Model

The regulatory control schemes described in this paper were applied to a pilot plant double effect evaporator at the University of Alberta. The unit operates with a nominal feedrate of 5.0 lb./min. of 3 percent aqueous triethylene glycol and produces a product of about 10 percent. A schematic diagram of the evaporator connected for

operation in a double effect, "forward feed" mode of operation is shown in Figure 1. The first effect is a calandria type unit with an 8 inch diameter tube bundle. The second effect is a long tube vertical unit with three 1" × 6' tubes and is operated with externally forced circulation. The second effect is operated under vacuum and utilizes the vapour from the first effect as a heating medium.

The nonlinear evaporator model was linearized and the variables transformed to a normalized perturbation form about a steady state operating set of conditions. The model equations are in the form of equation (1) and are presented in full in equation (14). The state, control, and load vectors are defined in terms of the process variables in the Nomenclature.

4.2. Design Parameters

The evaporator system has three "degrees of freedom" and the output vector was chosen as $\underline{y} = [W1, W2, C2]^T$, such that integral action will be applied to the two liquid holdups and the product concentration.

There were a number of other design parameters which required specification before the problem was solved.

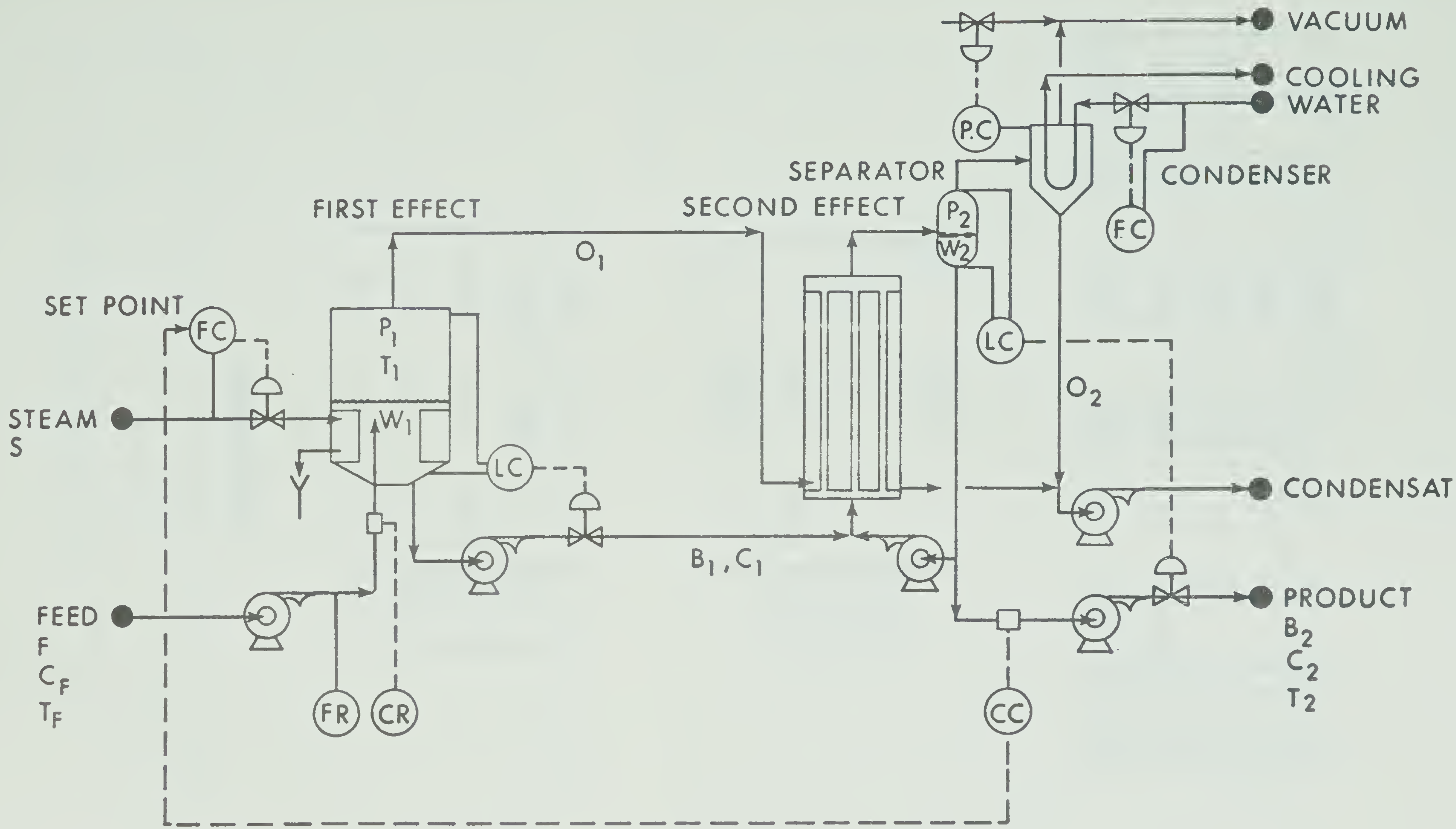


FIGURE 1. PILOT PLANT DOUBLE EFFECT EVAPORATOR

$$\begin{bmatrix} \dot{W}1 \\ \dot{C}1 \\ \dot{H}1 \\ \dot{W}2 \\ \dot{C}2 \end{bmatrix} = \begin{bmatrix} 0 & -.00156 & -.1711 & 0 & 0 \\ 0 & -.1419 & .1711 & 0 & 0 \\ 0 & -.00874 & -1.102 & 0 & 0 \\ 0 & -.00128 & -.1489 & 0 & .000126 \\ 0 & .0605 & .1489 & 0 & -.05911 \end{bmatrix} \begin{bmatrix} W1 \\ C1 \\ H1 \\ W2 \\ C2 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & -.1434 & 0 \\ 0 & 0 & 0 \\ .3921 & 0 & 0 \\ 0 & .1078 & -.05924 \\ 0 & -.04863 & 0 \end{bmatrix} \begin{bmatrix} S \\ B1 \\ B2 \end{bmatrix}$$

$$+ \begin{bmatrix} .2174 & 0 & 0 \\ -.07396 & .1434 & 0 \\ -.036 & 0 & .1814 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F \\ CF \\ HF \end{bmatrix}$$

Equation (14)

4.2.1. Control Interval

From the theoretical standpoint there are two expected effects due to the control interval. Firstly, as the control interval decreases and approaches continuous control, the gains increase (from 12 at 96 seconds to 300 at 4 seconds) since the control actions can correct deviations rapidly by overshooting and quickly returning to their new steady state values. Secondly, it follows that, since the control action cannot overreact during longer intervals and the disturbances have longer to make their presence felt, the quality of control will decrease with increasing control interval. The integral of the squared error of the product concentration as a function of the control interval is shown in Figure 2. Note that it increases from 0.074 at 4 seconds to 0.121 at 128 seconds.

Two practical considerations in choosing a control interval are the effect of high gains acting on noisy measurements and the increased computer system time usage as the interval decreases. Longer control intervals are an advantage in both these instances. There is obviously a compromise to be made and a figure of 64 seconds was chosen for use on the evaporator since it was compatible with DDC intervals as powers of two, did not produce excessive gains (1 to 20), was economical on computer usage (about 8 percent of the system time), and did not result in a very significant decrease in quality of control.

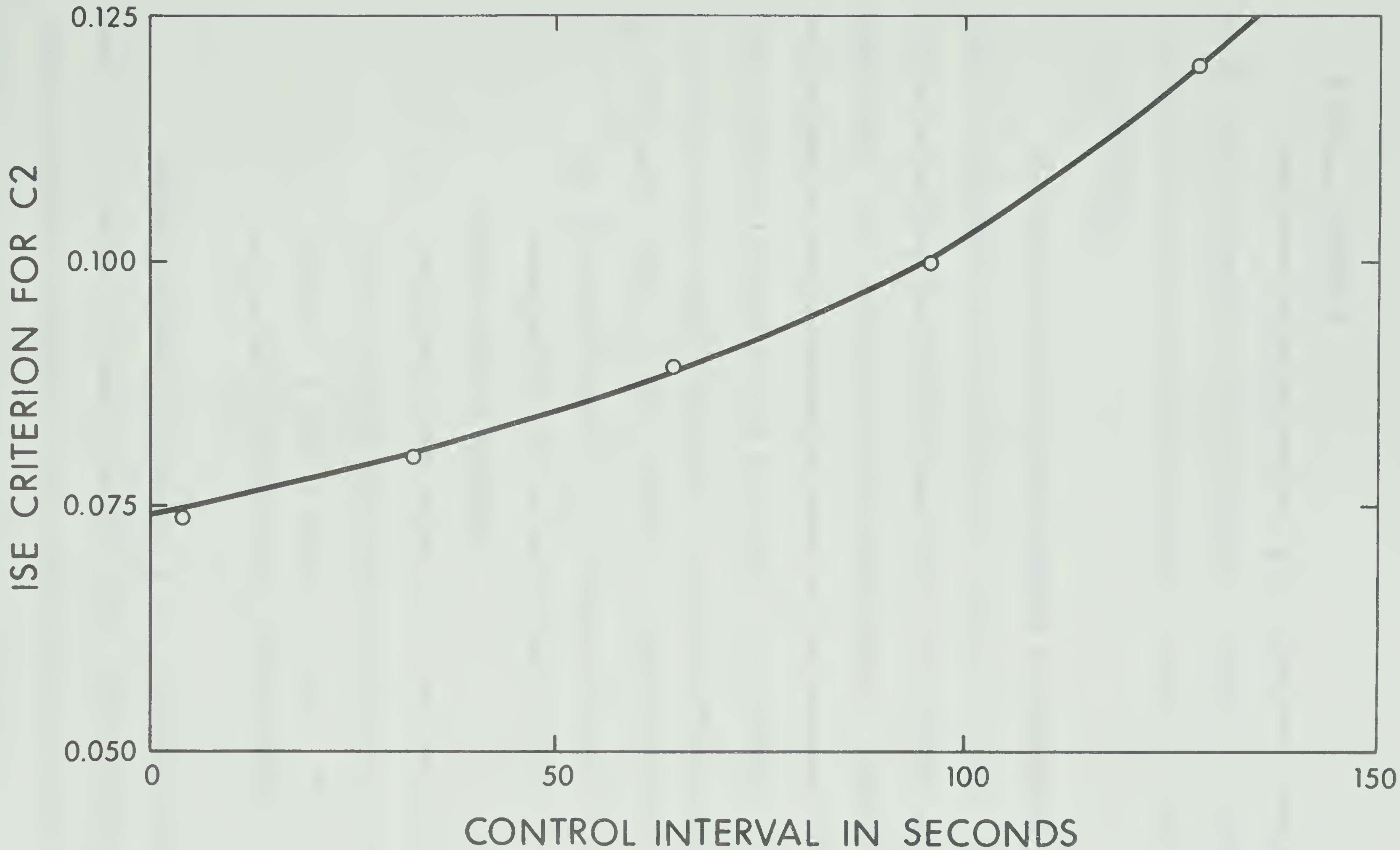


FIGURE 2. EFFECT OF CONTROL INTERVAL ON C2 CRITERION

4.2.2. "Q" Matrix

The state weighting matrix, $\underline{\underline{Q}}$, is the most important of the design parameters in the criterion. The matrix is chosen to be positive definite and usually diagonal in order to simplify its specification.

The choice of the diagonal elements is generally a matter of judgement on the basis of the "required control". The effects of varying the relative magnitude of the elements are predictable. An increase in relative magnitude will "improve" the response of the corresponding state and the response of the other states will generally not be as good. An examination of the control vector coefficient matrix in the difference equation ($\underline{\underline{B}}$ in equation (5)) will give guidance in some cases since:

(a) For each column of $\underline{\underline{B}}$ at least one state corresponding to a nonzero element of the column must be weighted.

(b) A relatively large weight on a state corresponding to a comparatively large coefficient in any column of $\underline{\underline{B}}$ will penalize the use of the control variable corresponding to that column.

Another approach to choosing $\underline{\underline{Q}}$ is to use the coefficient matrix of a quadratic Liapunov function [11]. This procedure ensures stability and appears to give reasonable, if not the "best", control.

In the proportional-plus-integral formulation weighting must also be placed on the "integral states". As might be expected increasing the relative weights on these states results in increased integral action but also slightly increased proportional action. As a result settling times will decrease and overshoots, compared to purely proportional action, will increase with increasing "integral" state weighting. Figure 3 shows results obtained using weights of zero (or straight proportional), one, and ten on the integral states. The solid and dashed lines from the same run should be compared for overshoots, and the extremely large scale of the figure should be noted.

While the amount of integral action on any variable can be varied and even removed by a zero weighting, the interaction between the proportional and integral modes inherent in the optimal formulation does not facilitate arbitrary "trade offs" between the modes. However, limited control over the contribution of each mode can be achieved through the relative weights in the $\underline{\underline{Q}}$ matrix.

4.2.3. Other Criterion Parameters

There are three other parameters in the general quadratic criterion (Equation (6)), the control vector weighting matrix $\underline{\underline{R}}$, the final state weighting matrix $\underline{\underline{S}}$, and the time weighting scalar β .

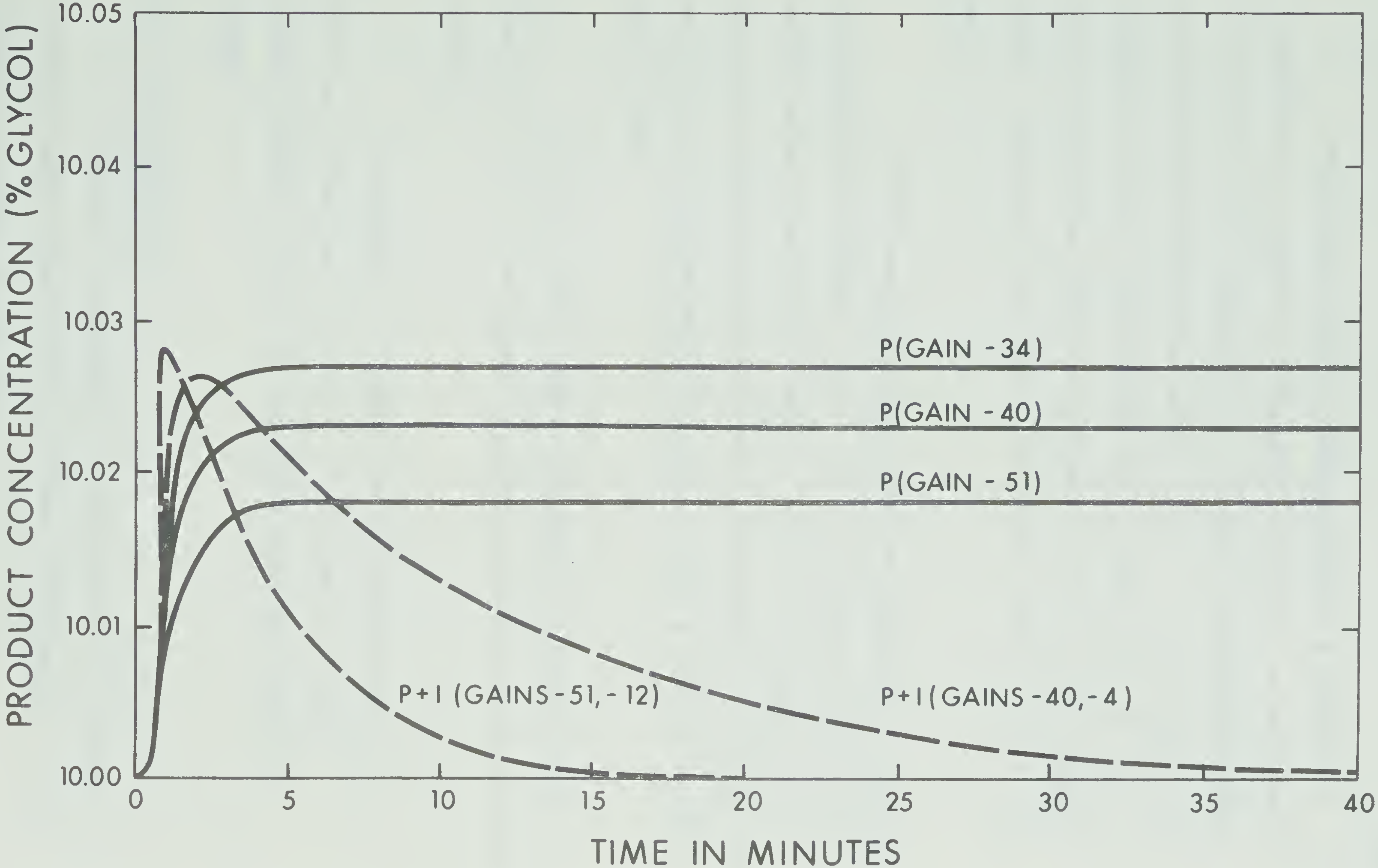
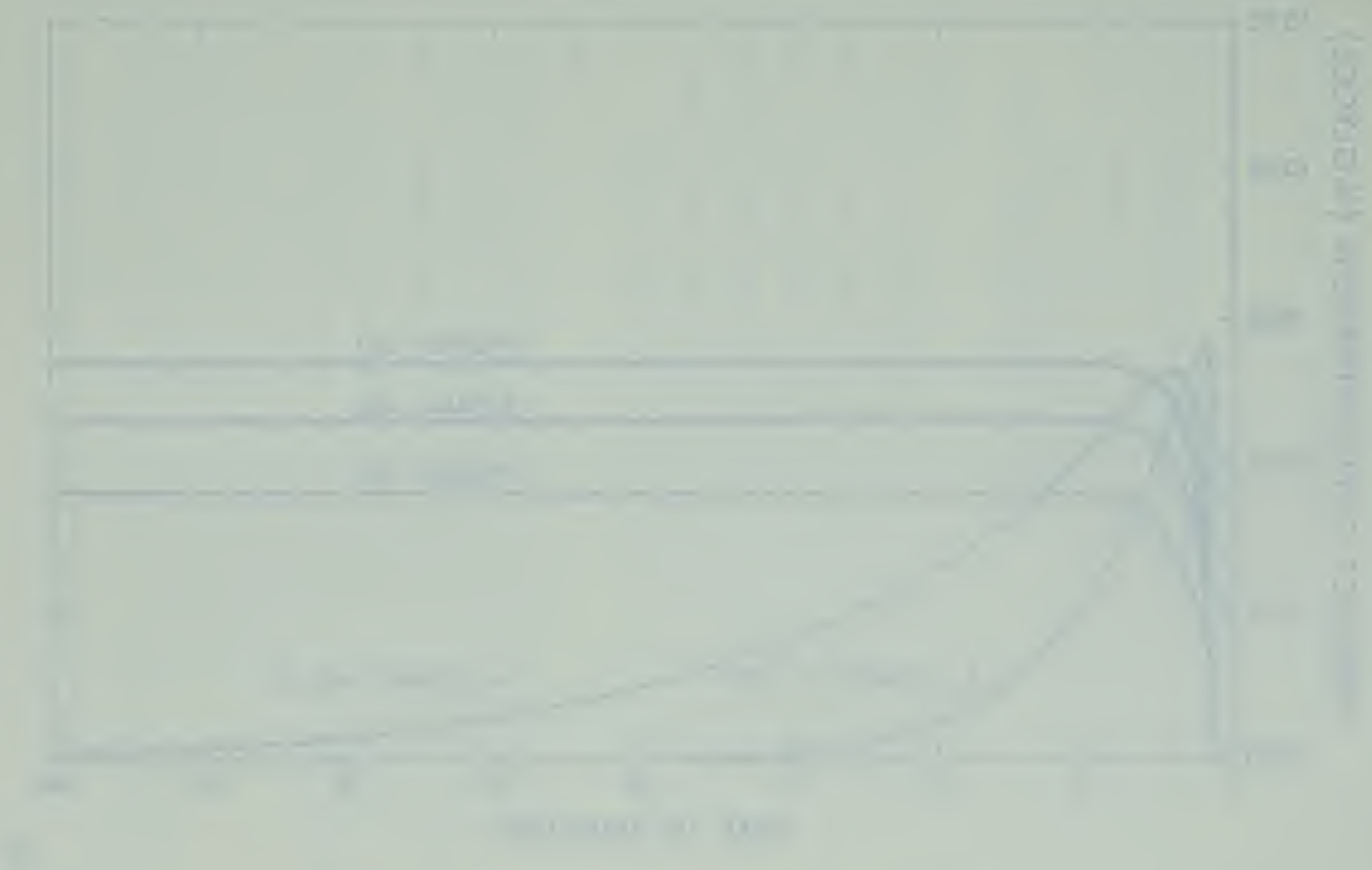


FIGURE 3. INTEGRAL WEIGHTING EFFECT ON C2 RESPONSE



The control variable weighting matrix $\underline{\underline{R}}$ cannot be considered as a completely independent weighting. As mentioned in Section 3.2.2, the elements of $\underline{\underline{Q}}$ can have a considerable effect in effectively weighting the control action. In these cases the weighting for that control action can be removed from $\underline{\underline{R}}$ or at least reduced. The choice of $\underline{\underline{R}}$ remains a matter for "judgement" with a knowledge of the "costs" of using the control actions available.

For the evaporator application the steady state values of the manipulated variables (S, B1, B2) are fixed by material and energy balance constraints. Hence no trade off is possible and since the transients are short there is no significant advantage to be gained by weighting these variables. Hence $\underline{\underline{R}}$ was set to zero for all runs.

The final state weighting by matrix $\underline{\underline{S}}$ has no effect when the infinite time dynamic programming solution of the formulation is used.

The parameter β weights deviations more heavily as the time increases and a value of 1.5 was found to "improve" control although to do so the controller gains were generally higher.

4.3. Simulated Runs

A number of simulated runs were carried out using the linearized model (Equation (14)) in state difference form. State weighting by matrix $\underline{\underline{Q}}$ was used.

$$\underline{\underline{Q}} = \text{diag}\ (10, 1, 1, 10, 100, 1, 1, 1)\ . \tag{15}$$

The product concentration was of prime interest and was heavily weighted while the liquid levels simply had to be controlled within physical limits. Nominal weights were placed on the remaining states and the "integral" states. No other weighting was used. Steam, a basic "cost" to the process, is effectually weighted through the first effect enthalpy H1 .

Figures 4a and 4b show a simulated comparison of multi-loop control, proportional multivariable control, and proportional-plus-integral multivariable control. The transients are a result of a ten percent increase in feed flowrate. The most noteworthy features of the comparison are the small deviations and the short transients of the multivariable control. The reasons for this "tight control" are the interacting nature of the controller and the high gains. Table 1 shows the gain matrices for multivariable proportional-plus-integral control. The gains vary from 0 to 19 compared with gains of 2 to 3 in the three conventional DDC loops. The integral constants were in the range 0 to 4.3 compared with conventional values of 0.01 to 0.04. Despite these high gains the control actions (Figure 4b) are not significantly larger than those due to the low gain conventional control scheme.

Since the levels, W1 and W2, just have to be maintained between limits rather than at a strict steady state, some deviation can be permitted with the levels if it leads to improved

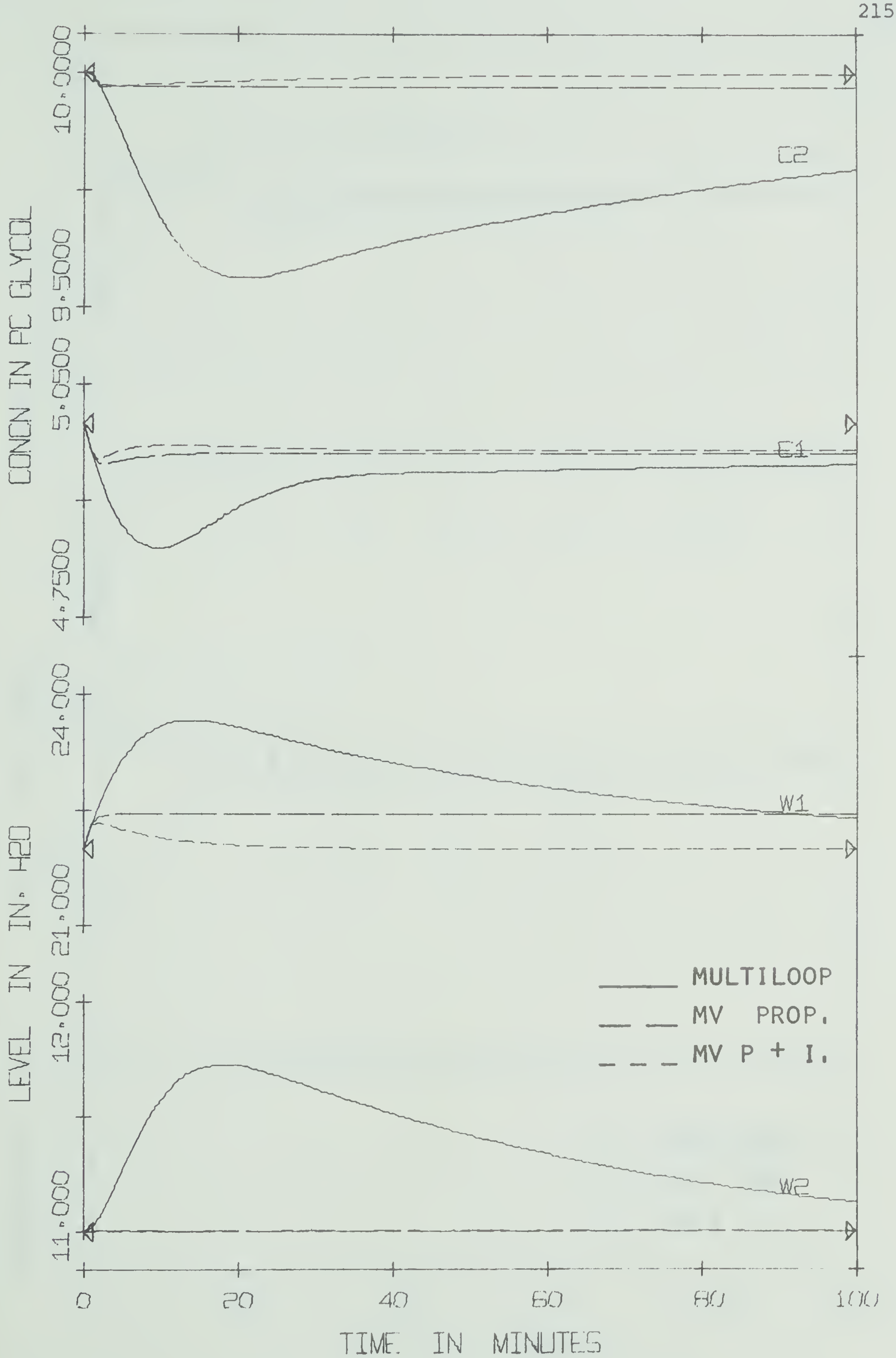


FIGURE 4a. SIMULATED COMPARISON OF CONTROL SCHEMES
(5L/+10%F/DDC, FB, P+I-O/T1/Q1/R1/D1)

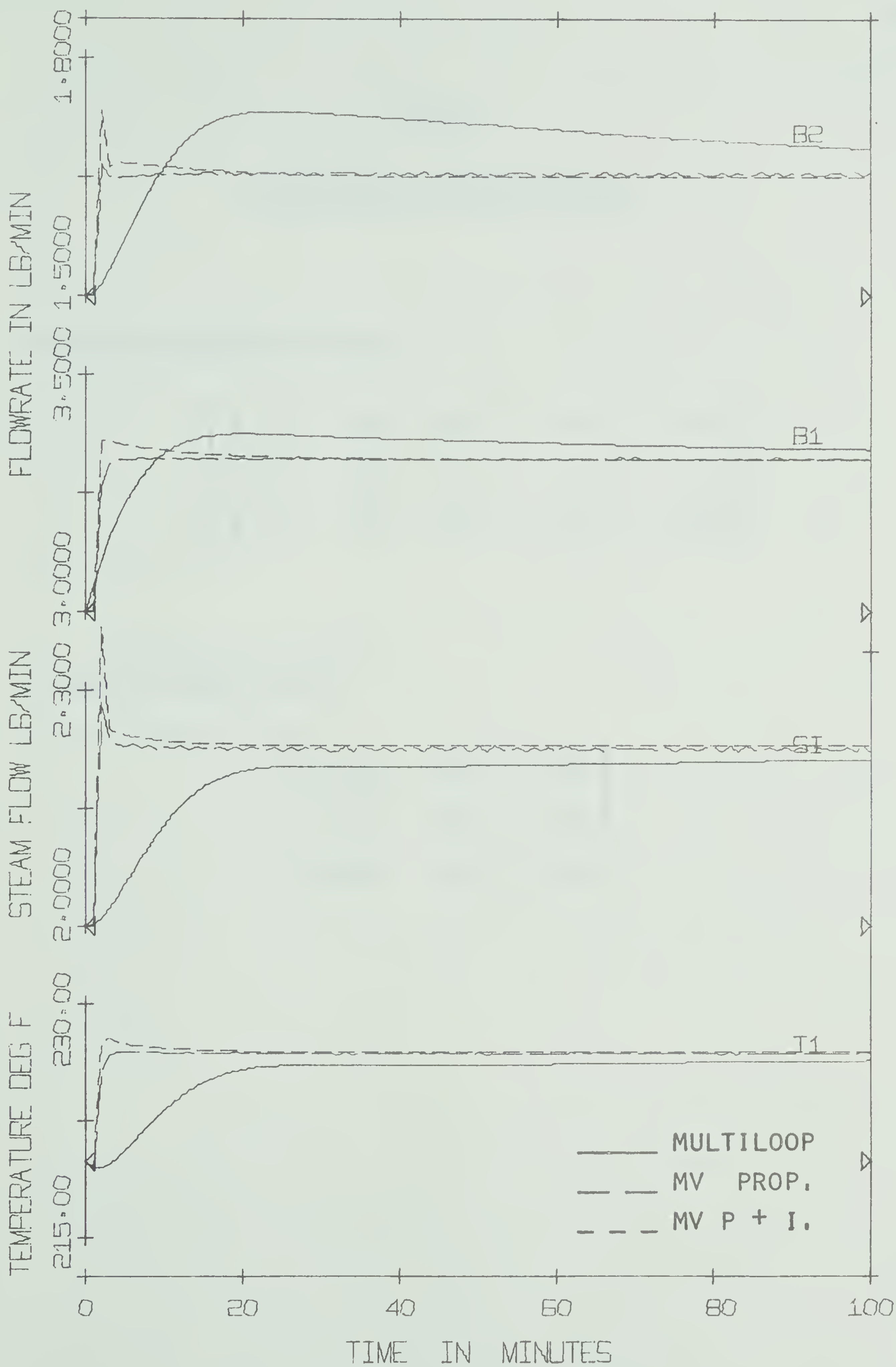


FIGURE 4b. SIMULATED COMPARISON OF CONTROL SCHEMES
(5L/+10%F/DDC, FB, P+I-O/T1/Q1/R1/D1)

TABLE 1

MULTIVARIABLE CONTROL MATRICES

Proportional Control Matrix:

$$\begin{bmatrix} 6.37 & -1.48 & -2.86 & -0.00 & -17.04 \\ 4.81 & 0.35 & 0.13 & 0.00 & 6.98 \\ 6.42 & 1.17 & -0.25 & 18.11 & 18.95 \end{bmatrix}$$

Integral Control Matrix:

$$\begin{bmatrix} 1.31 & -0.00 & -1.60 \\ 1.11 & 0.00 & 0.67 \\ 1.54 & 4.28 & 1.81 \end{bmatrix}$$

control of product composition, C2 . A simulated run was carried out with reduced weighting on the levels.

$$\underline{\underline{Q}} = \text{diag}\ (1,\ 1,\ 1,\ 1,\ 100,\ 0.01,\ 0.01,\ 1)\ .$$

Figures 5 compare this "averaging" control of levels (dashed lines) with the tight control (solid lines). The graphs show larger deviations and "slower" control for the levels and improved control for the product concentration. The control variables in the case of averaging control do not exhibit the overshoot required by tight control.

5. EXPERIMENTAL RESULTS

5.1. Implementation

The multivariable control system was implemented with an IBM 1800 digital control computer which is interfaced with the pilot plant evaporator. The process runs under Direct Digital Control under a time-sharing executive system which permits simultaneous execution of off-line jobs such as the plotting of the figures for this paper.

Multivariable control calculations are carried out by a queued process coreload written in FORTRAN which executes every control interval. System time for the coreload varies from two to five seconds in every 64 second control interval, depending on the disk operations. Actual CPU time would be considerably less but was not readily available. The program obtains state variable measurements from DDC data acquisition loops and makes control variable changes by adjusting the setpoints of DDC flow control loops.

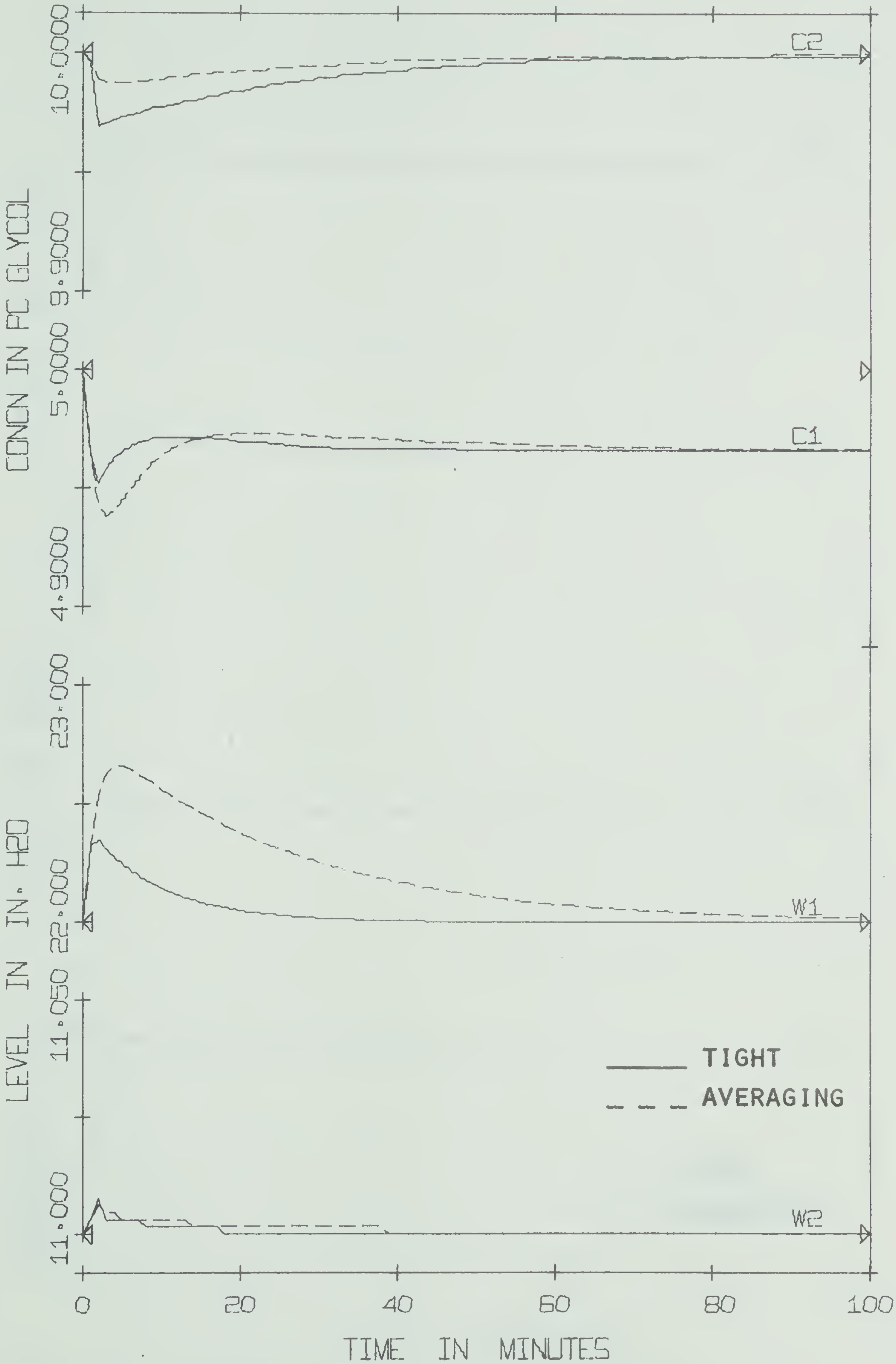


FIGURE 5a. SIMULATED EFFECT OF INTEGRAL WEIGHTING
(5L/+10%F/P+I-1, P+I-O/Q1/R1/D1)

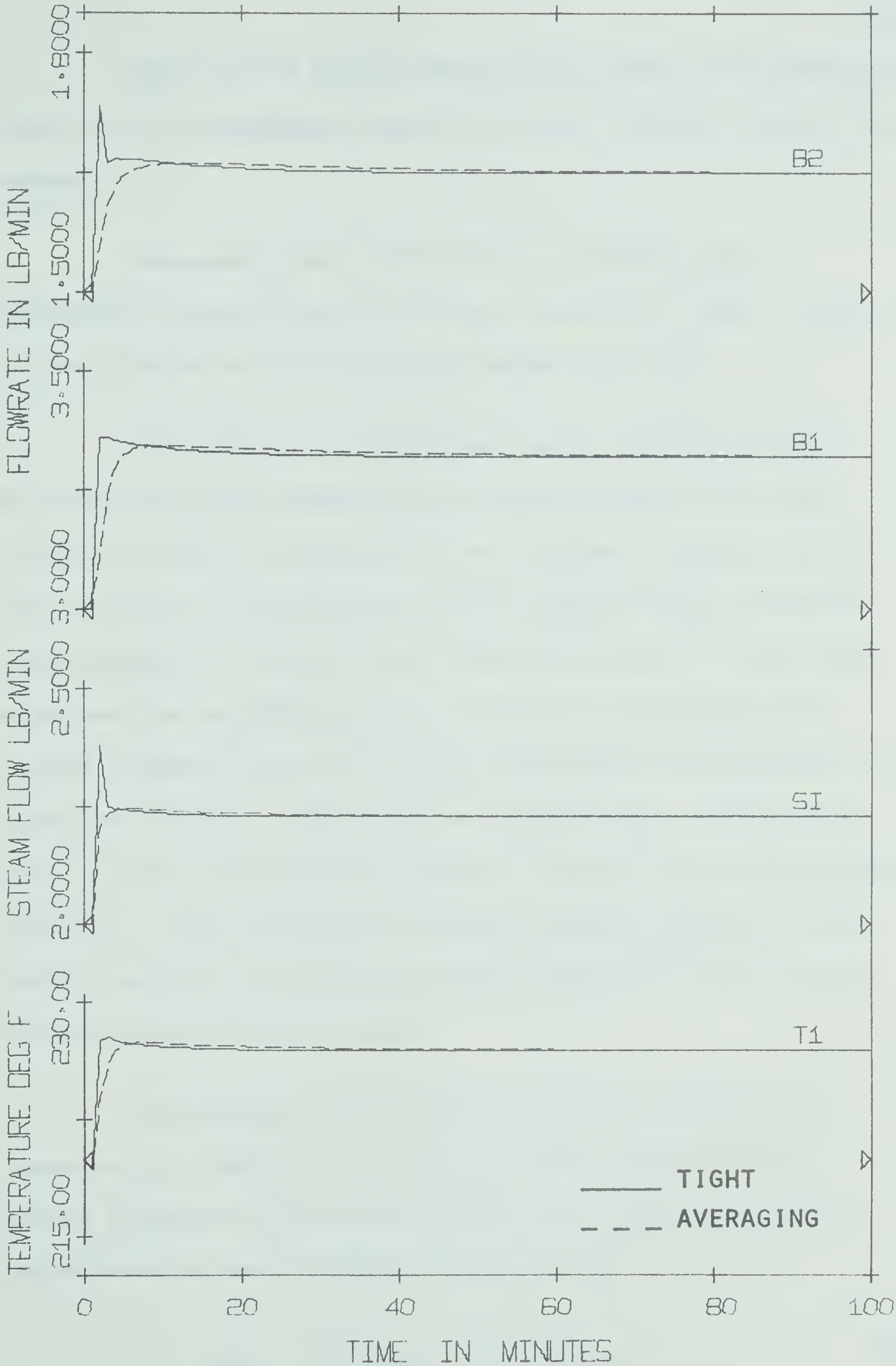


FIGURE 5b. SIMULATED EFFECT OF INTEGRAL WEIGHTING
(5L/+10%F/P+I-1, P+I-O/Q1/R1/D1)

There are two basic problems to be faced in the implementation. These are noisy measurement signals and state variables that are not measured.

Measurements were conditioned by standard digital exponential filters within the DDC data acquisition loops. Filtering was light so as not to introduce undue phase lags.

The linear process model was used to predict the state of the process and is supplied with measured values of the load variables, control variables, and the "combined" estimate of the state variables, obtained from the last predicted value and those states measured. Because of the integrating nature of their model equations, the two liquid holdups drifted when measurements of bottoms flowrates were used. Noisy measurements and calibration errors caused the drifting. When these two flowrates were eliminated by using two rows of the control matrices, the model gave much improved estimates. Model predictions were good probably because with the process under the multivariable control deviations of the process from its steady state were small.

The measured states and their predicted values were "exponentially combined" [12] to improve the estimate both for control purposes and for the next step in the model calculations. The adjusted estimate is given by the relation

$$\underline{x}_{est} = \underline{\underline{\alpha}}\,\underline{x}_{meas} + (\underline{\underline{I}} - \underline{\underline{\alpha}})\underline{x}_{calc} \tag{16}$$

where $\underline{\underline{\alpha}}$ is a diagonal matrix of "filter constants". A diagonal element of zero indicates a predicted value, and of unity indicates a measured value. Work is also under way on using a Kalman optimal filter to predict the values of the state variables.

5.2. Results

A number of experimental runs have been carried out to test the multivariable control derived from the linearized model and to compare this type of control with conventional multiloop control in a practical environment.

The response of the evaporator to a 20 percent increase in feed flowrate while under conventional DDC multiloop control is shown in Figures 6. The controller constants for the standard DDC loops were not optimal values but were chosen by "experience" gained over two years of operation of the evaporator. Figures 7 and 8 show evaporator responses to the same magnitude of disturbance while under multivariable proportional and proportional-plus-integral control respectively. As with the simulated results the small deviations and short transients are clearly shown.

The effects of noise become apparent in the experimental runs. Figures 7b and 8b show 20 to 40 percent fluctuations in the control variables resulting from the action of the high gains on noisy measurements. The liquid holdup, W2, produced by proportional control alone (Figure 7a) is much smoother than the comparable response using multivariable P + I control. This is attributed to the higher gains used in the latter case. However, despite the

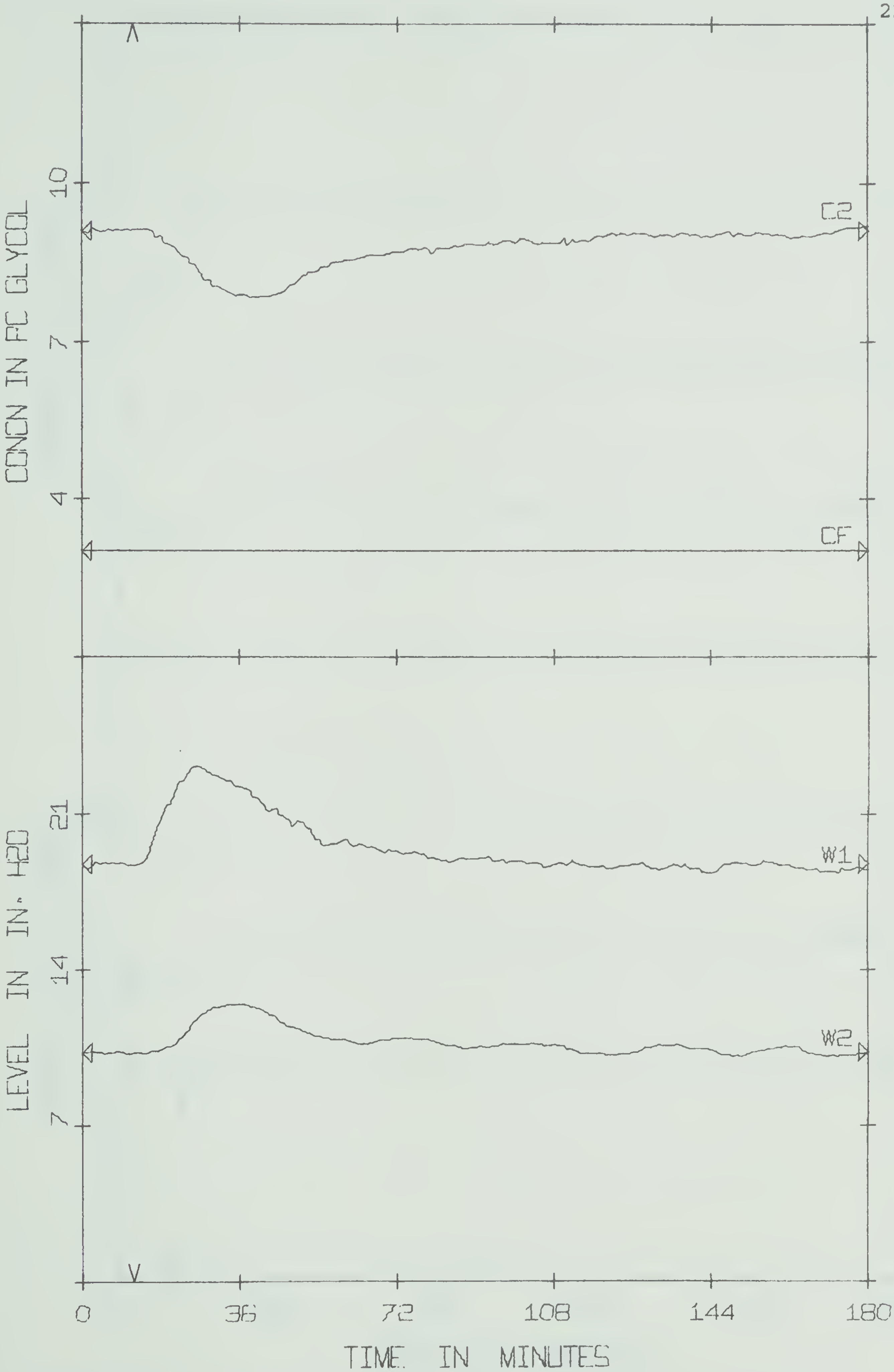


FIGURE 6a. EXPERIMENTAL MULTILOOP CONTROL
(EXP/+20%F/DDC/T1/PRED1)

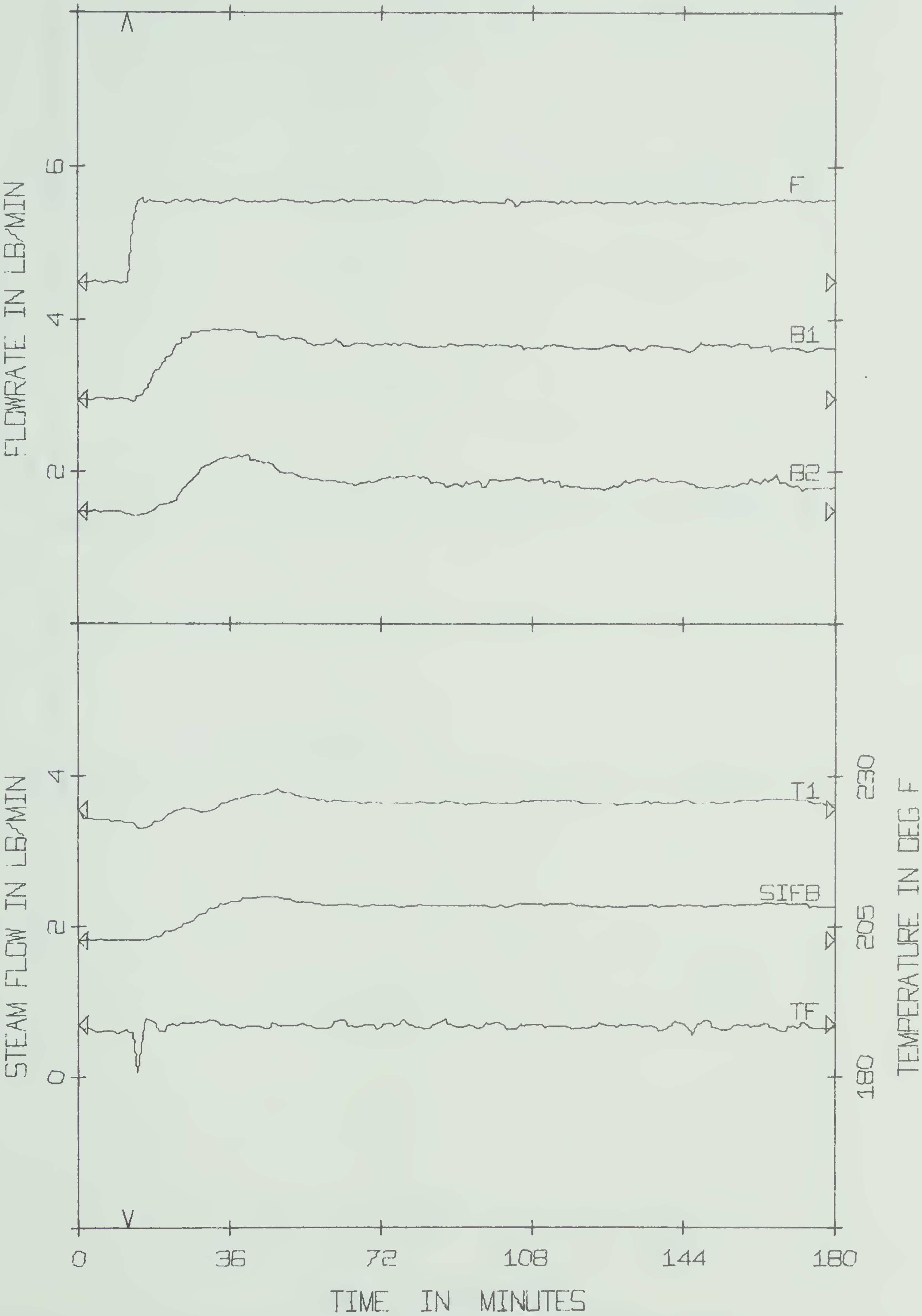


FIGURE 6b. EXPERIMENTAL MULTILOOP CONTROL
(EXP/+20%F/DDC/T1/PRED1)

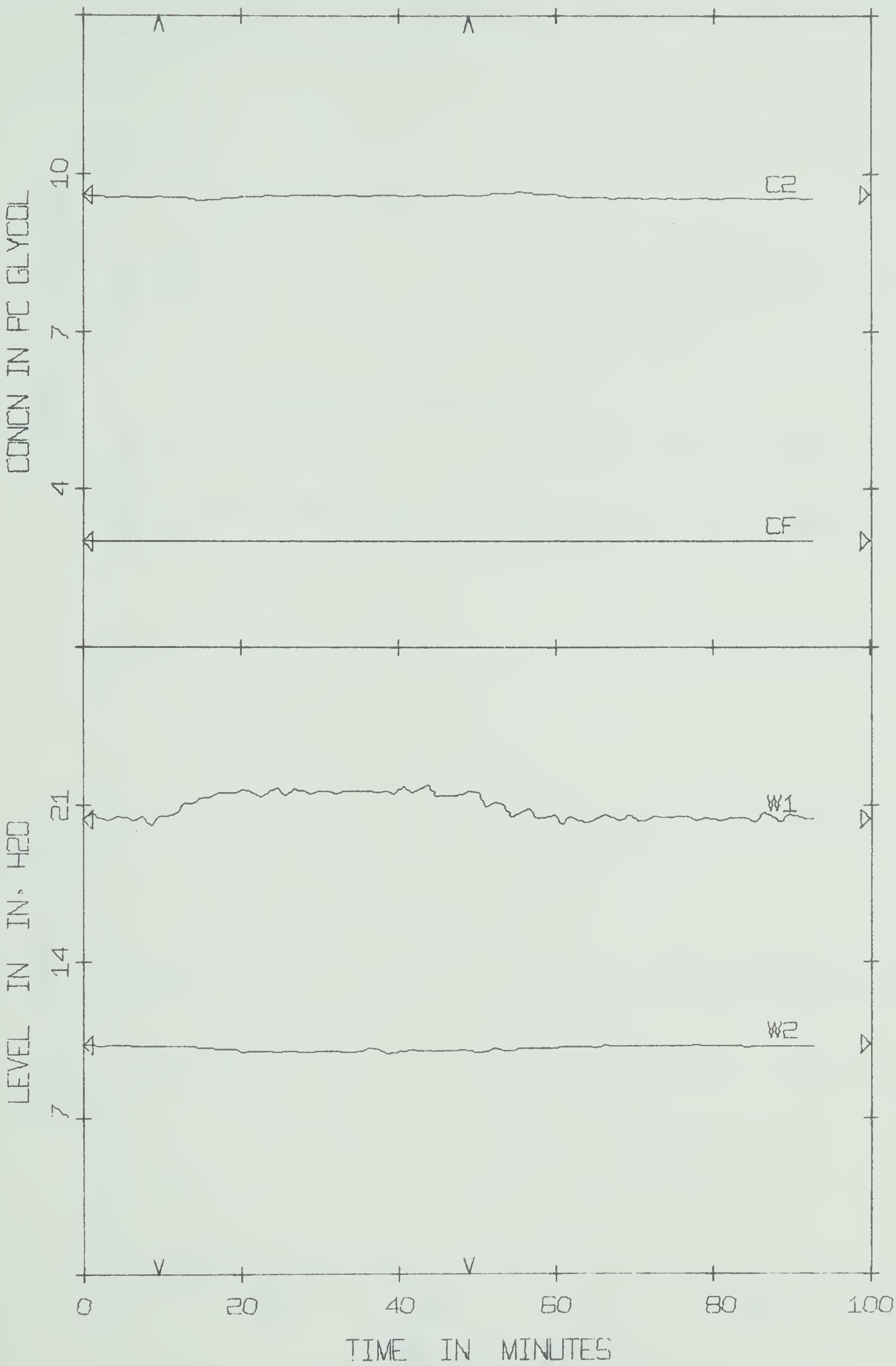


FIGURE 7a. EXPERIMENTAL MULTIVARIABLE PROPORTIONAL CONTROL (EXP/20%F/FB/Q1/R1/D1/A1/MVC41)

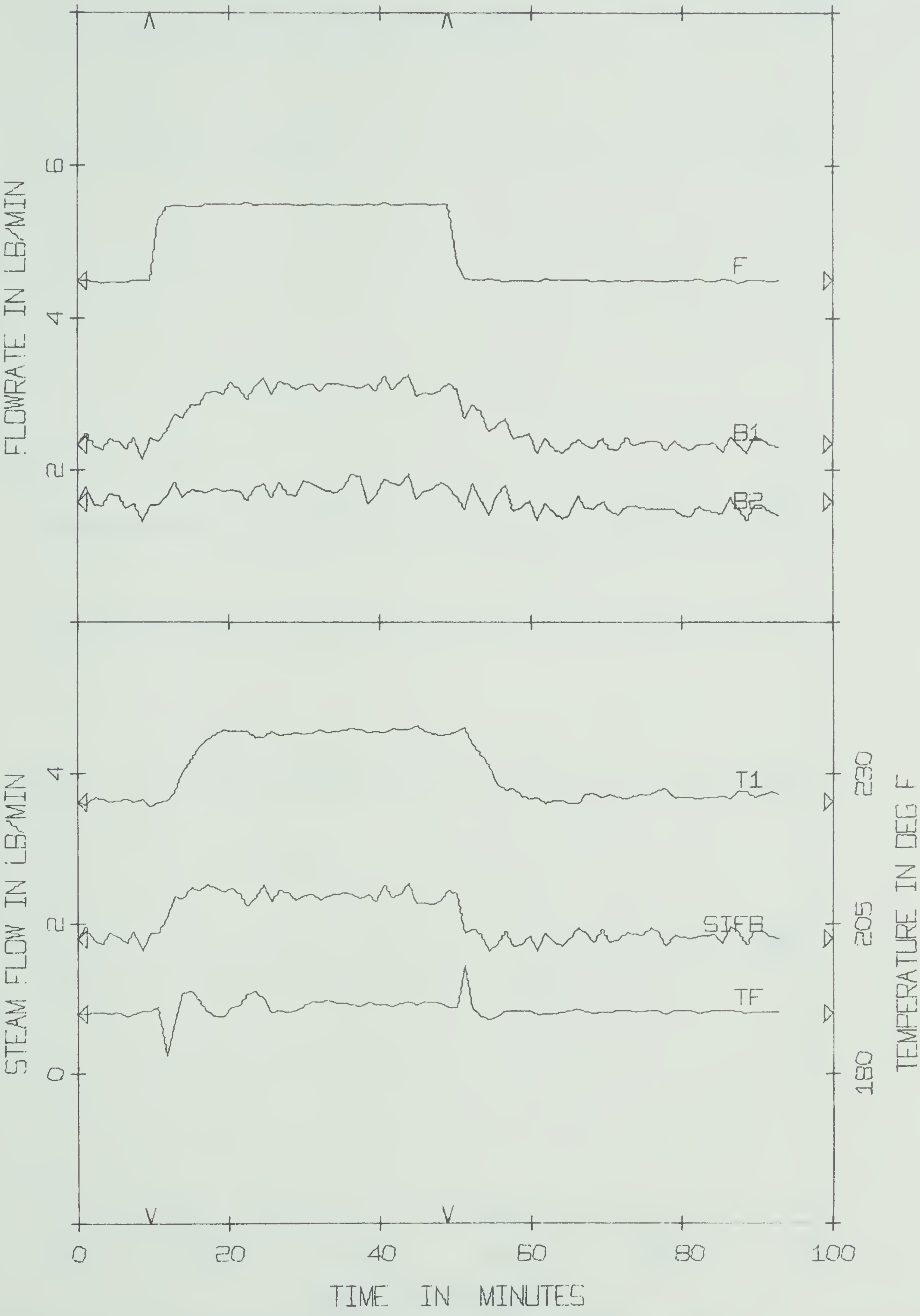


FIGURE 7b. EXPERIMENTAL MULTIVARIABLE PROPORTIONAL CONTROL (EXP/20%F/FB/Q1/R1/D1/A1/MVC41)

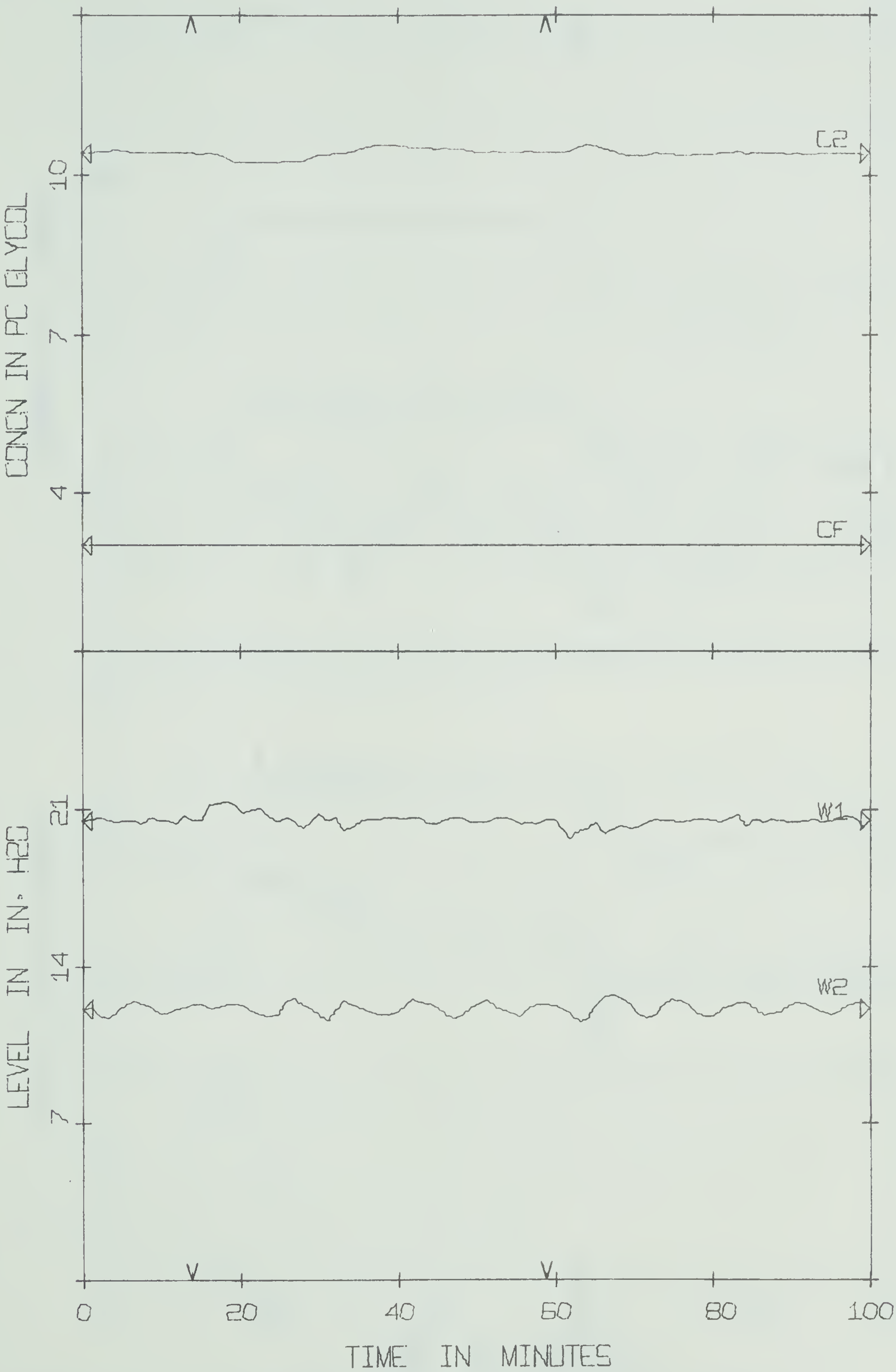


FIGURE 8a. EXPERIMENTAL MULTIVARIABLE INTEGRAL CONTROL I
(EXP/20%F/P+I-1/Q1/R1/D1/A1/MVC5)

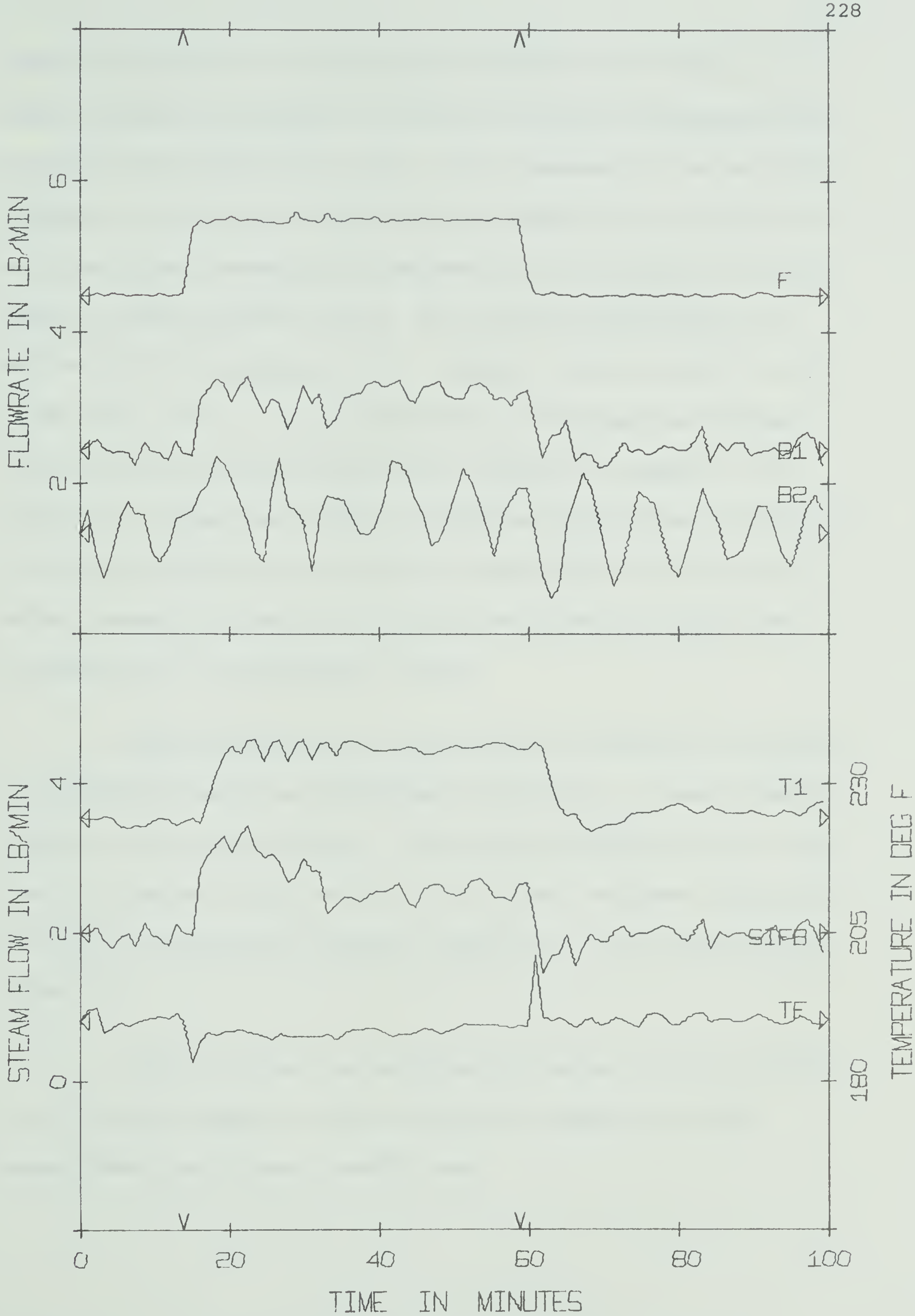


FIGURE 8b. EXPERIMENTAL MULTIVARIABLE INTEGRAL CONTROL I
(EXP/20%F/P+I-1/Q1/R1/D1/A1/MVC5)

large fluctuations in the manipulated variables the control of the output variables in Figures 7a and 8a is a distinct improvement over the conventional multiloop control. The responses of the controlled variables to a step up in feed flow were affected by the pressure in the first effect exceeding its maximum value and blowing the safety valve for short periods of time. The resulting disturbances are most obvious in the graph of T1 . However, the net result as far as W1, W2, and C2 were concerned was to decrease the available control action and hence prolong the transient as compared to the response to a step down in feed. (Note that constraints on the state and control variables are not easily incorporated into the optimal control formulation used in this study and hence the control law is derived on an "unconstrained" basis).

Figure 9 presents the results from an optimal multivariable run carried out using the "averaging" control (i.e., lower weighting factors) on the liquid levels. The transient responses of the levels show the expected larger deviations and slower responses (Figure 9a). The product concentration response improved as in the simulated run (Figure 5).

Figure 10 shows the experimental response of the evaporator to a 27 percent change in feed concentration under the "tight" proportional-plus-integral control law.

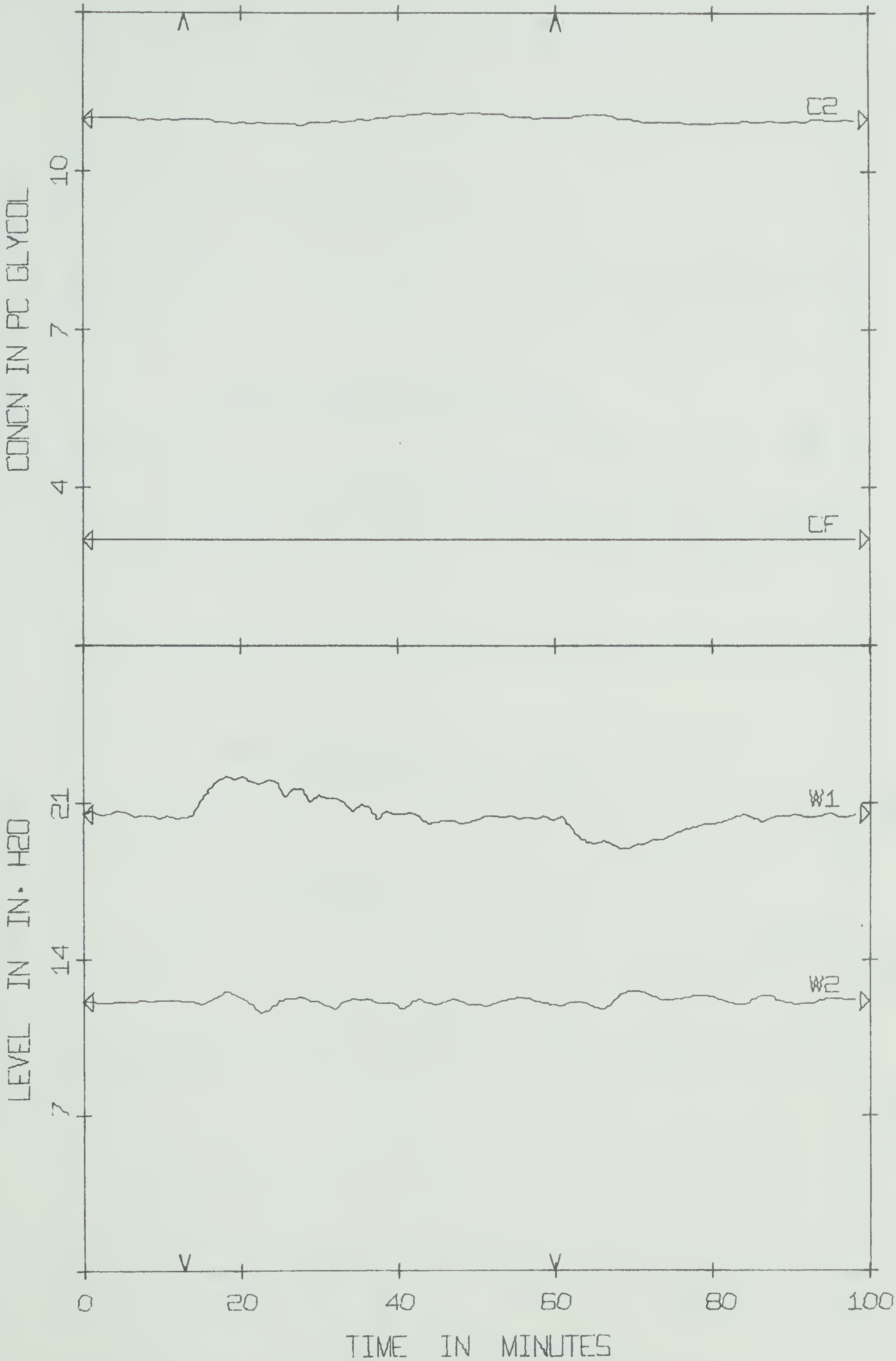


FIGURE 9a. EXPERIMENTAL MULTIVARIABLE INTEGRAL CONTROL II
(EXP/20%F/P+I-O/Q1/R1/D1/A1/MVC6)

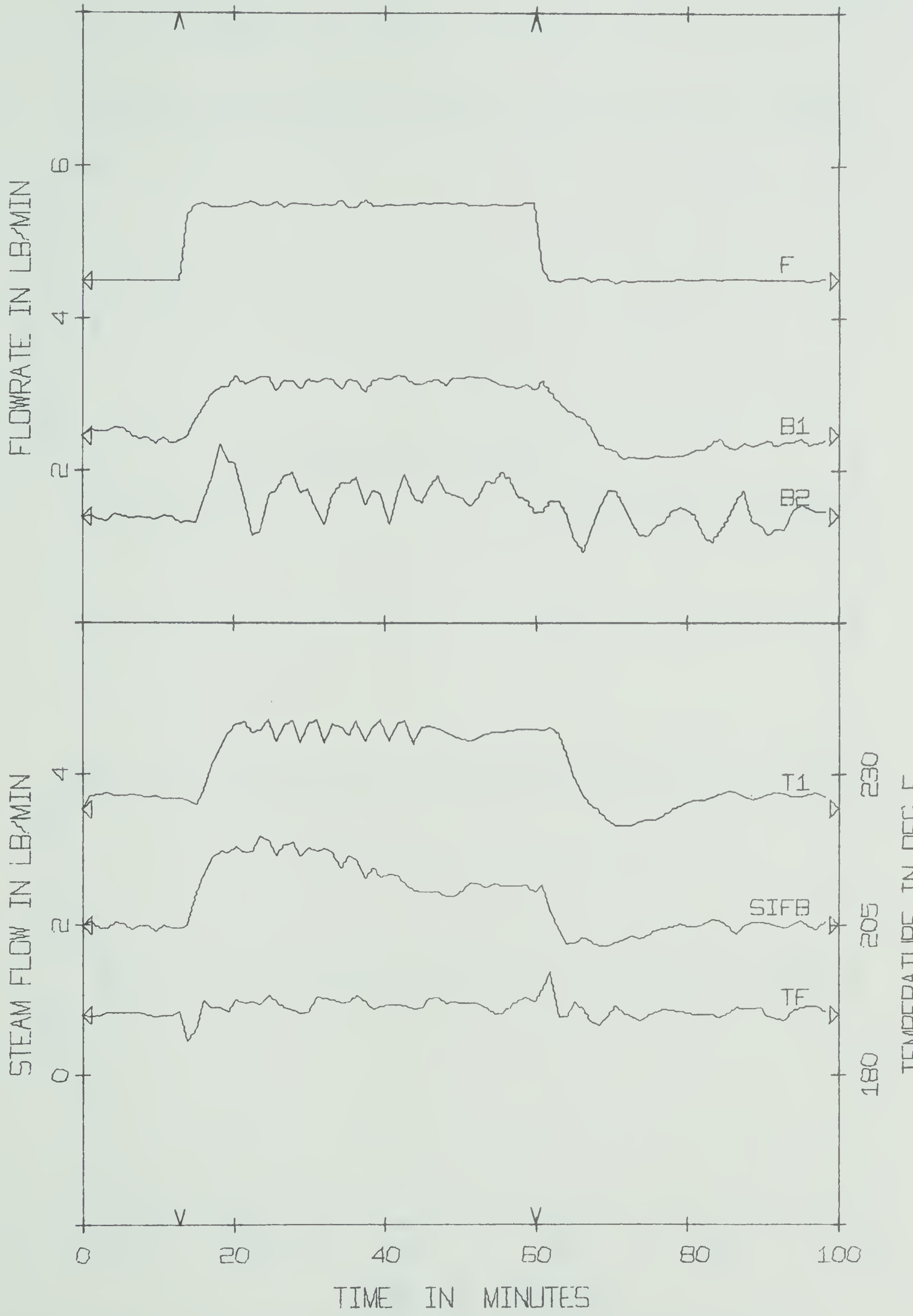


FIGURE 9b. EXPERIMENTAL MULTIVARIABLE INTEGRAL CONTROL II
(EXP/20%F/P+I-O/Q1/R1/D1/A1/MVC6)

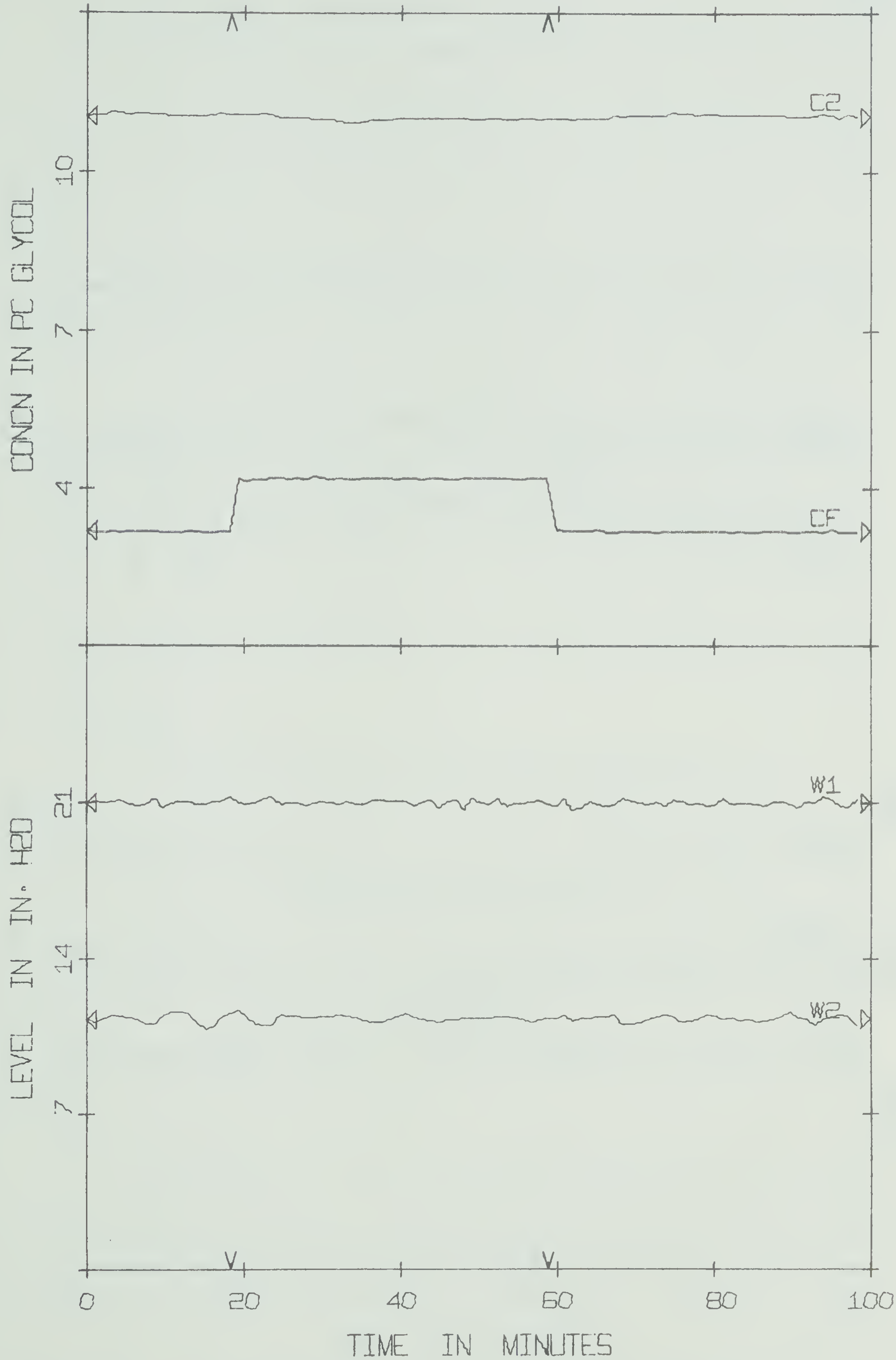


FIGURE 10a. EXPERIMENTAL MULTIVARIABLE INTEGRAL CONTROL III
(EXP/28%CF/P+I-1/Q1/R1/D1/A1/MVC7)

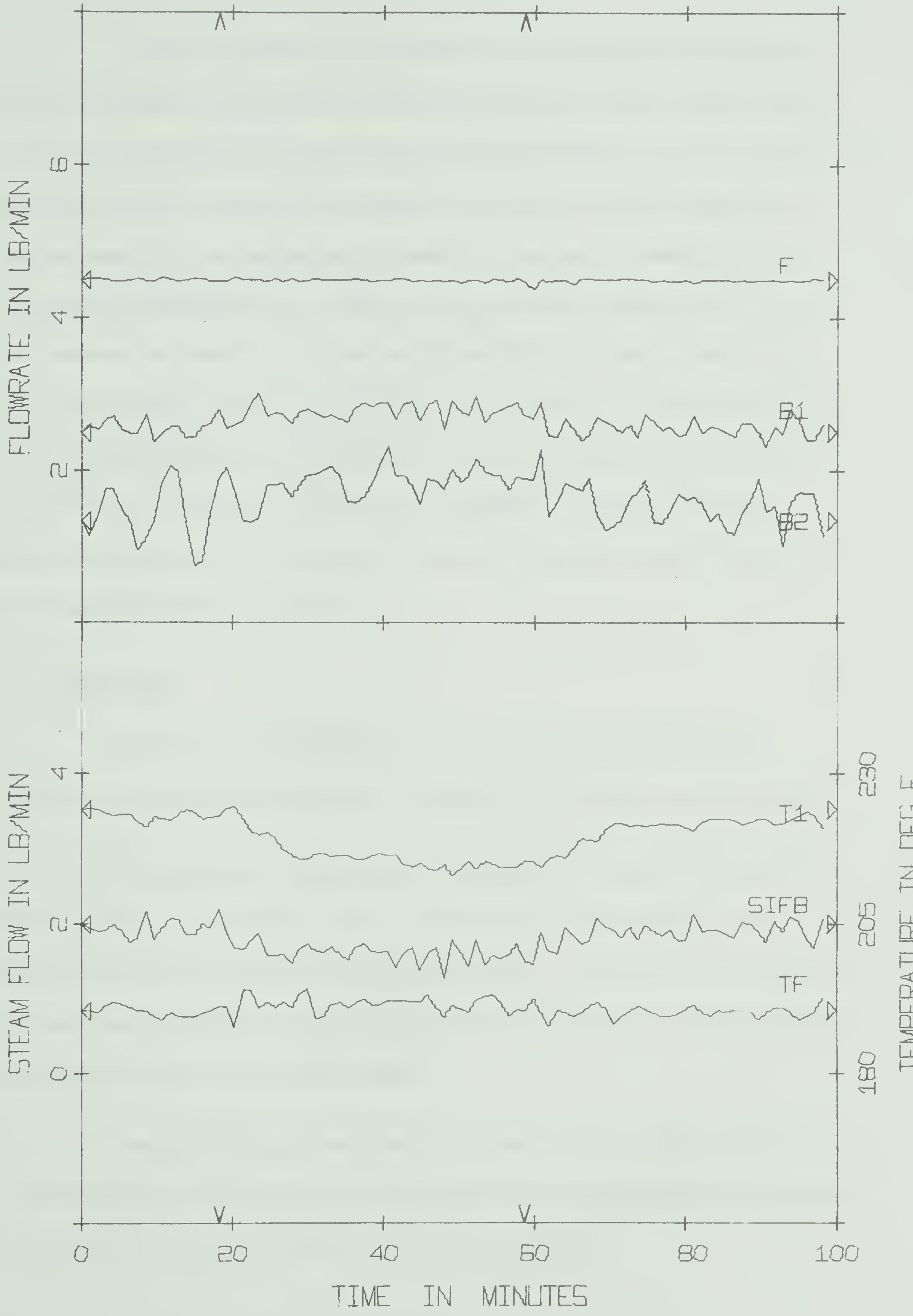


FIGURE 10b. EXPERIMENTAL MULTIVARIABLE INTEGRAL CONTROL III
(EXP/28%CF/P+I-1/Q1/R1/D1/A1/MVC7)

It is interesting to examine the accuracy of the model used in deriving the optimal control law used in these experimental runs. The linearized model used in the formulation of the optimal control problem adequately represented the open-loop response of the process for disturbances of about 10 percent. However, for 20 percent disturbances in feed flow the model response led that of the process by several minutes and predicted a final steady state value that was in error by as much as 25 percent. The nonlinearity of the process was also revealed by the significant differences in response to a step up in feed vs a stepdown. However, despite these limitations of the model, the control laws derived from the model gave excellent results.

6. CONCLUSIONS

The work illustrates a successful implementation of optimal multivariable regulatory control in a process environment.

The dynamic programming technique was easily formulated and the off-line solution gave a stable high gain control law without excessive computational requirements. Unlike state driving, the optimal regulatory control laws did not result in large control actions in spite of the high gains.

Implementation, was easily accomplished using a digital control computer, and did not suffer from the excessive computational requirements of many optimal control techniques.

Both simulated and experimental results showed improved control compared to a conventional multiloop control scheme. The experimental results also showed the model accuracy was not as critical as might be expected and that noise, while making its presence felt, particularly in the manipulated variables, did not seriously affect the improved control.

The work has illustrated that optimal regulatory control can be recommended for practical implementation, particularly where processes interact strongly and product quality is important.

CHAPTER EIGHT

MULTIVARIABLE SETPOINT CONTROL

ABSTRACT

Two design approaches to optimal multivariable setpoint control are demonstrated experimentally on a pilot plant evaporator.

The two designs are based upon the dynamic programming solution of different formulations of the optimal control problem. The first formulation includes a setpoint vector in the quadratic criterion and results in an optimal approach to new setpoint values. The second formulation involves optimal model following and gives the designer control over the form of the setpoint response.

Both approaches are implemented by a digital control computer on a pilot plant double effect evaporator at the University of Alberta. Process responses to setpoint changes are much better under multivariable control than they are under conventional multiloop control.

1. INTRODUCTION

It is advantageous to be able to make small changes in setpoints while a process is under regulatory control. This is possible in conventional multiloop control systems although the controllers are normally tuned for regulatory control rather than for setpoint disturbances. Re-tuning the loops for setpoint control is usually detrimental to regulatory control. This paper presents optimal multivariable setpoint control which does not require changes to the normal regulatory multivariable feedback control matrices.

The inclusion of setpoints in the quadratic criterion results in fast system response to setpoint changes. To accommodate downstream processes it is often desirable to change conditions gradually and a model following formulation is presented which gives the designer control over the form of the response to a setpoint change. Discrete dynamic programming is used to solve both formulations of the optimal control problem generating control laws with constant coefficient matrices.

Implementation on a pilot plant evaporator gives experimental data which are used to compare the two formulations and examine the effects of simultaneous load changes. The responses are also compared to those resulting from a setpoint change under conventional multiloop control and from a simple state driving technique.

2. LITERATURE SURVEY

The inclusion of setpoint control in optimal control problems has received little attention. Kalman and Koepcke [1] included setpoints in a quadratic criterion but did not consider the degrees of freedom available to drive states to specific values [2].

Model following techniques were examined in some detail by Tyler [3]. The model considered was homogeneous and the Ricatti equation was used in the solution of the optimal control problem. The two criteria considered were summed quadratic functions of firstly the error, the difference between the outputs of the process model and the desired model, and secondly the derivative of the error. Markland [4] solved the same problem by minimizing the error in the least squares sense. Yore [5] introduced a model with inputs and considered a configuration with feedback from the process state, and feedforward from the model state and the inputs. The optimal control problem with a summed quadratic error index was solved to give the feedback and model state feedforward control matrices. The input feedforward was evaluated separately and a variety of criteria were mentioned.

3. SETPOINT DESIGN METHODS

Two design approaches are presented in the following subsections based upon the discrete dynamic programming solution of different formulations of the optimal control problem including loads.

3.1. Optimal Setpoint Control

Consider a linear time-invariant state space process model of the following form.

$$\underline{\dot{x}} = \underline{\underline{A}}_c \underline{x} + \underline{\underline{B}}_c \underline{u} + \underline{\underline{D}}_c \underline{d} \tag{1}$$

where

$\underline{x}$ is an n dimensional state vector,

$\underline{u}$ is an m dimensional control vector,

$\underline{d}$ is a p dimensional load vector,

$\underline{\underline{A}}_c$, $\underline{\underline{B}}_c$, $\underline{\underline{D}}_c$ are constant coefficient matrices of appropriate dimensions.

The optimal control problem is to minimize the following criterion with respect to the control variables, $\underline{u}$.

$$J = \sum_{i=1}^{N} [(\underline{x}_i - \underline{\underline{C}}^T \underline{x}')^T \underline{\underline{Q}} (\underline{x}_i - \underline{\underline{C}}^T \underline{x}') + \underline{u}_{i-1}^T \underline{\underline{R}}\, \underline{u}_{i-1}] \tag{2}$$

where

$$\underline{x}' = \underline{\underline{C}}\, \underline{x} \tag{3}$$

is a subset of the process state, $\underline{x}$, of dimension q . It can be shown that the degrees of freedom for driving states or outputs to specific values is the dimension of the control vector [2] . Hence the dimension of $\underline{x}'$ must be less than or equal to that of $\underline{u}$ $(q \leq m)$.

In general the actual outputs, or setpoints, $\underline{y}_{sp}$ also of dimension q , can be any unique function of the subset $\underline{x}'$ (that

is, $\underline{x}'$ can be determined given $\underline{y}_{sp}$). The following analyses will assume $\underline{x}' = \underline{y}_{sp}$.

Applying discrete dynamic programming to the solution of this formulation results in the following control law.

$$\underline{u} = \underline{\underline{K}}_{FB}\underline{x} + \underline{\underline{K}}_{FF}\underline{d} + \underline{\underline{K}}_{SP}\underline{y}_{sp} \; . \tag{4}$$

Recursive relations can be developed which define the control matrices as follows.

$$\underline{\underline{K}}_{FB}^{N-i} = -(\underline{\underline{B}}^T\underline{\underline{P}}^{i-1}\underline{\underline{B}} + \underline{\underline{R}})^{-1}\underline{\underline{B}}^T\underline{\underline{P}}^{i-1}\underline{\underline{A}} \tag{5}$$

$$\underline{\underline{K}}_{FF}^{N-i} = -(\underline{\underline{B}}^T\underline{\underline{P}}^{i-1}\underline{\underline{B}} + \underline{\underline{R}})^{-1}\underline{\underline{B}}^T(\underline{\underline{P}}^{i-1}\underline{\underline{D}} + \underline{\underline{O}}^{i-1}) \tag{6}$$

$$\underline{\underline{K}}_{SP}^{N-i} = -(\underline{\underline{B}}^T\underline{\underline{P}}^{i-1}\underline{\underline{B}} + \underline{\underline{R}})^{-1}\underline{\underline{B}}^T(\underline{\underline{M}}^{i-1} - \underline{\underline{N}}^{i-1}\underline{\underline{C}}^T) \tag{7}$$

where

$$\underline{\underline{P}}^{i} = (\underline{\underline{T}}^{N-i})^T\underline{\underline{P}}^{i-1}\underline{\underline{T}}^{N-i} + (\underline{\underline{K}}_{FB}^{N-i})^T\underline{\underline{R}}\,\underline{\underline{K}}_{FB}^{N-i} + \underline{\underline{Q}} \tag{8}$$

$$\underline{\underline{M}}^{i} = (\underline{\underline{T}}^{N-i})^T(\underline{\underline{P}}^{i-1}\underline{\underline{B}}\,\underline{\underline{K}}_{SP}^{N-i} - \underline{\underline{M}}^{i-1}) + (\underline{\underline{K}}_{FB}^{N-i})^T\underline{\underline{R}}\,\underline{\underline{K}}_{SP}^{N-i} \tag{9}$$

$$\underline{\underline{N}}^{i} = (\underline{\underline{T}}^{N-i})^T\underline{\underline{N}}^{i-1} + \underline{\underline{Q}} \tag{10}$$

$$\underline{\underline{O}}^{i} = (\underline{\underline{T}}^{N-i})^T(\underline{\underline{O}}^{i-1} + \underline{\underline{P}}^{i-1}(\underline{\underline{B}}\,\underline{\underline{K}}_{FF}^{N-i} + \underline{\underline{D}})) + (\underline{\underline{K}}_{FB}^{N-i})^T\underline{\underline{R}}\,\underline{\underline{K}}_{FF}^{N-i} \tag{11}$$

with the counter $i = 1,2,3,\ldots$ and initial conditions

$$\underline{\underline{P}}_o = \underline{\underline{Q}}, \quad \underline{\underline{M}}_o = 0, \quad \underline{\underline{N}}_o = \underline{\underline{Q}}, \quad \underline{\underline{O}}_o = 0 \; . \tag{12}$$

These relations have been found to converge rapidly to constant values for the control matrices. It was assumed in the derivation that $\underset{\sim}{y}_{sp}$ and $\underline{d}$ were constant over the time period of the derivation although this does not restrict implementation in any way.

Since the setpoint control law has constant coefficient matrices and the setpoint vector $\underset{\sim}{y}_{sp}$, if non-zero, constitutes a constant load to the closed loop system, there will be a resulting offset from $\underset{\sim}{y}_{sp}$. This offset is, practically speaking, negligible although it can be compensated for as follows.

Consider the closed loop system with no load disturbances.

$$\underline{\dot{x}} = (\underline{\underline{A}}_c + \underline{\underline{B}}_c \underline{\underline{K}}_{FB})\underline{x} + \underline{\underline{B}}_c \underline{\underline{K}}_{SP} \underset{\sim}{y}_{sp} \; . \tag{13}$$

The final steady state can be evaluated by the final value theorem.

$$\underset{\sim}{y}_{ss} = \underline{\underline{C}}\,\underline{x}_{ss} = -\underline{\underline{C}}(\underline{\underline{A}}_c + \underline{\underline{B}}_c \underline{\underline{K}}_{FB})^{-1} \underline{\underline{B}}_c \underline{\underline{K}}_{SP} \underset{\sim}{y}_{sp} = \underline{\underline{E}}\,\underset{\sim}{y}_{sp} \; . \tag{14}$$

It follows that, in order to obtain exact correspondence between $\underset{\sim}{y}_{ss}$ and $\underset{\sim}{y}_{sp}$, the setpoint control matrix $\underline{\underline{K}}_{SP}$ should be "corrected" by postmultiplication by the inverse of matrix $\underline{\underline{E}}$.

Design parameters involved in the formulation are the control interval, Δt, and the state and control weighting matrices, $\underline{\underline{Q}}$ and $\underline{\underline{R}}$. Guidelines on the effects of different weighting are available [2] to aid in the choice of $\underline{\underline{Q}}$ and $\underline{\underline{R}}$.

An alternative approach to formulating the setpoint problem exists. Given the subset $\underline{x}'$ the steady state model relations can be used to evaluate $\underline{x}$ and $\underline{u}$ at the new conditions. The problem can then be linearized about these new conditions resulting in the optimal control problem with non-zero initial conditions. This is the state driving formulation which has been examined by Kalman and Koepcke [1] and Lapidus and Luus ([6] Chapter 3). However this approach is computationally inefficient with the reformulation and recalculation of control matrices for every setpoint change.

Even with the present formulation it would be desirable to relinearize and recalculate the control matrices if the new conditions were "permanent" and far enough away from the original ones. In the extreme case of sizeable changes in conditions or a very nonlinear process relinearization may be necessary along the trajectory as performed by Choquette, Noton, and Watson [7].

3.2. Optimal Model Following

In the model following formulation a model is selected (in the standard state space form) which exhibits the desired output response, $\underline{y}_m(t)$. The optimal control objective is then to have the process output approximate $\underline{y}_m(t)$ as closely as possible.

Consider the desired "setpoint model" defined as follows.

$$\dot{\underline{y}}_m = \underline{\underline{H}}_c \underline{y}_m + \underline{\underline{G}}_c \underline{y}_{sp} \quad . \tag{15}$$

The model output $\underline{y}_m$ is assumed to be equivalent to the subset $\underline{x}'$ defined in Section 3.1. Hence the criterion can be written in the

following form.

$$J = \sum_{i=1}^{N} [(\underline{x}_i - \underline{\underline{C}}^T \underline{y}_{mi})^T \underline{\underline{Q}}(\underline{x}_i - \underline{\underline{C}}^T \underline{y}_{mi}) + \underline{u}_{i-1}^T \underline{\underline{R}}\, \underline{u}_{i-1}] \; . \tag{16}$$

This form of the criterion and the models (Equations (1) and (15)) can be rearranged as follows.

$$\begin{bmatrix} \dot{\underline{x}} \\ \dot{\underline{y}}_m \end{bmatrix} = \begin{bmatrix} \underline{\underline{A}}_c & 0 \\ 0 & \underline{\underline{H}}_c \end{bmatrix} \begin{bmatrix} \underline{x} \\ \underline{y}_m \end{bmatrix} + \begin{bmatrix} \underline{\underline{B}}_c \\ 0 \end{bmatrix} \underline{u} + \begin{bmatrix} \underline{\underline{D}}_c & 0 \\ 0 & \underline{\underline{G}}_c \end{bmatrix} \begin{bmatrix} \underline{d} \\ \underline{y}_{sp} \end{bmatrix} \tag{17}$$

$$J = \sum_{i=1}^{N} \left[\begin{bmatrix} \underline{x}_i \\ \underline{y}_{mi} \end{bmatrix}^T \begin{bmatrix} \underline{\underline{Q}} & -\underline{\underline{Q}}\,\underline{\underline{C}}^T \\ -\underline{\underline{C}}\,\underline{\underline{Q}} & \underline{\underline{C}}\,\underline{\underline{Q}}\,\underline{\underline{C}}^T \end{bmatrix} \begin{bmatrix} \underline{x}_i \\ \underline{y}_{mi} \end{bmatrix} + \underline{u}_{i-1}^T \underline{\underline{R}}\, \underline{u}_{i-1} \right] \; . \tag{18}$$

This augmented formulation of the optimal control problem is equivalent to the formulation in Section 3.1 without the subset $\underline{x}'$ in the criterion. Feedback and feedforward control matrices, $\underline{\underline{K}}^*_{FB}$ and $\underline{\underline{K}}^*_{FF}$, can be evaluated from recursive relations, the augmented equivalents of Equations (5) and (6), resulting from the application of discrete dynamic programming.

$$\underline{u} = \underline{\underline{K}}^*_{FB} \begin{bmatrix} \underline{x} \\ \underline{y}_m \end{bmatrix} + \underline{\underline{K}}^*_{FF} \begin{bmatrix} \underline{d} \\ \underline{y}_{sp} \end{bmatrix} . \tag{19}$$

These control matrices can be partitioned to give the control law in the usual form where $\underline{\underline{K}}_{FB}$ and $\underline{\underline{K}}_{FF}$, being independent of the inclusion of the model following (for example if $\underline{y}_{sp} = 0$), have the same numerical values as the feedback/feedforward formulation.

$$\underline{u} = K_{FB}\underline{x} + K_{FF}\underline{d} + K_{M}\underline{y}_m + K_{SP}\underline{y}_{sp} \quad . \tag{20}$$

The implementation is illustrated by a schematic flow diagram in Figure 1.

The "setpoint model" parameters, the elements of G_c and H_c, are chosen according to the application. A number of useful points are listed below.

(a) Non-interaction between setpoint changes can be approached by making G_c and H_c diagonal.

(b) The diagonal elements of H_c can be chosen to give the desired rate of change of setpoint (a value of $-1/\tau$ will result in a first order response with time constant τ).

(c) Once H_c is chosen, G_c can be evaluated to give the desired model gain matrix (usually $-H_c^{-1}G_c = I$).

As was the case for direct setpoint control some offset will occur with setpoints away from the original steady state. The final value theorem gives the following relation.

$$\begin{aligned} \underline{y}_{ss} = C\,\underline{x}_{ss} &= C(A_c + B_c K_{FB})^{-1} B_c (K_M H_c^{-1} G_c - K_{SP})\underline{y}_{sp} \\ &= E\,\underline{y}_{sp} \quad . \end{aligned} \tag{21}$$

If it is desirable to correct for the offset, the matrices G_c and K_{SP} can be "corrected" by postmultiplication by the inverse of E .

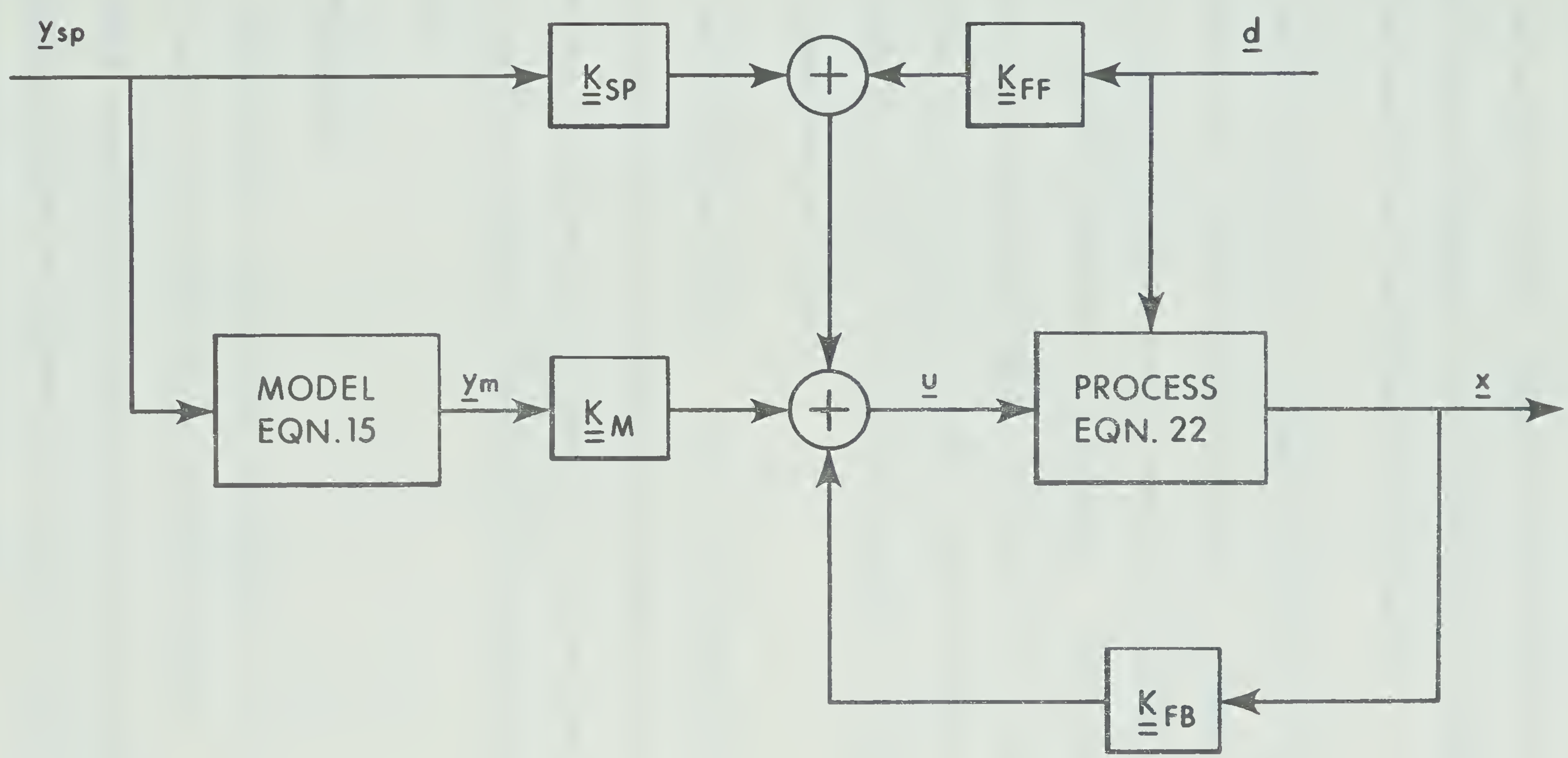


FIGURE 1. SCHEMATIC OF MODEL FOLLOWING CONTROL
(See Tables 1 and 2 for Control Matrices)

4. THE EVAPORATOR MODEL

The setpoint control schemes were evaluated on a pilot plant double effect evaporator in the Department of Chemical and Petroleum Engineering at the University of Alberta.

The first effect is a calandria type unit with an eight inch tube bundle and a three gallon capacity. It produces about 4½ percent triethylene glycol from a feed of 5 lb./min. of 3 percent glycol and a nominal steamrate of 2 lb./min.. The vapour from the first effect heats the second effect, a long tube vertical unit with forced circulation. This unit is under about 15 inches of vacuum and produces about 1½ lb./min. of 10 percent product. A schematic flow diagram is presented as Figure 2.

The evaporator model is a five dimensional state equation with variables in normalized perturbation form. The model equations are presented as Equation (22) and the state, control, and load vectors are defined in terms of the process variables in the Nomenclature.

5. IMPLEMENTATION

Multivariable control was implemented on the pilot plant evaporator using an IBM 1800 digital control computer. The process operates under Direct Digital Control (DDC) and the computer uses a multiprogramming executive system which permits simultaneous execution of DDC, control programs, and off-line jobs.

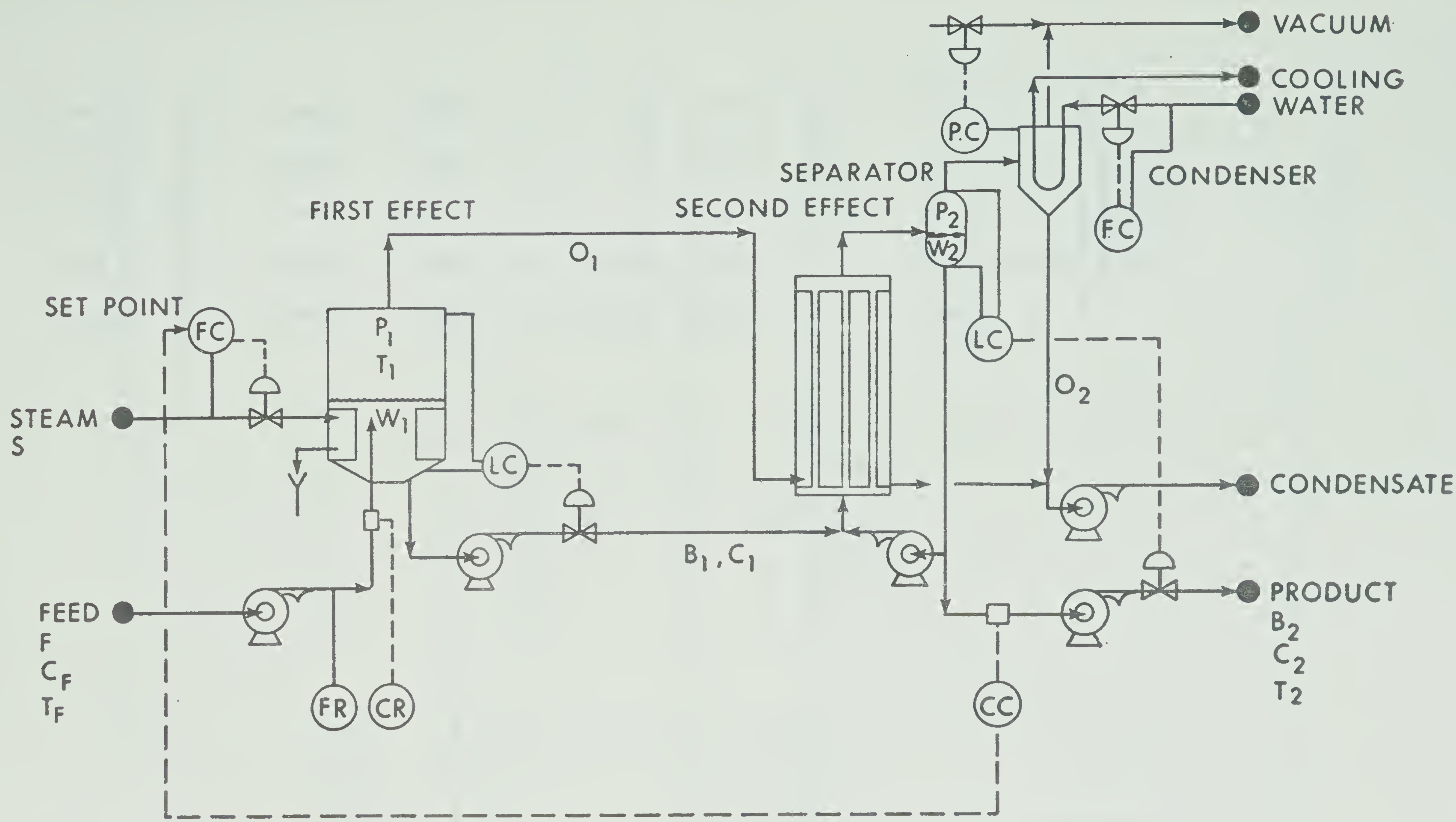


FIGURE 2. PILOT PLANT DOUBLE EFFECT EVAPORATOR

$$\overset{\cdot}{\begin{bmatrix} W1 \\ C1 \\ H1 \\ W2 \\ C2 \end{bmatrix}} = \begin{bmatrix} 0 & -.00156 & -.1711 & 0 & 0 \\ 0 & -.1419 & .1711 & 0 & 0 \\ 0 & -.00875 & -1.102 & 0 & 0 \\ 0 & -.00128 & -.1489 & 0 & .00013 \\ 0 & .0605 & .1489 & 0 & -.0591 \end{bmatrix} \begin{bmatrix} W1 \\ C1 \\ H1 \\ W2 \\ C2 \end{bmatrix} + \begin{bmatrix} 0 & -.143 & 0 \\ 0 & 0 & 0 \\ .392 & 0 & 0 \\ 0 & .108 & -.0592 \\ 0 & -.0486 & 0 \end{bmatrix} \begin{bmatrix} S \\ B1 \\ B2 \end{bmatrix}$$

$$+ \begin{bmatrix} .2174 & 0 & 0 \\ -.074 & .143 & 0 \\ -.036 & 0 & .181 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F \\ CF \\ HF \end{bmatrix}$$

$$\begin{bmatrix} W1 \\ W2 \\ C2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} W1 \\ C1 \\ H1 \\ W2 \\ C2 \end{bmatrix} \qquad \text{Equation (22)}$$

5.1. Control System

Control calculations, including the control algorithm and "setpoint model" calculations, are carried out by a periodically executed program written in FORTRAN. The computation time for the control algorithm is essentially that for fifteen multiplications compared to that for three multiplications for conventional DDC. In addition there are "overhead" calculations required for state estimation and model calculations. System usage has not exceeded eight percent for the usual 64 second control interval. The program obtains state variable measurements from DDC data acquisition loops and makes control variable changes by adjusting the setpoints of DDC flow control loops.

Details are available in Chapter 9 on the implementation and some of the design considerations.

5.2. Control Matrices

The design parameters for the problem formulations used in this paper were investigated in Chapter 5 and the "best" values are listed below.

$$\underline{\underline{Q}} = \text{diag}\ (10,1,1,10,100)$$

$$\underline{\underline{R}} = 0\ , \quad \Delta t = 64 \text{ seconds}\ . \tag{23}$$

The setpoint control approach requires no other parameters and the resulting control matrices are listed in Table 1.

TABLE 1

SETPOINT CONTROL MATRICES

$$\underline{\underline{K}}_{FB} = \begin{bmatrix} 5.095 & -1.475 & -2.68 & 0 & -14.56 \\ 3.95 & .36 & .21 & 0 & 7.39 \\ 5.31 & 1.19 & -.11 & 15.83 & 18.81 \end{bmatrix}$$

$$\underline{\underline{K}}_{FF} = \begin{bmatrix} 2.047 & -.136 & -.463 \\ 1.019 & .037 & 0 \\ 1.135 & .116 & 0 \end{bmatrix} \qquad \underline{\underline{K}}_{SP} = \begin{bmatrix} -5.10 & 0 & 16.08 \\ -3.95 & 0 & -7.77 \\ -5.31 & -15.83 & -20.06 \end{bmatrix}$$

$$\underline{\underline{E}} = \begin{bmatrix} 1. & .9 \times 10^{-7} & -.4 \times 10^{-3} \\ .9 \times 10^{-4} & 1. & -.2 \times 10^{-3} \\ -.3 \times 10^{-3} & -.5 \times 10^{-7} & .999 \end{bmatrix} \approx \underline{\underline{I}}$$

A non-interacting first-order model was chosen to demonstrate the model following technique. The selection of time constants for this model depends upon both the process and the desired rate of change of the process outputs. If the model time constants are smaller than those of the closed loop process, that is the process cannot physically keep up, then it will change conditions at its "maximum" rate. Therefore it is only when the desired rate of change is slower than the process' "maximum", that the increased complexity of the model following formulation and implementation is worthwhile. Model time constants of one and five minutes were chosen to try and illustrate these effects and the model gain was chosen to be unity. The control matrices resulting from these choices of a model are shown in Tables 1 and 2. The control matrices $\underline{\underline{K}}_{FB}$ and $\underline{\underline{K}}_{FF}$ evaluated from the augmented model following formulation are numerically equivalent to those evaluated from the direct setpoint formulation since they are functions of the weighting parameters only and not the formulations.

Once the model transient has died out the model following formulation is equivalent to the direct setpoint formulation and as would be expected the control matrices $\underline{\underline{K}}_{M}$ and $\underline{\underline{K}}_{SP}$ in Table 2 sum to equal $\underline{\underline{K}}_{SP}$ in Table 1.

An examination of the $\underline{\underline{E}}$ matrix in Table 1 shows that it is very close to being a unit matrix indicating that the offsets due to proportional control matrices are extremely small.

TABLE 2

MODEL FOLLOWING CONTROL MATRICES

(a) 1 minute time constant. ($\underline{\underline{H}} = -\underline{\underline{I}}$, $\underline{\underline{G}} = \underline{\underline{I}}$)

$$\underline{\underline{K}}_M = \begin{bmatrix} -1.15 & 0 & 3.68 \\ -1.26 & 0 & -2.99 \\ -1.86 & -5.45 & -6.80 \end{bmatrix} \qquad \underline{\underline{K}}_{SP} = \begin{bmatrix} -3.95 & 0 & 12.42 \\ -2.70 & 0 & -4.77 \\ -3.45 & -10.38 & -13.26 \end{bmatrix}$$

(b) 5 minute time constant. ($\underline{\underline{H}} = -0.2\ \underline{\underline{I}}$, $\underline{\underline{G}} = 0.2\ \underline{\underline{I}}$)

$$\underline{\underline{K}}_M = \begin{bmatrix} -3.67 & 0 & 11.61 \\ -3.12 & 0 & -6.51 \\ -4.32 & -12.79 & -16.13 \end{bmatrix} \qquad \underline{\underline{K}}_{SP} = \begin{bmatrix} -1.43 & 0 & 4.48 \\ -.84 & 0 & -1.25 \\ -1.00 & -3.04 & -3.93 \end{bmatrix}$$

6. EXPERIMENTAL RESULTS

A number of experimental runs were carried out in order to evaluate the setpoint control methods on a real process. Table 3 summarizes these runs.

Figures 3 to 5 compare the three setpoint control schemes for a ten percent change in product concentration, C2 , setpoint. There are a number of points arising from the comparison.

(a) The C2 responses in Figures 3 and 4 are better for a decrease in setpoint than an increase because of more available control action. Physical constraints result in limits of zero and 3 lb./min. on steam flowrate, S . Since the normal operating rate is 2 lb./min. there is twice as much control action available for decreases in setpoint than there is for increases in setpoint.

(b) The process does not keep up with the model in Figure 4 indicating that the model time constant was smaller than that of the process. As a result the process responds as fast as it can giving comparable results to Figure 3, the direct scheme. In Figure 5 the process follows the "slower" model closely.

(c) Figure 5 where the model following operates successfully shows smaller deviations in levels, W1 and W2 , as a result of the non-interacting model.

TABLE 3

SUMMARY OF EXPERIMENTAL RUNS

Figure	Run	Control	Disturbance	State Estimation
3	83	Direct	$\pm$10% C2	A2
4	84	MF(τ = 1)	$\pm$10% C2	A2
5	76	MF(τ = 5)	$\pm$10% C2	A1
6	166	Direct	$\pm$15% W1	A1
7	72	MF(τ = 1)	$\pm$15% W1	A1
8	77	MF(τ = 5)	$\pm$15% W1	A1
9	168	Direct	$\pm$10% C2	A1
10	69	Direct	$\pm$10% C2, $\pm$10% F	A1
11	79	Direct + FF	$\pm$10% C2, $\pm$10% F	A1

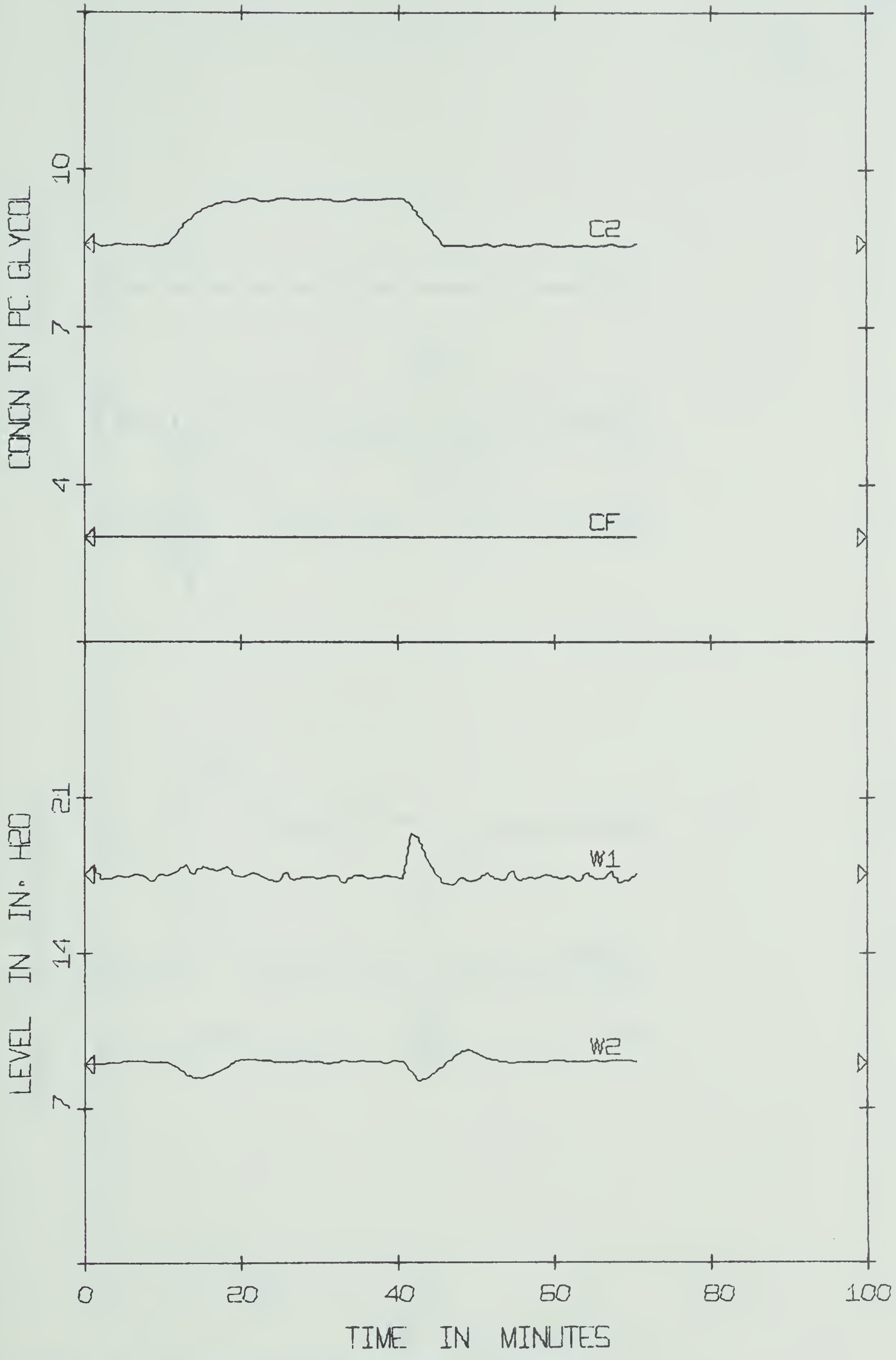


FIGURE 3a. DIRECT SETPOINT CHANGE IN C2. I
(EXP/10%C2/SP/Q1/R1/D1/A2/MVC83)

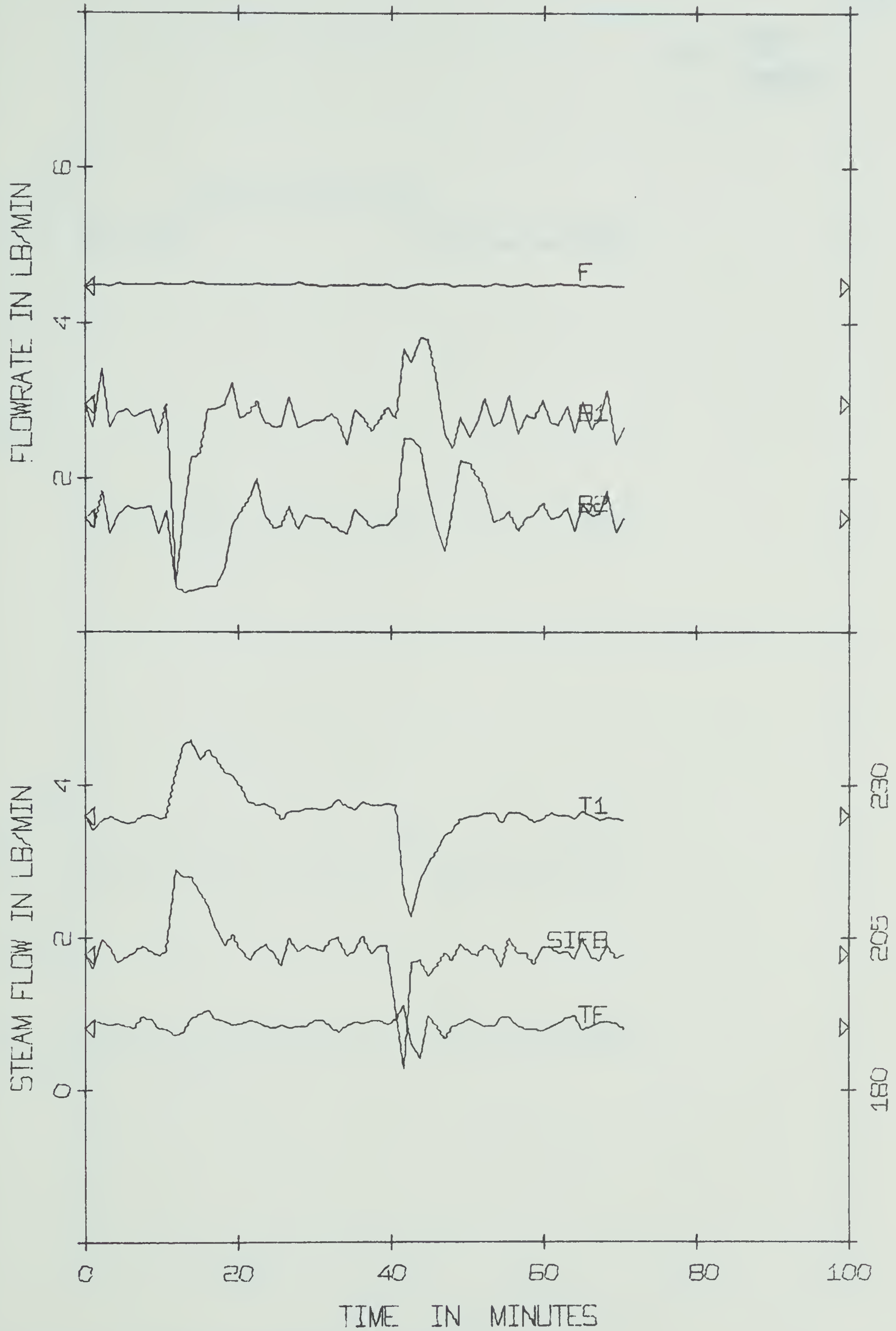


FIGURE 3b. DIRECT SETPOINT CHANGE IN C2. I
(EXP/10%C2/SP/Q1/R1/D1/A2/MVC83)

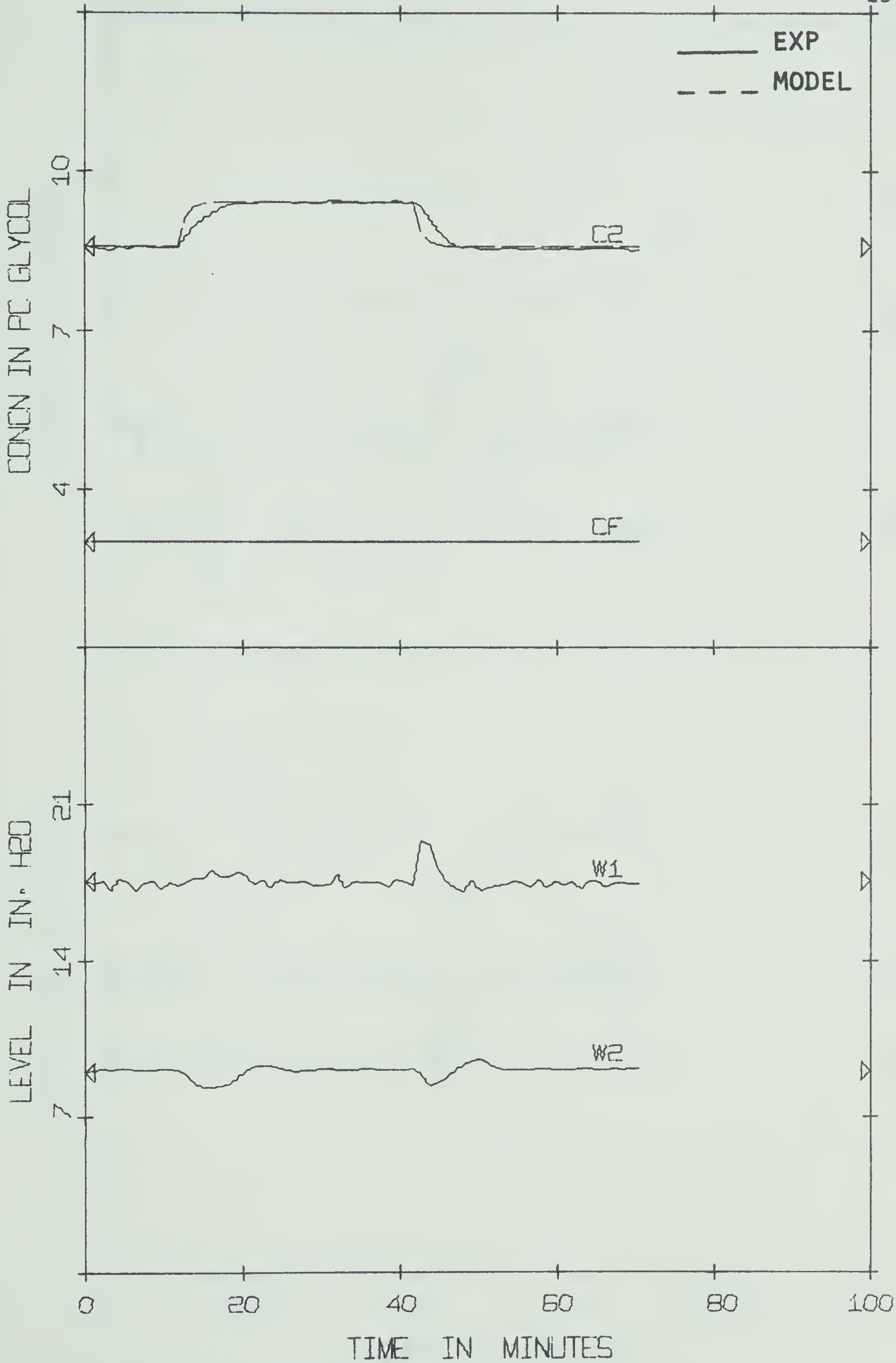


FIGURE 4a. MODEL FOLLOWING CHANGE IN C2. I
(EXP/10%C2/MF1/Q1/R1/D1/A2/MVC84)

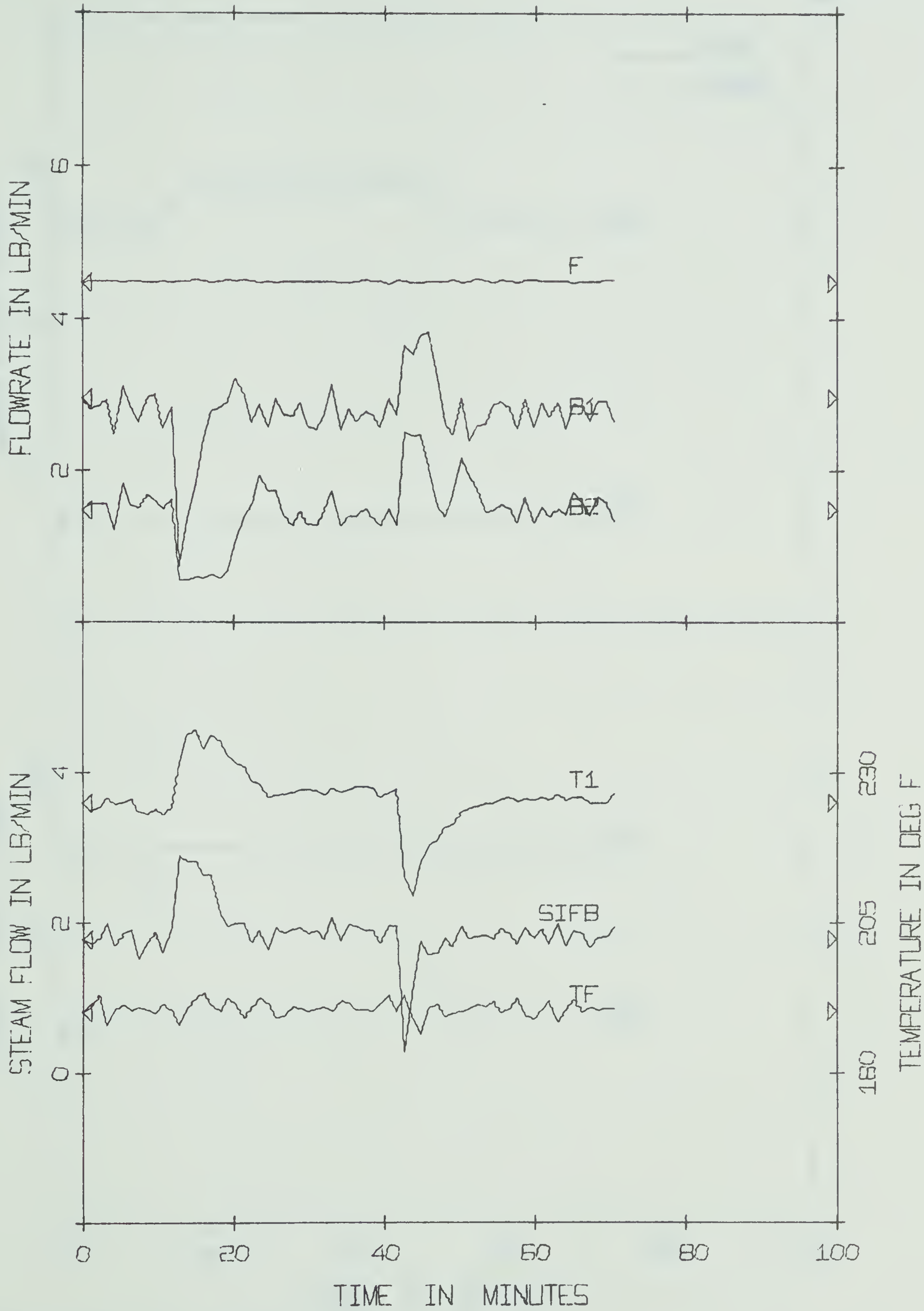


FIGURE 4b. MODEL FOLLOWING CHANGE IN C2. I
(EXP/10%C2/MF1/Q1/R1/D1/A2/MVC84)

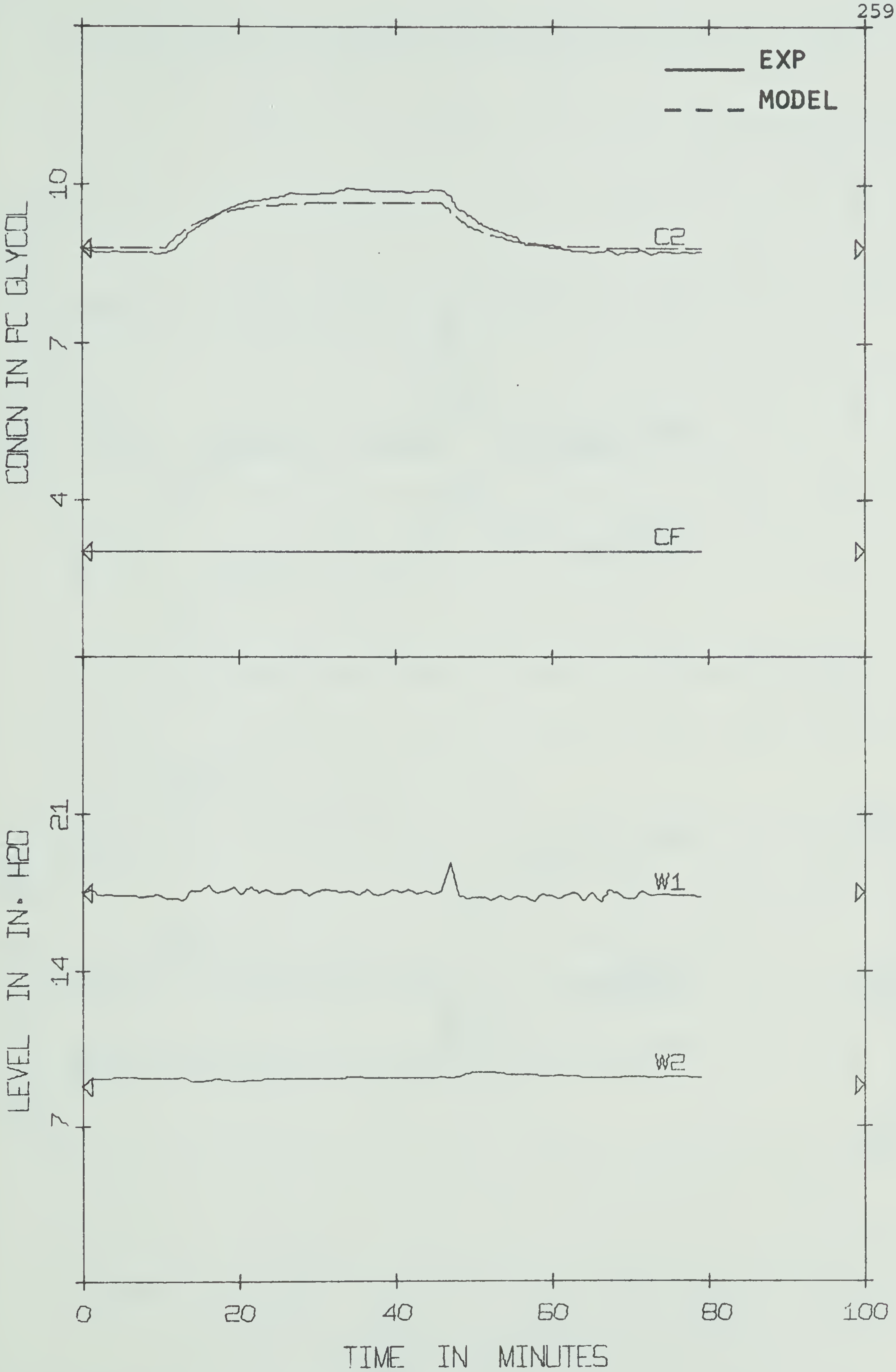
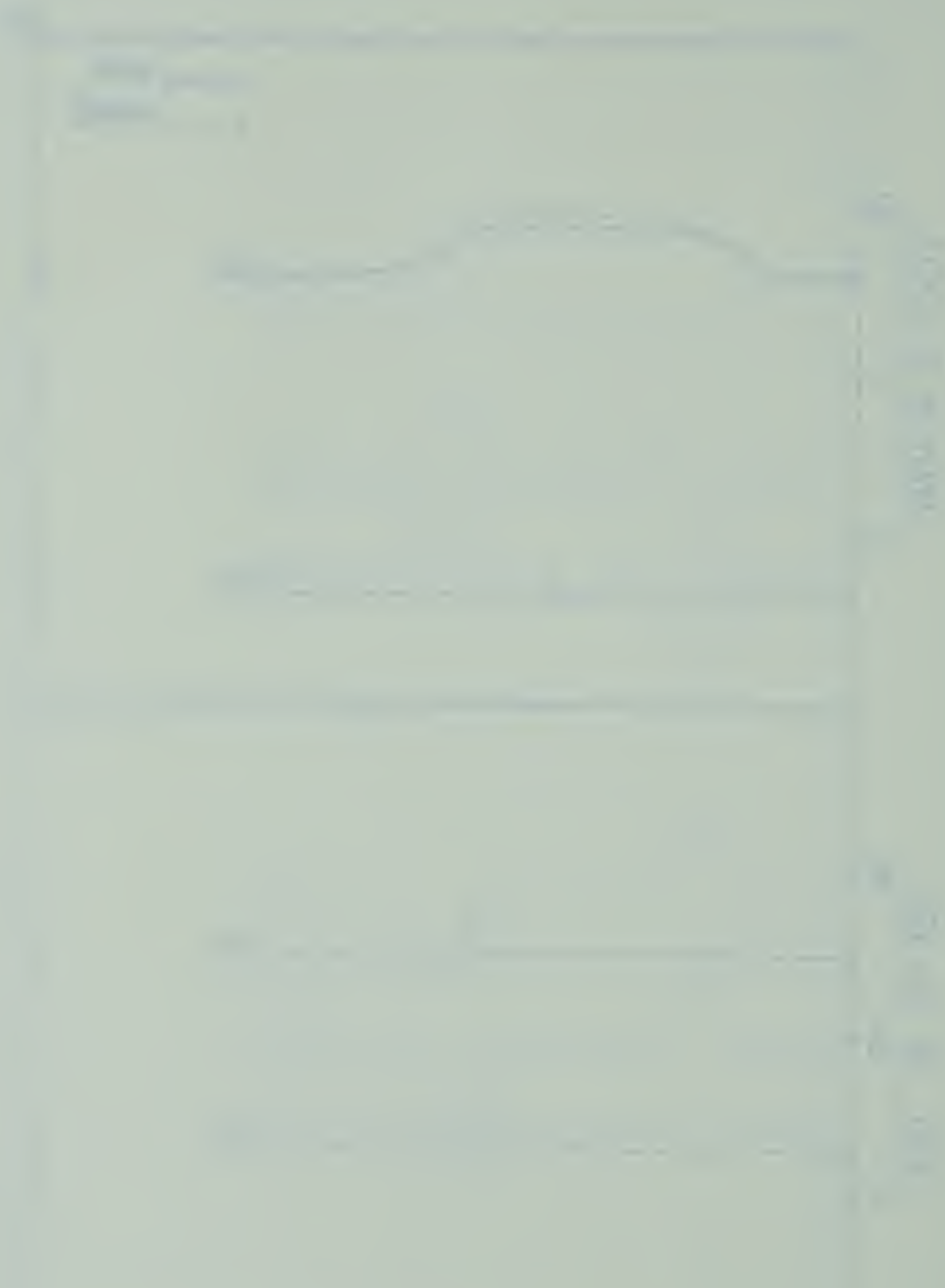


FIGURE 5a. MODEL FOLLOWING CHANGE IN C2. II
(EXP/10%C2/MF5/Q1/R1/D1/A1/MVC76)



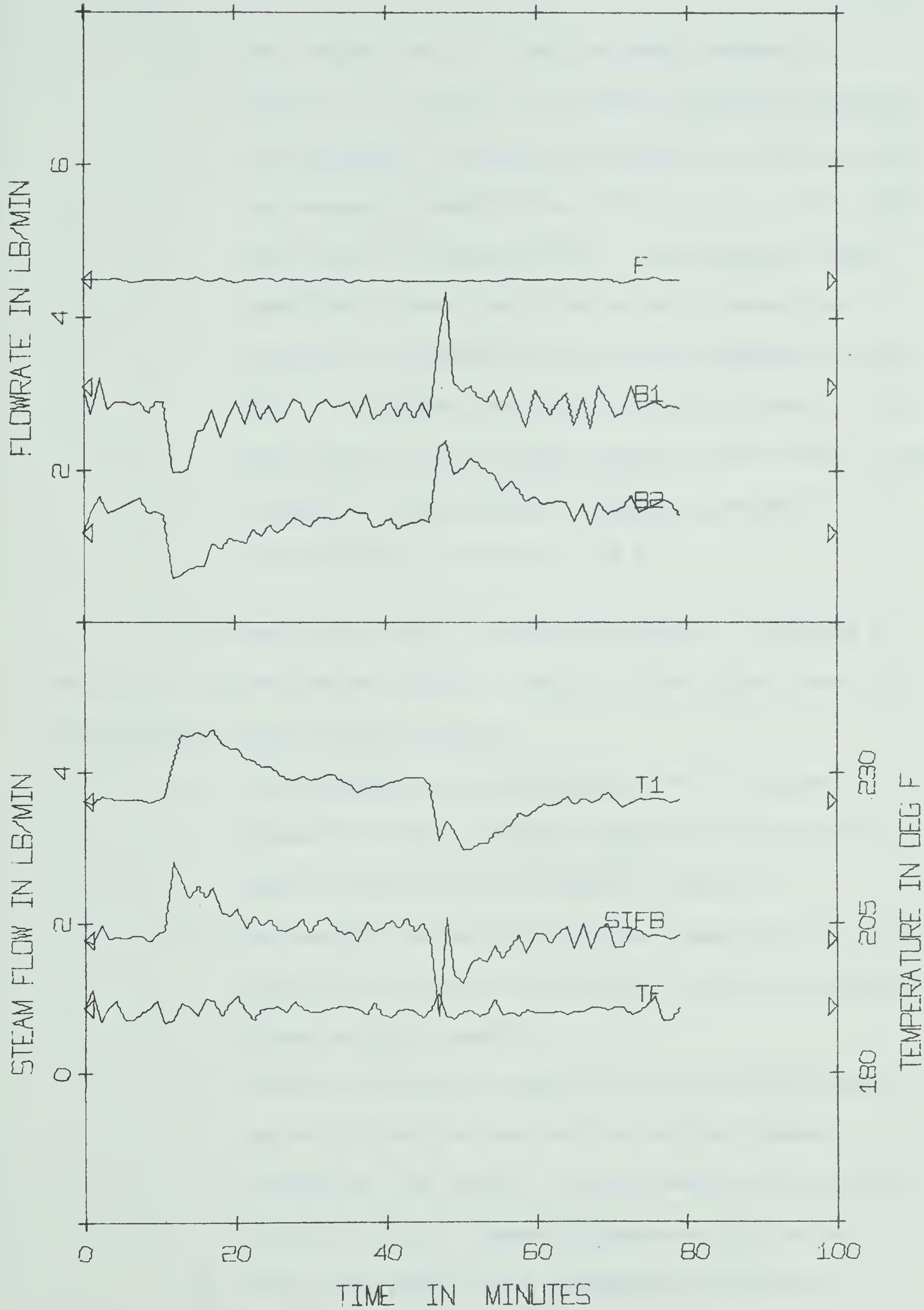


FIGURE 5b. MODEL FOLLOWING CHANGE IN C2. II
(EXP/10%C2/MF5/Q1/R1/D1/A1/MVC76)

(d) The "offset" of C2 from the model response in Figure 5 is a result of the state estimation procedure of "averaging" the predicted (from the evaporator model) and measured concentrations and of errors in the model gains (due to linearization). The evaporator model predicted a lower concentration which caused the process to "overshoot" so that their weighted average, the C2 estimate, followed the setpoint model. This effect does not occur when measured values alone are used (Figure 4). This effect is even more evident in the W1 responses in Figures 7 and 8.

The same three control schemes are compared in Figures 6 to 8 for a fifteen percent setpoint change in first effect level, W1. The following points are of interest.

(a) The overshoot and oscillation in W1 in Figure 6 (under the direct setpoint scheme) is avoided by the model following control schemes (Figure 7).

(b) The smaller process time constant associated with W1 allows the level to keep up with both the one and five minute setpoint models.

(c) The model following schemes, particularly the slower one which requires less control action (compare Figures 6b, 7b, and 8b), produce smaller interactions in W2 and C2 (compare Figures 6a, 7a, and 8a).

(d) The effect of the state estimation procedure and incorrect evaporator model gains is again evident in Figures 7 and 8.

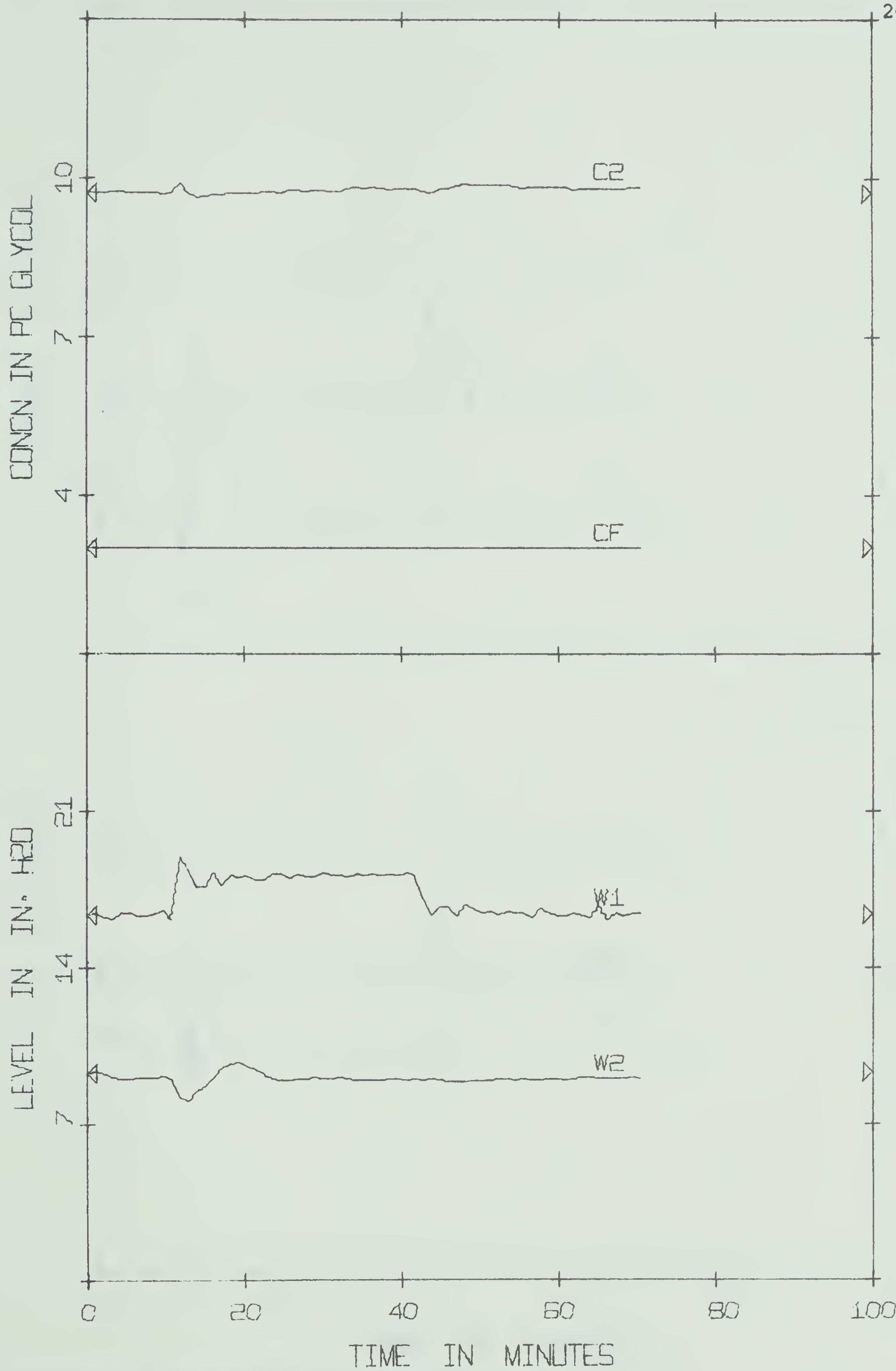


FIGURE 6a. DIRECT SETPOINT CHANGE IN W1
(EXP/15%W1/SP/Q1/R1/D1/A1/MVC166)

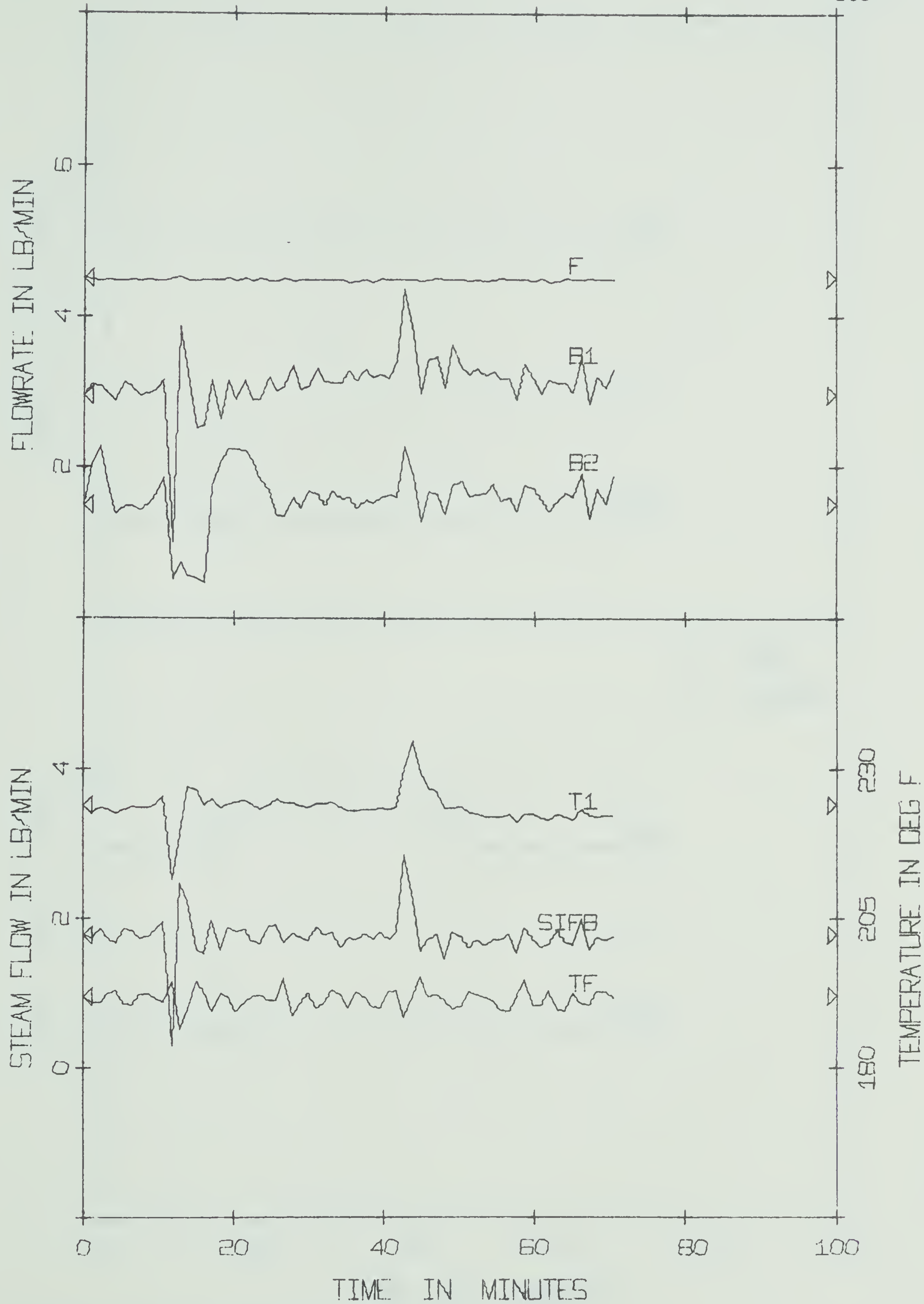


FIGURE 6b. DIRECT SETPOINT CHANGE IN W1
(EXP/15%W1/SP/Q1/R1/D1/A1/MVC166)

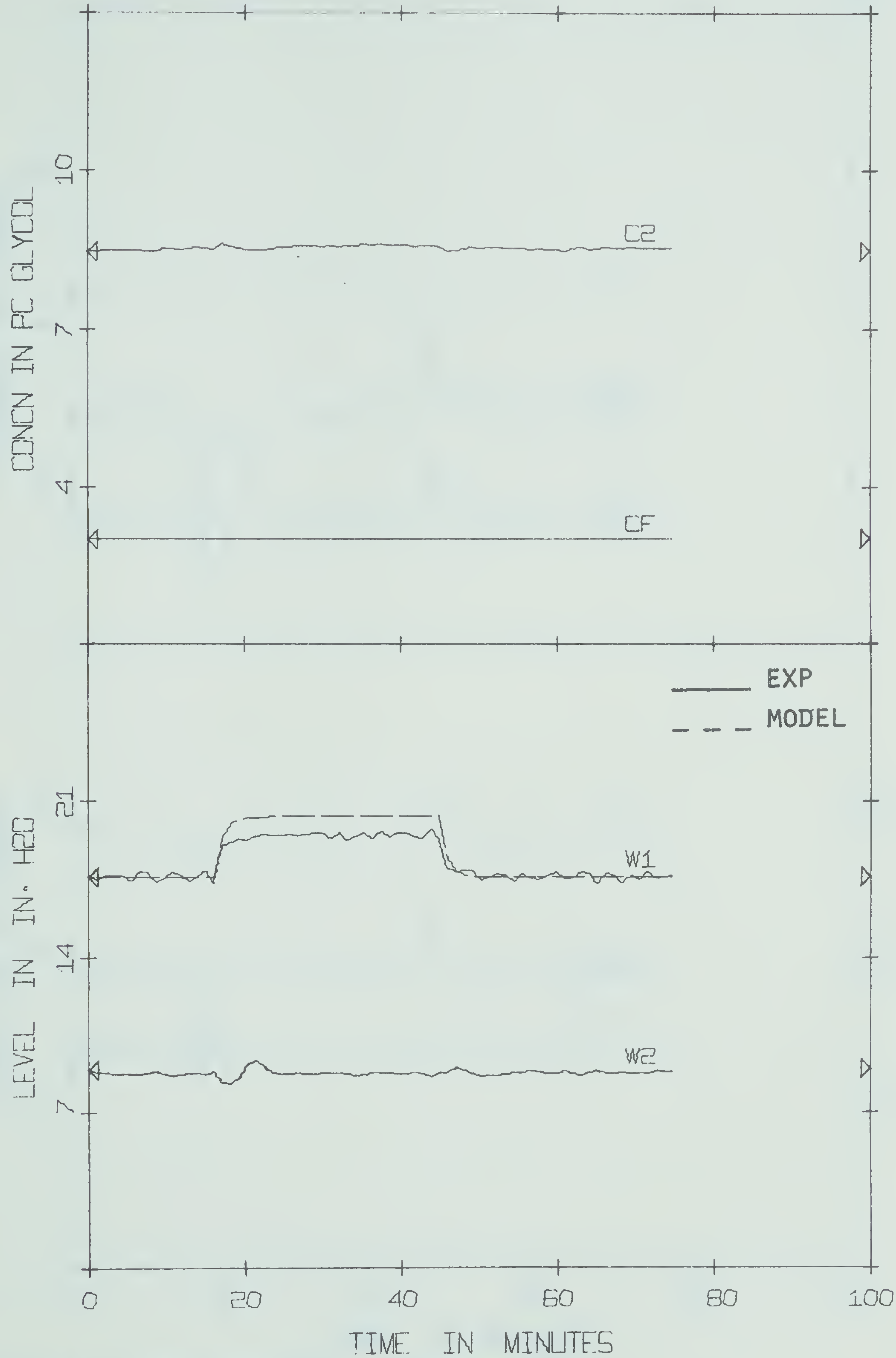


FIGURE 7a. MODEL FOLLOWING CHANGE IN W1. I
(EXP/15%W1/MF1/Q1/R1/D1/A1/MVC72)

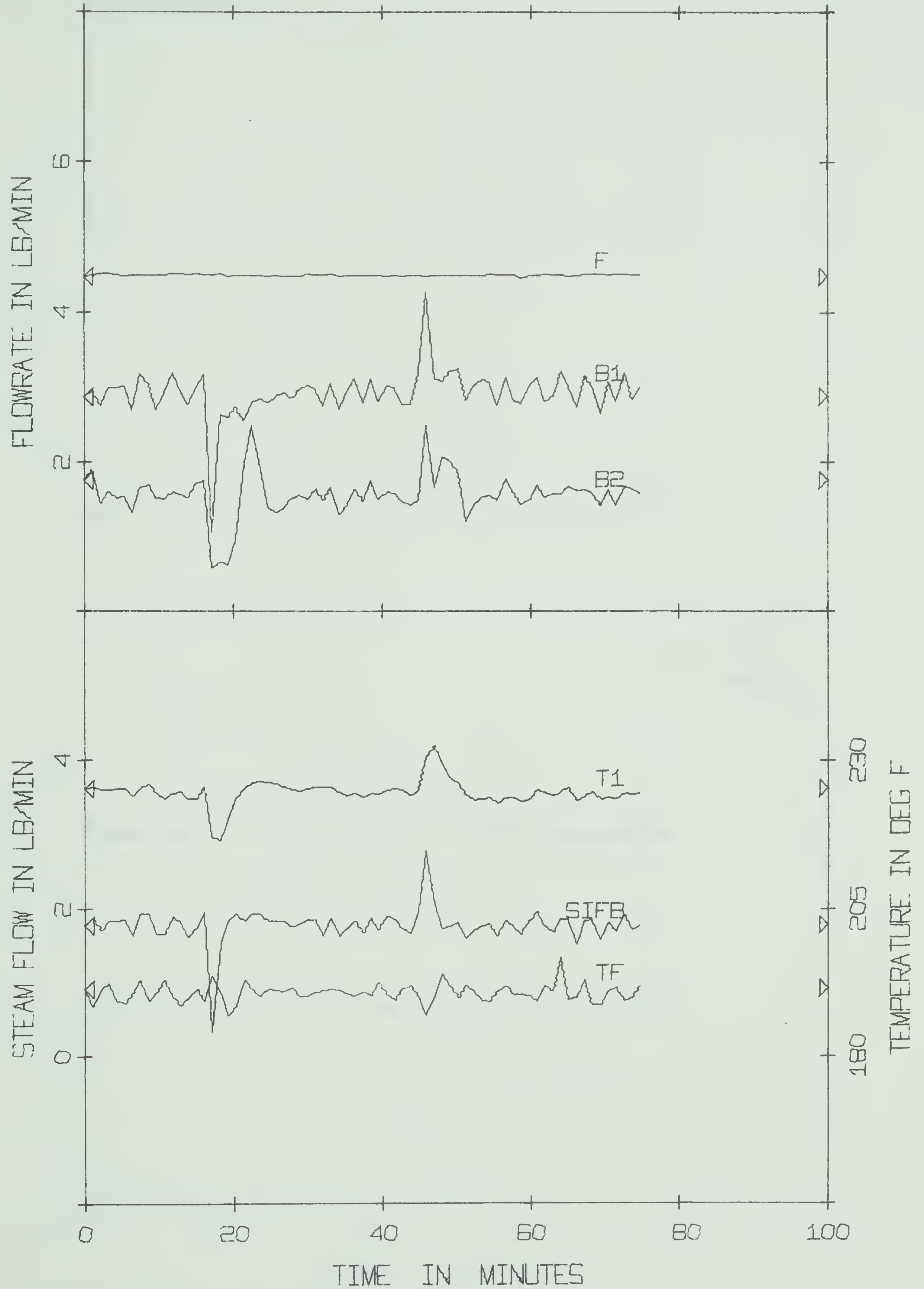


FIGURE 7b. MODEL FOLLOWING CHANGE IN W1. I
(EXP/15%W1/MF1/Q1/R1/D1/A1/MVC72)

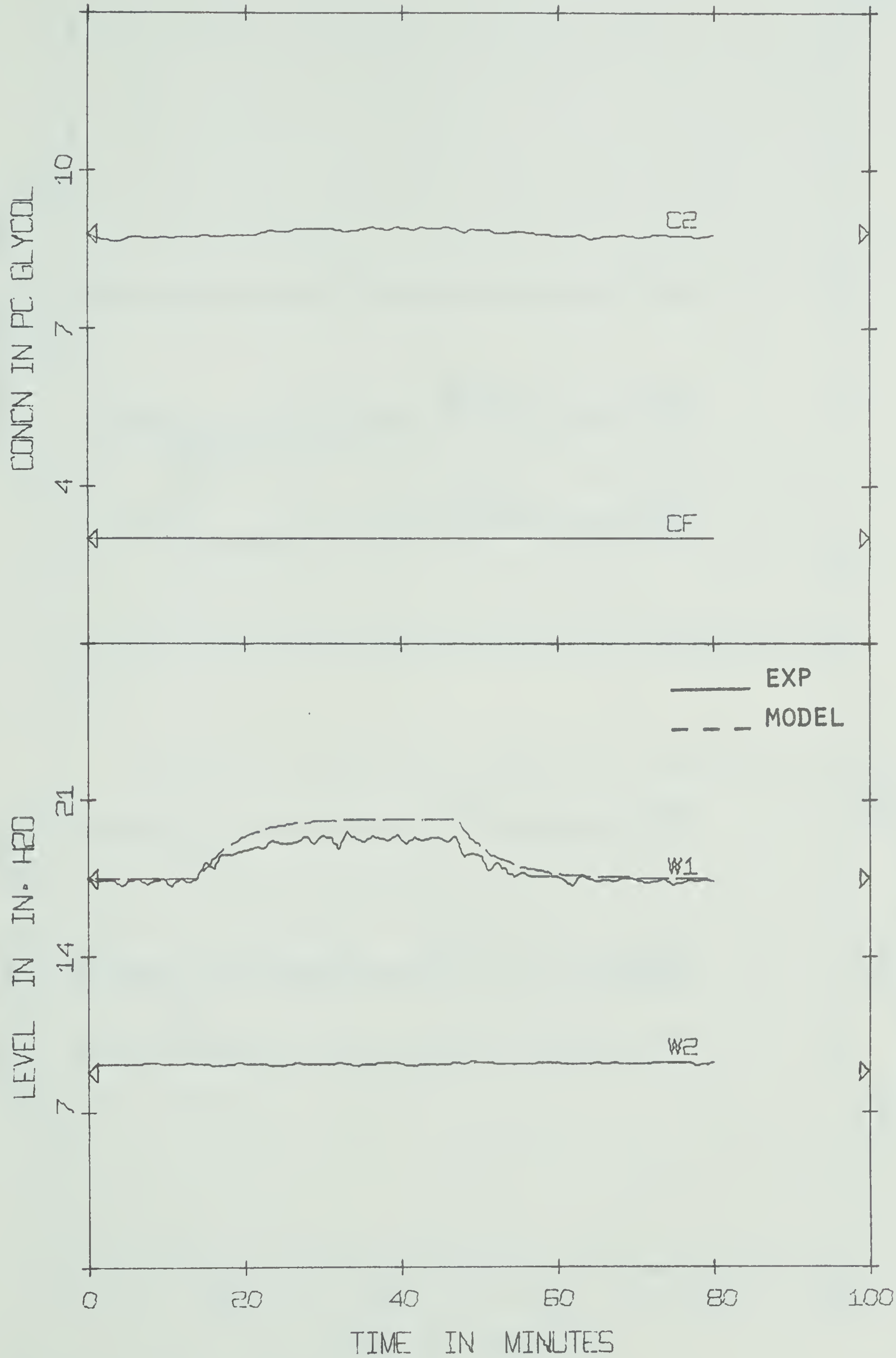


FIGURE 8a. MODEL FOLLOWING CHANGE IN W1. II
(EXP/15%W1/MF5/Q1/R1/D1/A1/MVC77)

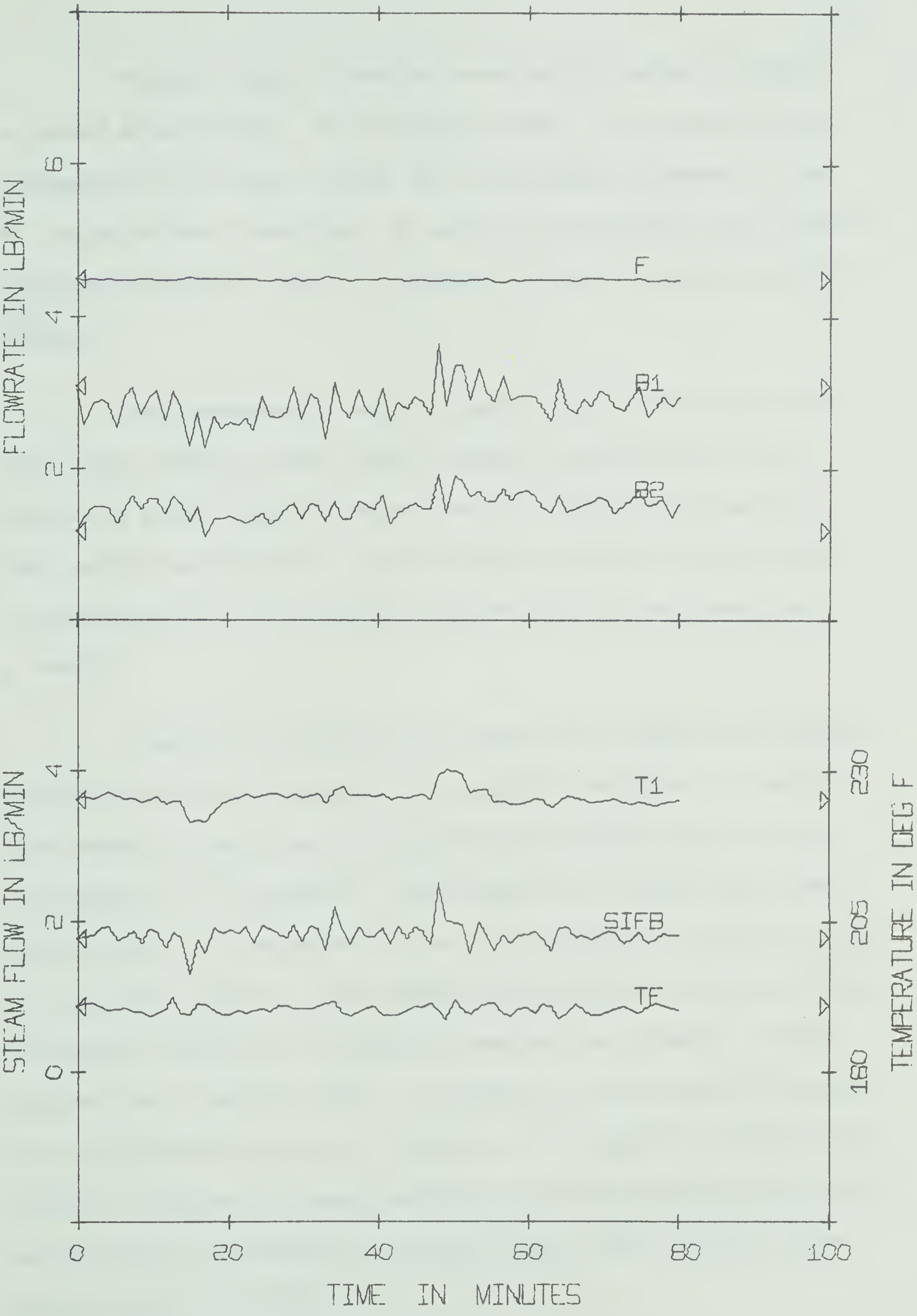


FIGURE 8b. MODEL FOLLOWING CHANGE IN W1. II
(EXP/15%W1/MF5/Q1/R1/D1/A1/MVC77)

The same type of behavior occurred for setpoint changes in second effect level, W2, although, since W2 is essentially independent of the other states (the off-diagonal elements in the W2 column of the closed-loop $\underline{\underline{A}}$ matrix are negligible) very little interaction with W1 and C1 occurred in any of the three control schemes.

The experimental model following runs showed that a setpoint model with one minute time constants corresponding to the levels and a five minute constant for C2 would have given the best overall performance. This conclusion may also have been made by examining the time constants (eigenvalues) of the closed loop $\underline{\underline{A}}$ matrix.

Figures 9 to 11 show the results of a study of the effects of simultaneous load changes on the setpoint responses. A ten percent change in feed flowrate was introduced at the same time as a 10% change in C2 setpoint. The evaporator was under direct setpoint control. A comparison of the C2 responses in Figures 9 and 10 shows that there is a much slower rise to the new setpoint (about 30 minutes compared to 15 minutes) when the feed flowrate "worked against" the setpoint change. A decrease in feed flowrate showed no noticeable effect on the C2 transient. The addition of feedforward action (in Figure 11) almost nullified the effects of the disturbance, particularly on a decrease in setpoint where there was more steam control action available.

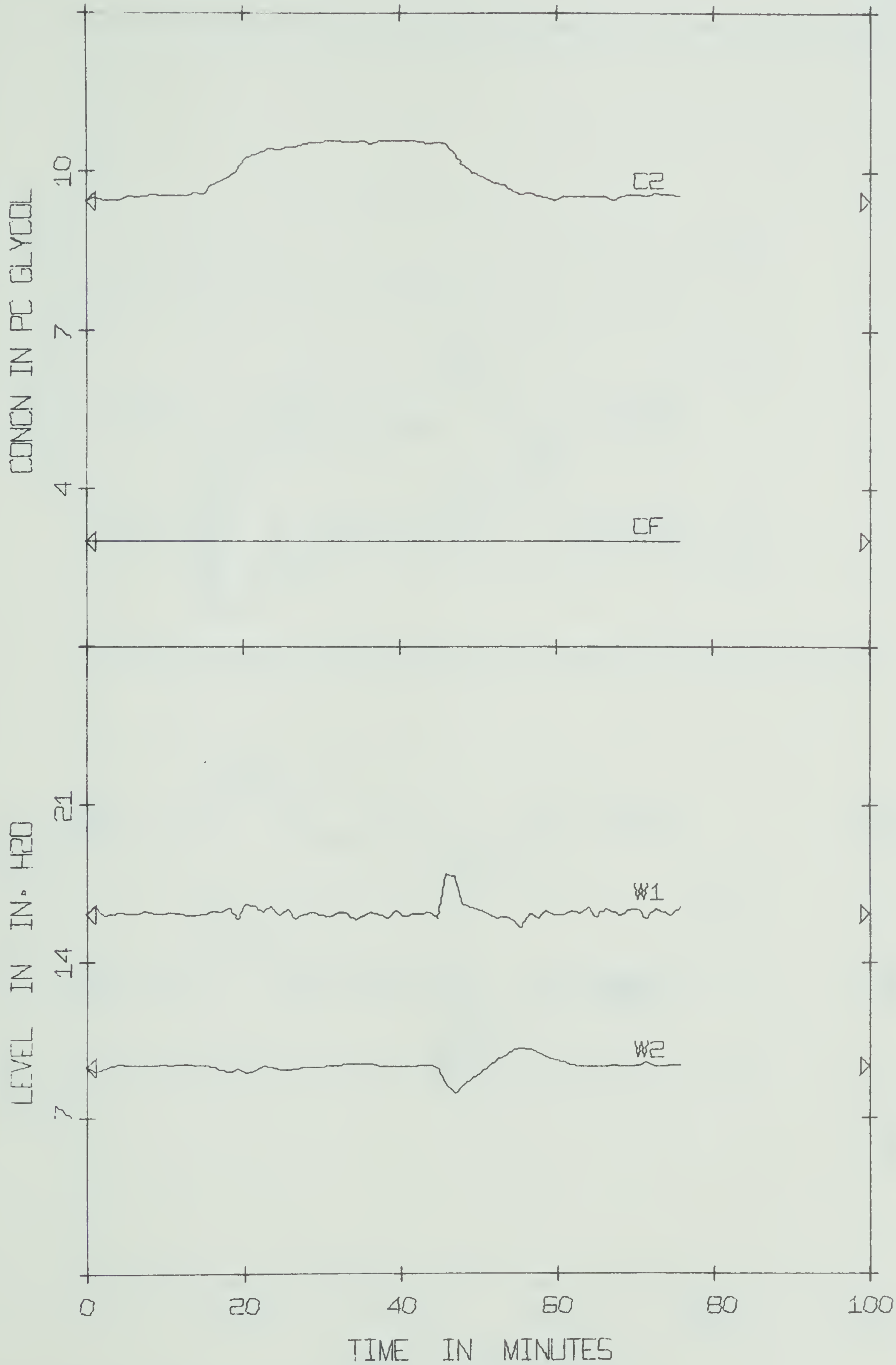
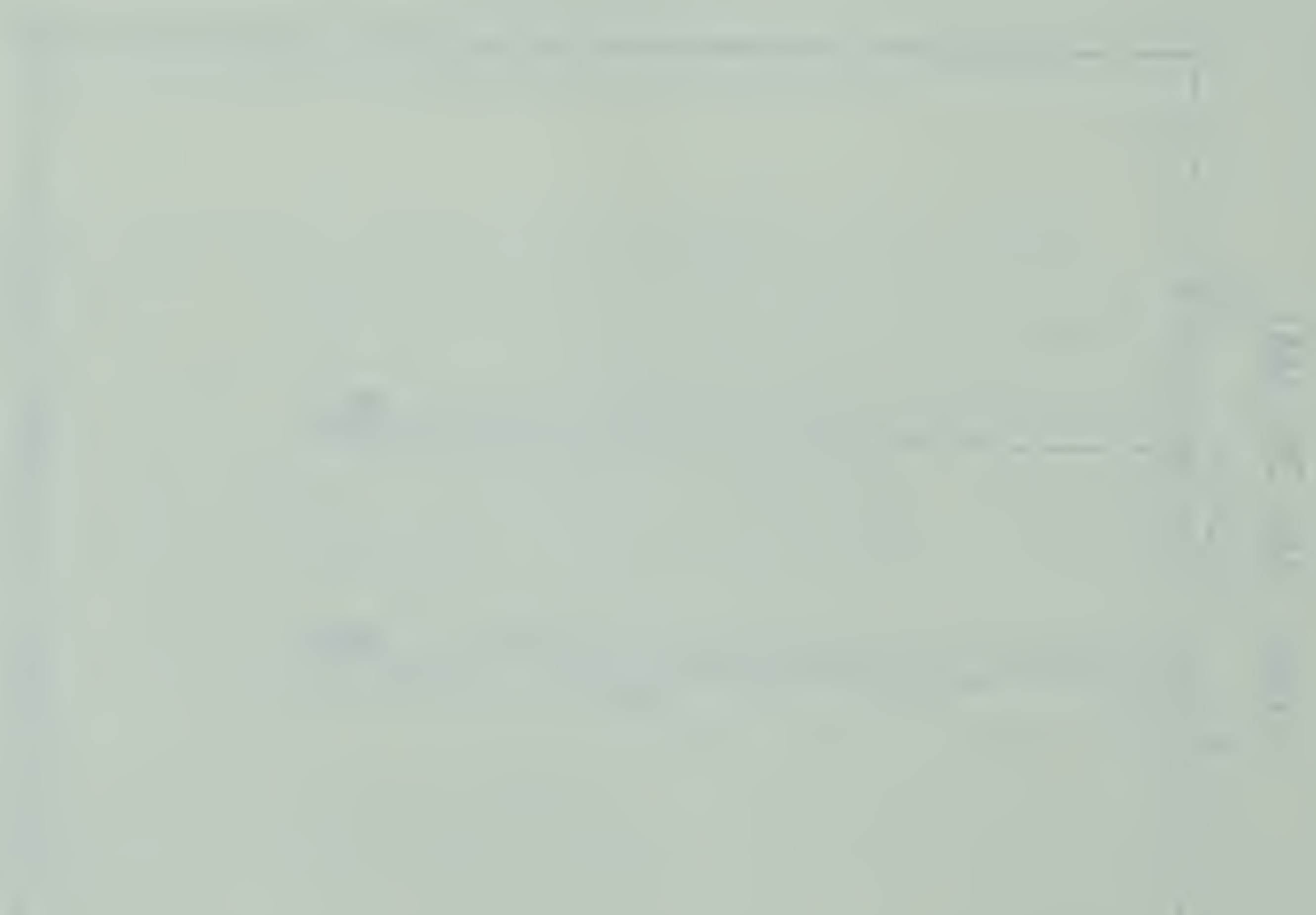


FIGURE 9a. DIRECT SETPOINT CHANGE IN C2. II
(EXP/10%C2/SP/Q1/R1/D1/A1/MVC168)



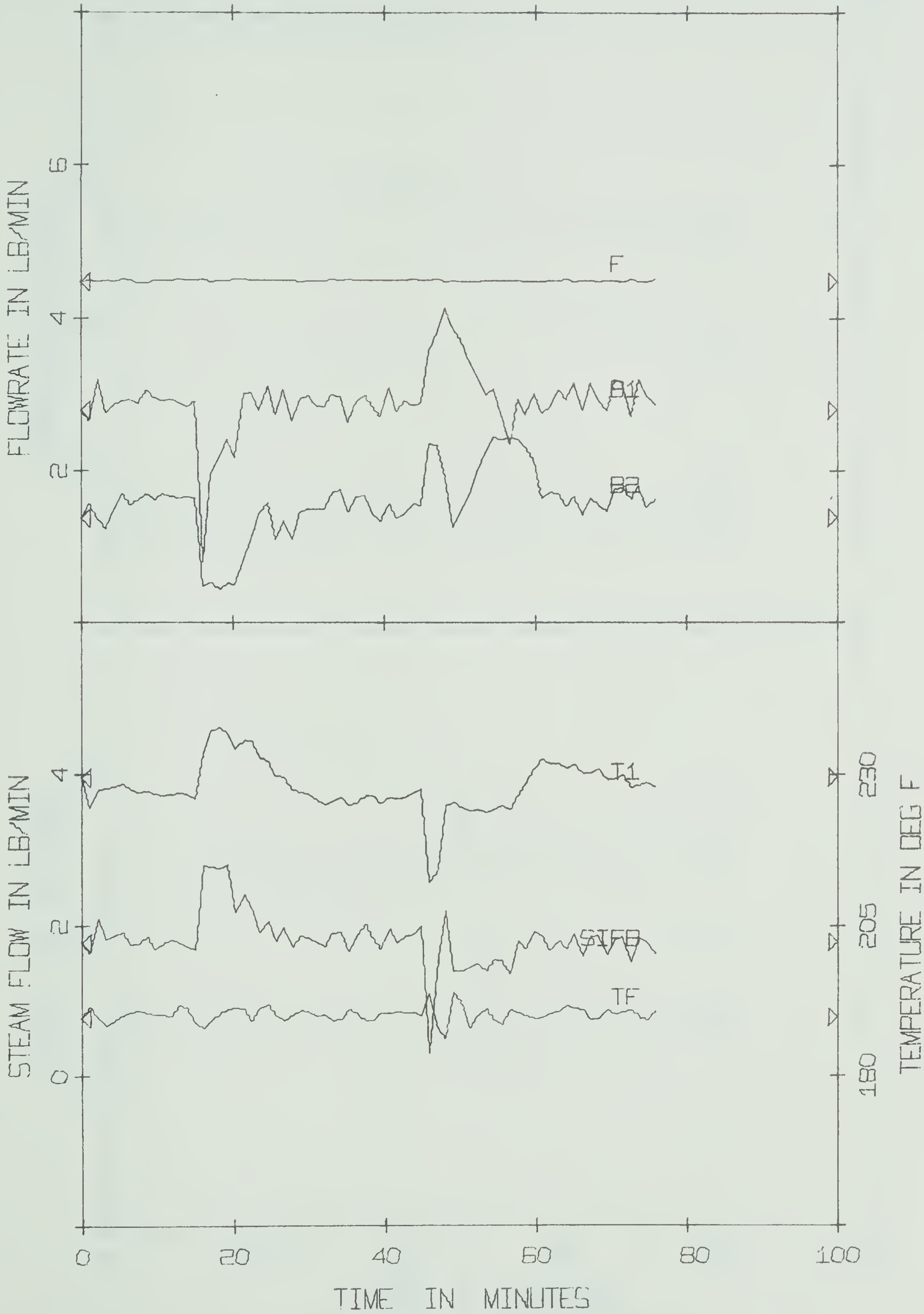


FIGURE 9b. DIRECT SETPOINT CHANGE IN C2. II
(EXP/10%C2/SP/Q1/R1/D1/A1/MVC168)

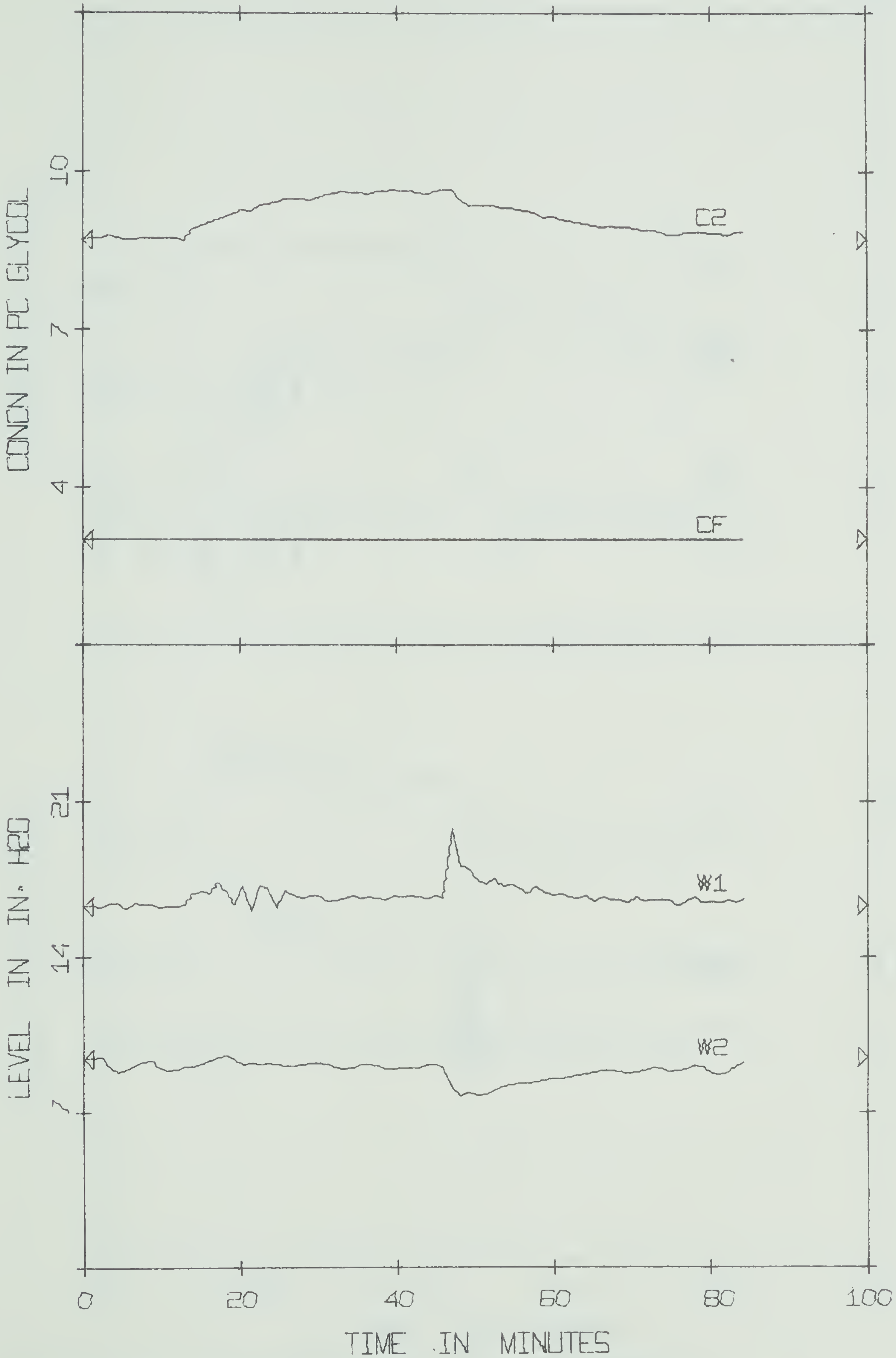


FIGURE 10a. SIMULTANEOUS SETPOINT AND LOAD CHANGES. I
(EXP/10%C2, 10%F/SP/Q1/R1/D1/A1/MVC69)

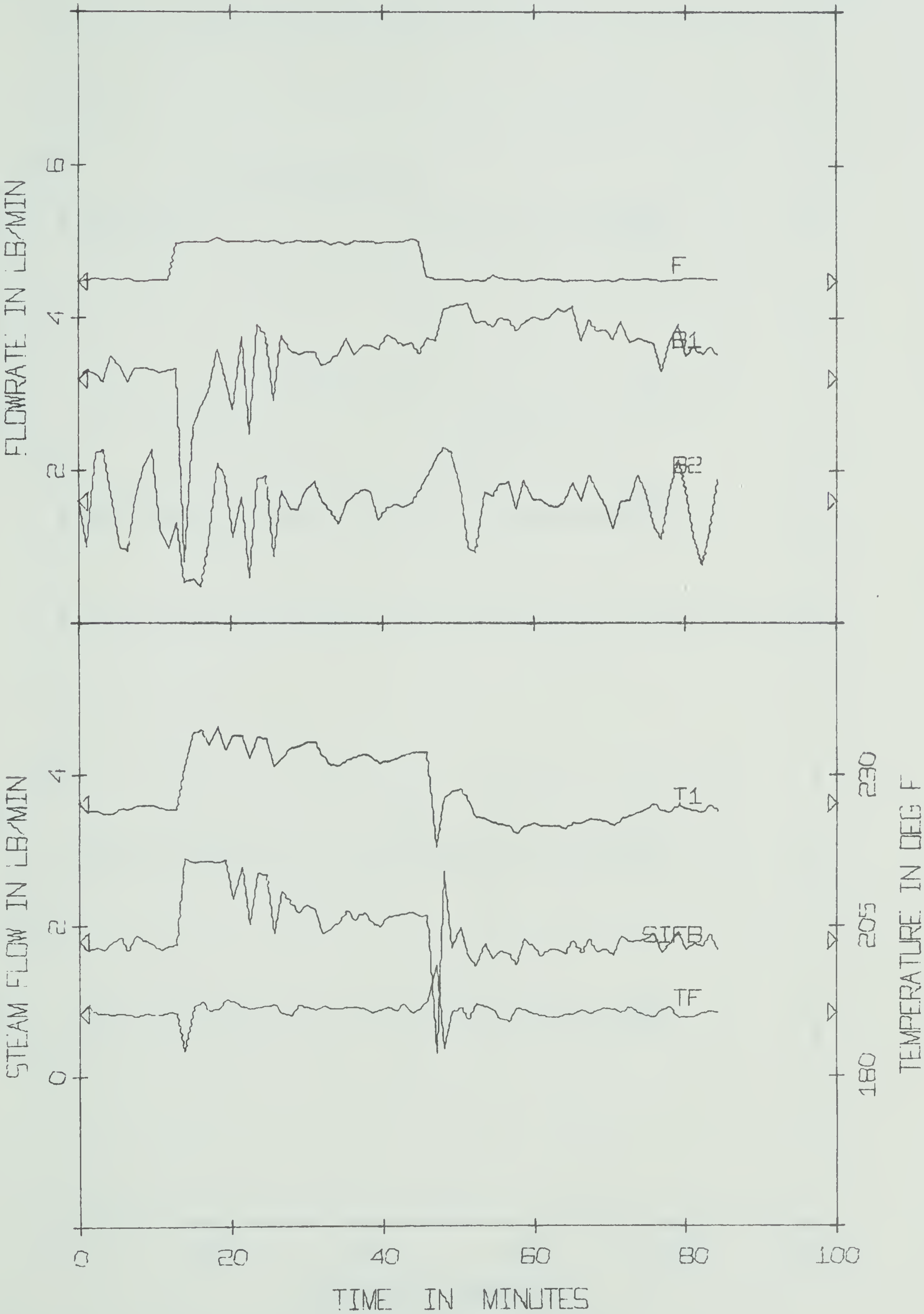


FIGURE 10b. SIMULTANEOUS SETPOINT AND LOAD CHANGES. I
(EXP/10%C2, 10%F/SP/Q1/R1/D1/A1/MVC69)

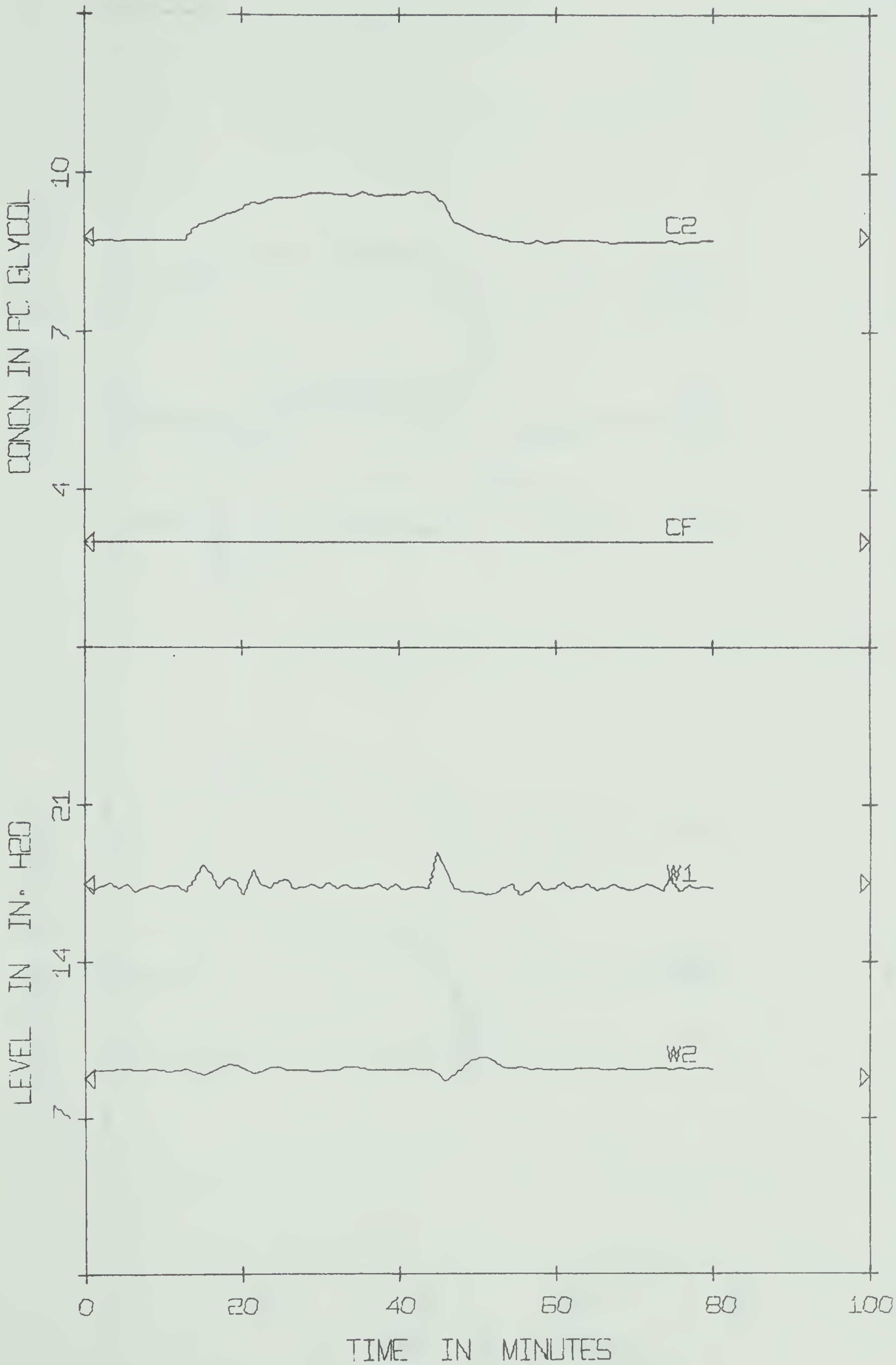


FIGURE 11a. SIMULTANEOUS SETPOINT AND LOAD CHANGES. II
(EXP/10%C2, 10%F/SP, FF-QI/Q1/R1/D1/A1/MVC79)

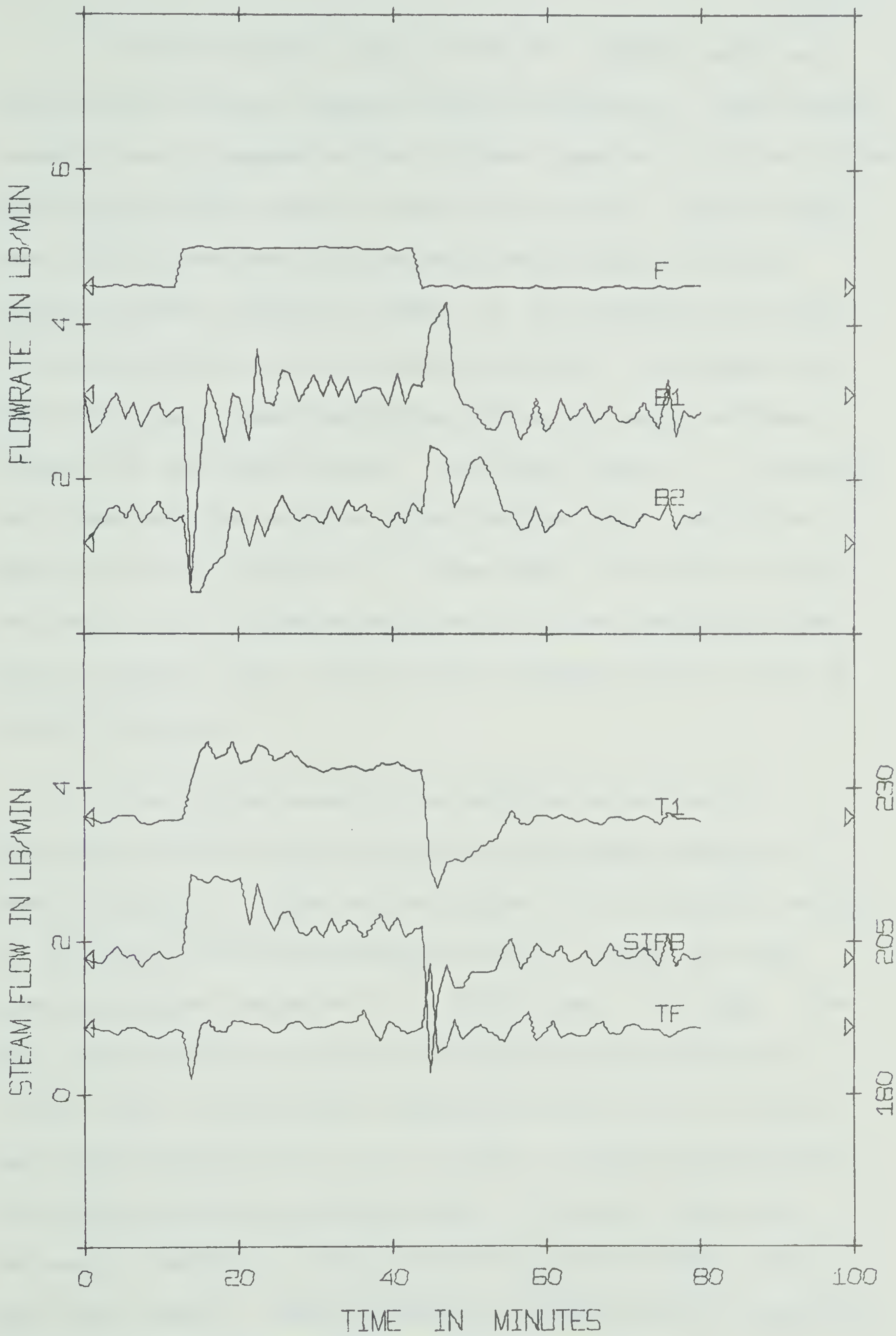


FIGURE 11b. SIMULTANEOUS SETPOINT AND LOAD CHANGES. II
(EXP/10%C2, 10%F/SP, FF-QI/Q1/R1/D1/A1/MVC79)

A twenty percent change in the C2 setpoint under conventional DDC multiloop control is shown in Figure 12. The 60 minute transient can be compared to the 20 minute transient in Figure 13 where direct multivariable setpoint control is in effect. Note the sharp level disturbances under multivariable control where all three control variables combined to change C2 and sacrifice the control of the levels which are not as heavily weighted. In contrast the level disturbances under conventional DDC were a result of interactions with the steam flowrate. The initial climb in C2 occurred while steam was high and first effect bottoms, B1, was low and then the rate of increase of C2 slowed when B1 acted to control the level, W1 . The faster reaction of C2 to a decrease in setpoint in Figure 13 was a result of more available control action as already discussed.

Time optimal state driving with the optimal policy analytically derived from an empirical second-order model [8] is shown in Figure 14. There are two reasons for the longer (37 minute) C2 transients for the same setpoint change. Firstly the steam flowrate had hard contraints of 1 lb./min. and 2.2 lb./min.. The high constraint was to maintain first effect pressure below its 10 psig. limit. Multivariable regulatory control will not handle hard constraints and used up to 3 lb./min. of steam maintaining the first effect pressure by venting vapour. Secondly, steam alone was used to "drive" C2 with bottoms flows used for tight single loop level control. Multivariable state driving with hard constraints has also been implemented [8] with improved results.

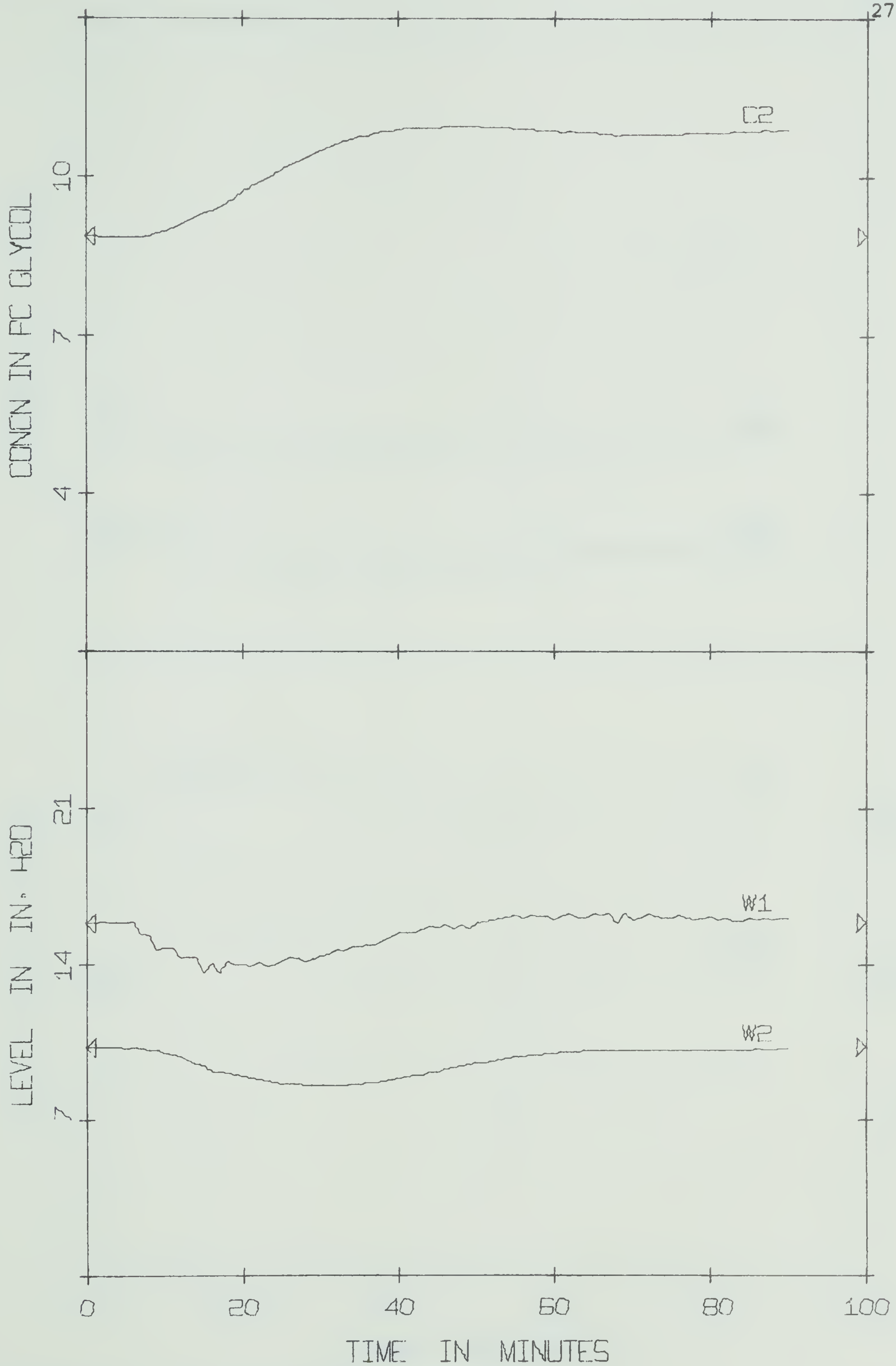


FIGURE 12a. DDC SETPOINT CHANGE IN C2
(EXP/+20%C2/DDC/T1/JC14)

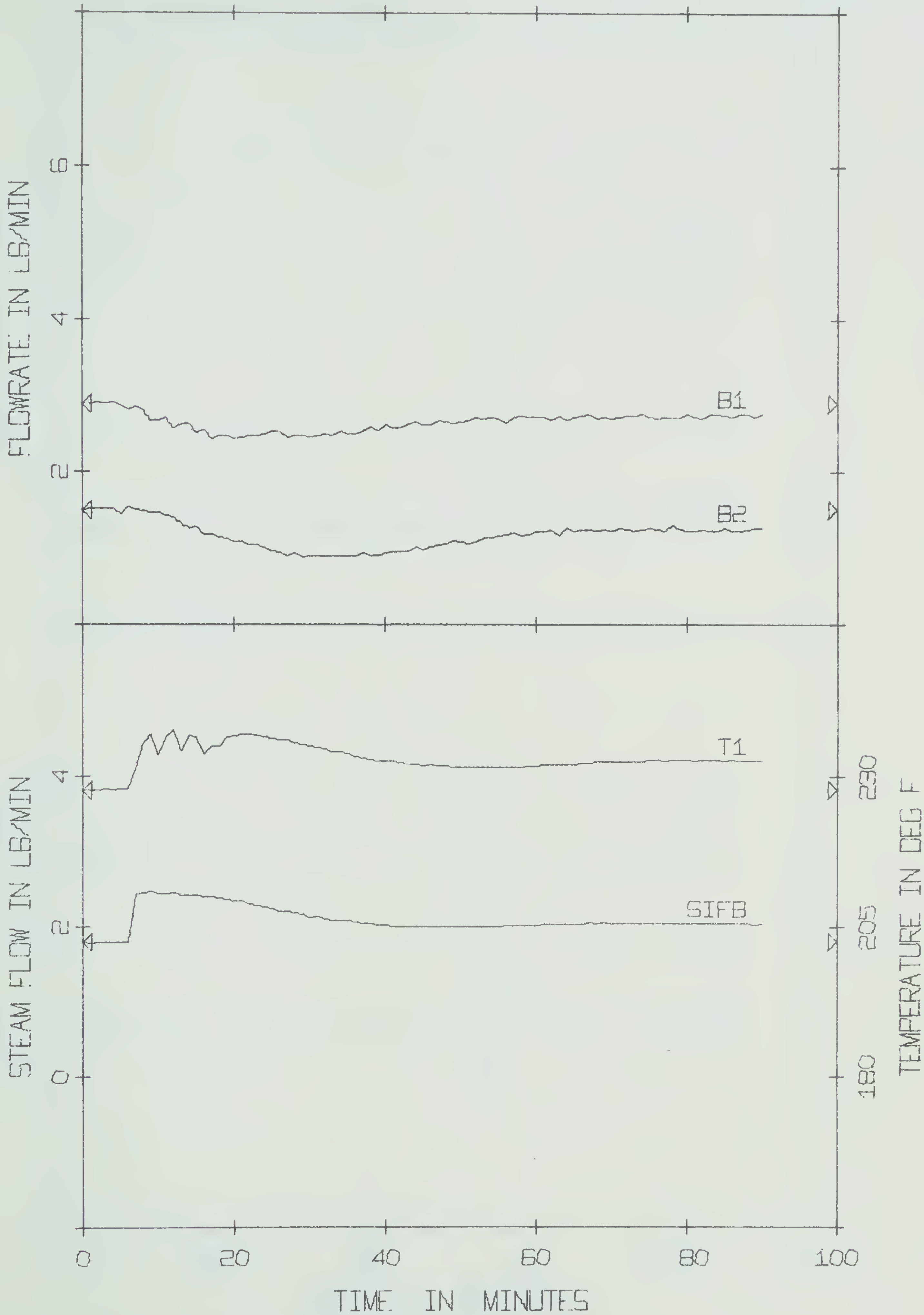


FIGURE 12b. DDC SETPOINT CHANGE IN C2
(EXP/+20%C2/DDC/T1/JC14)

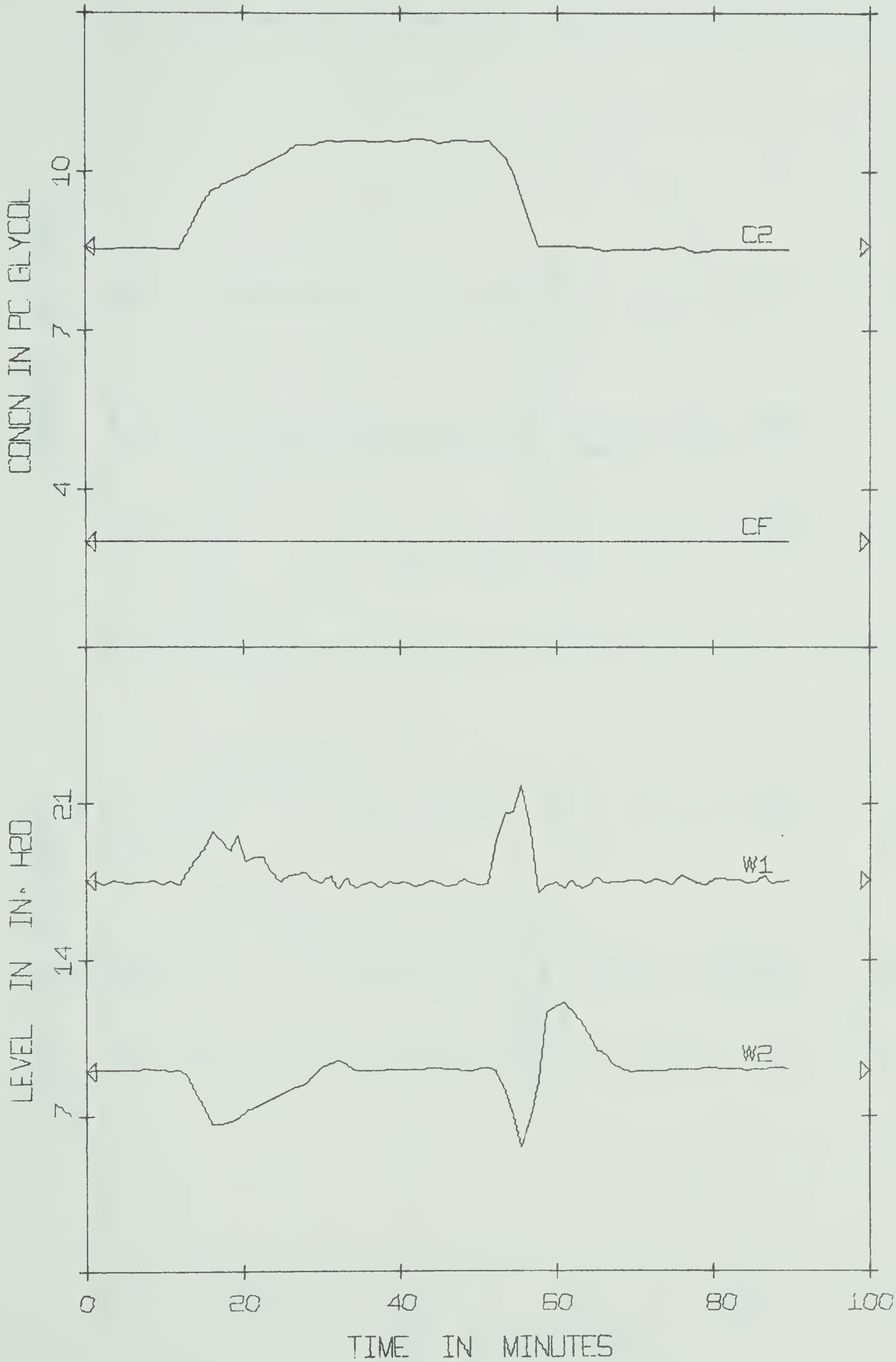


FIGURE 13a. DIRECT SETPOINT CHANGE IN C2. III
(EXP/20%C2/SP/Q1/R1/D1/A2/MVC81)

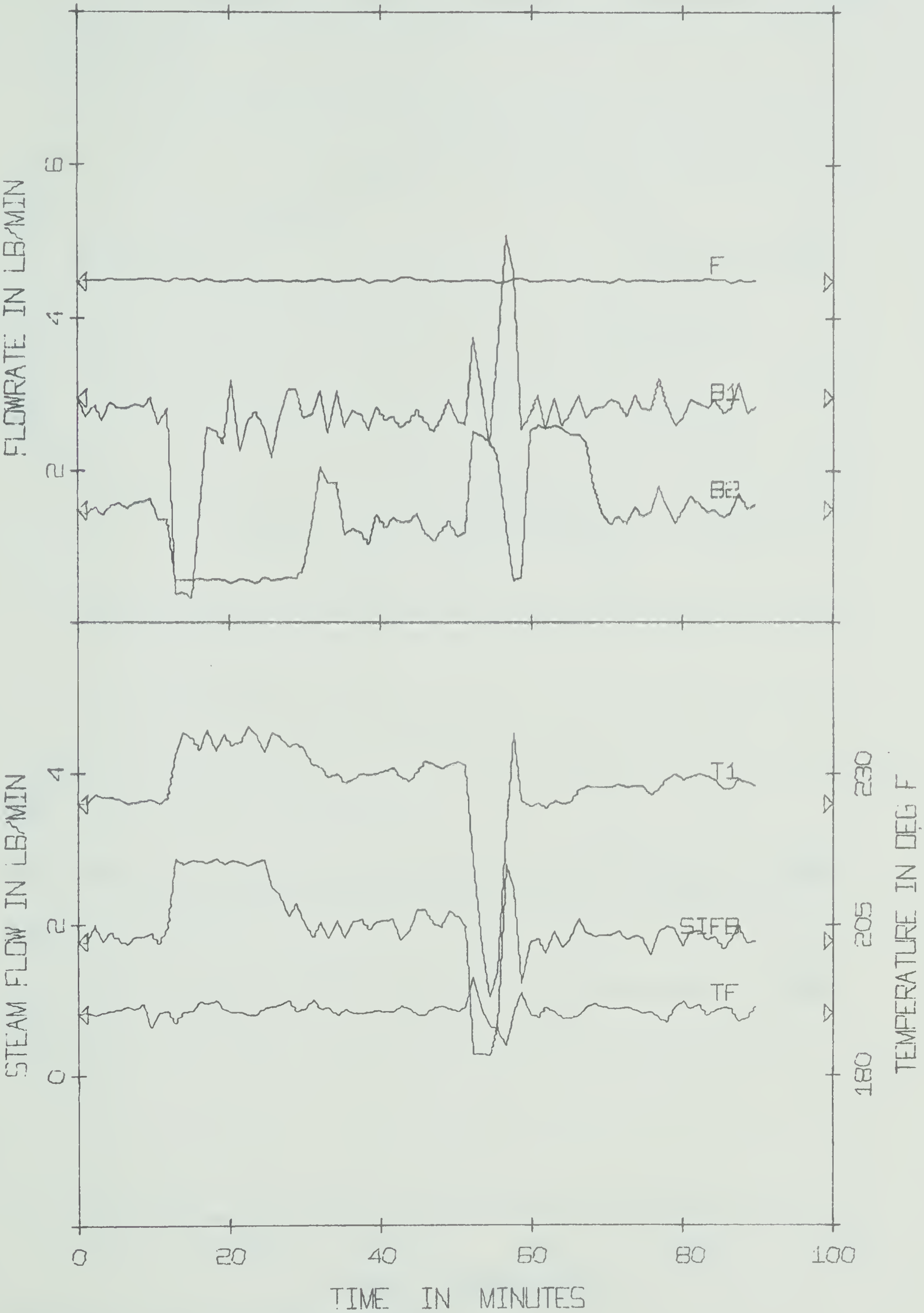


FIGURE 13b. DIRECT SETPOINT CHANGE IN C2. III
(EXP/20%C2/SP/Q1/R1/D1/A2/MVC81)

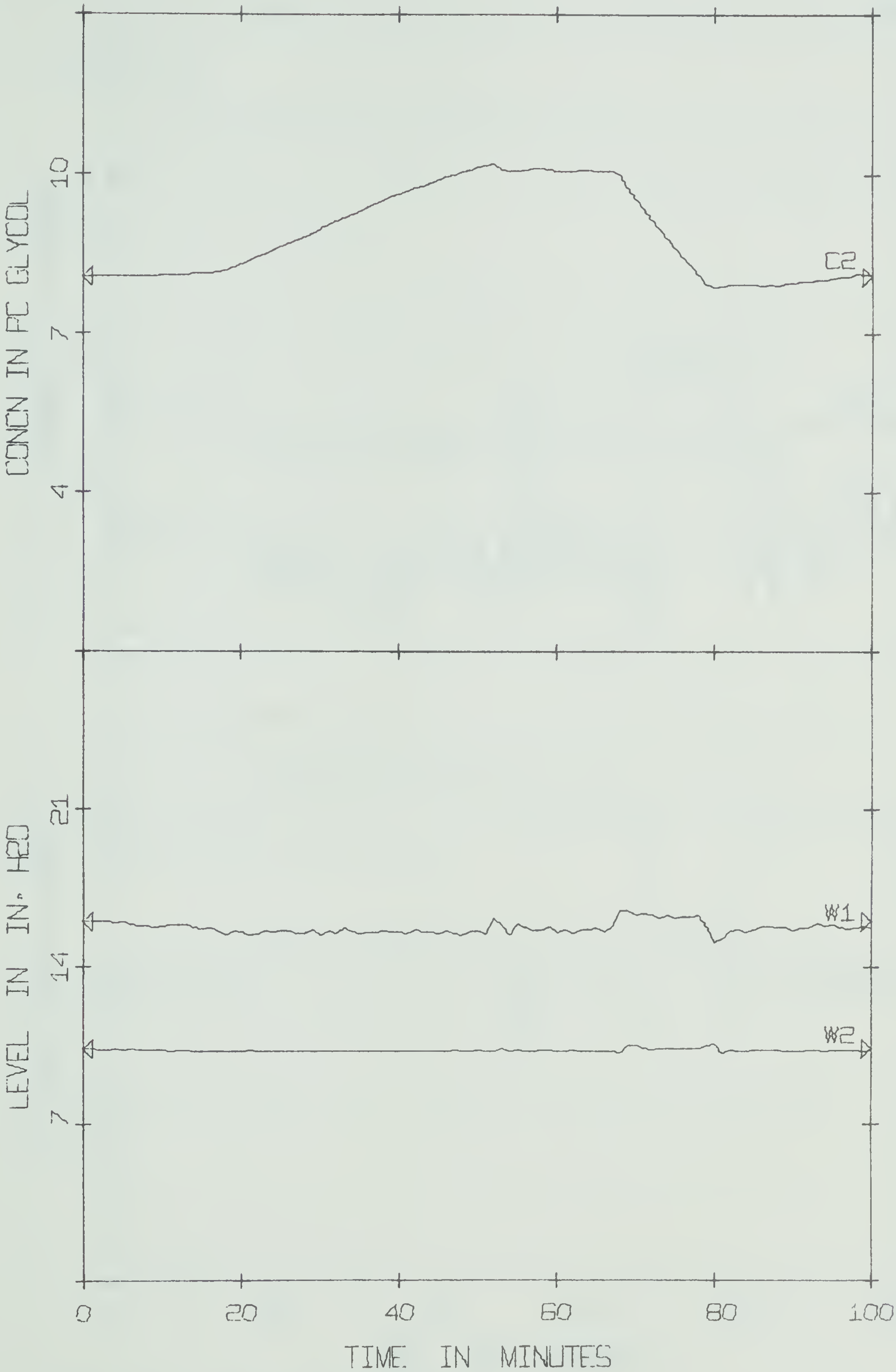
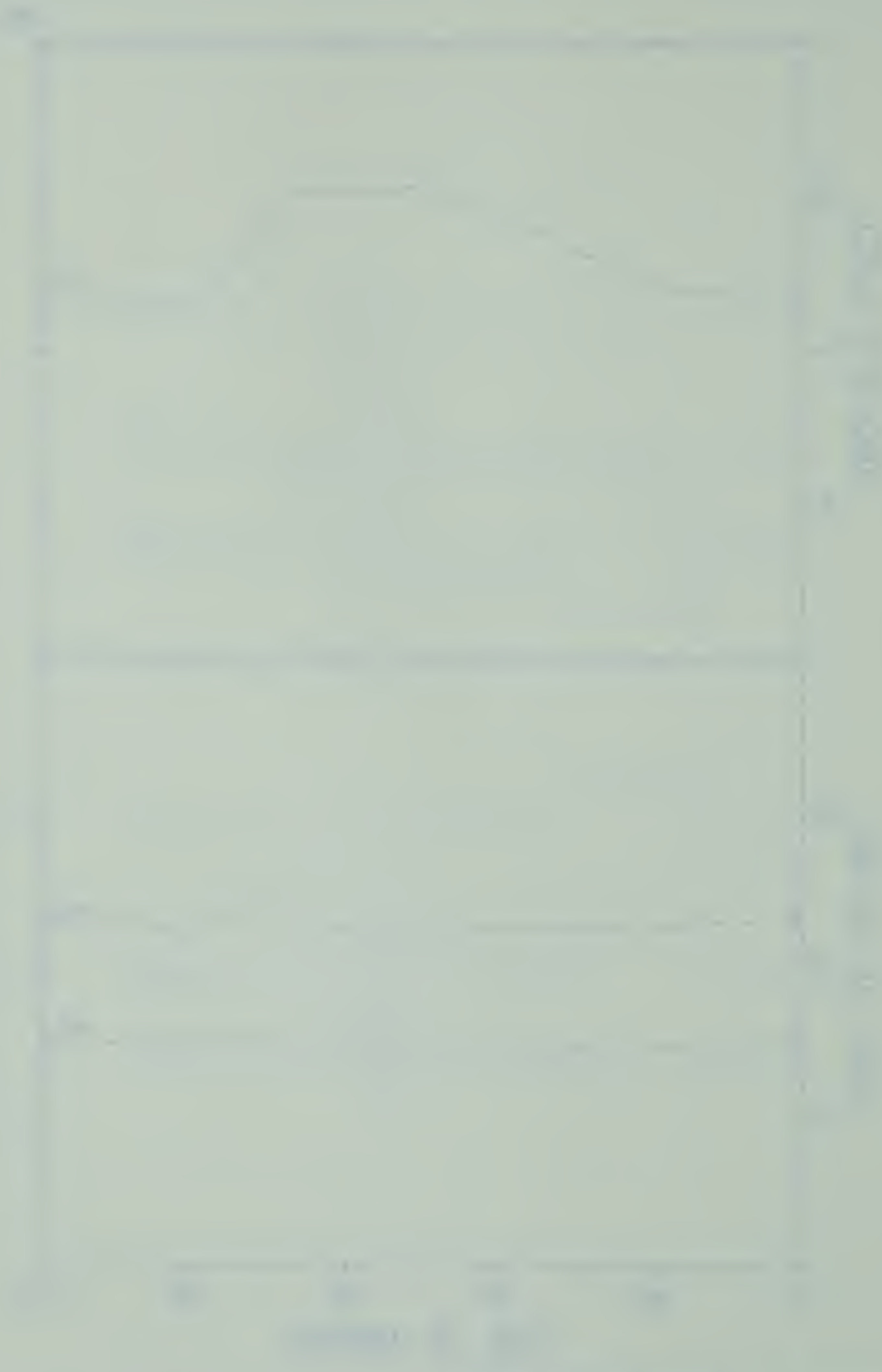


FIGURE 14a. STATE DRIVING CONTROL
(EXP/20%C2/SD/T2/JC32)



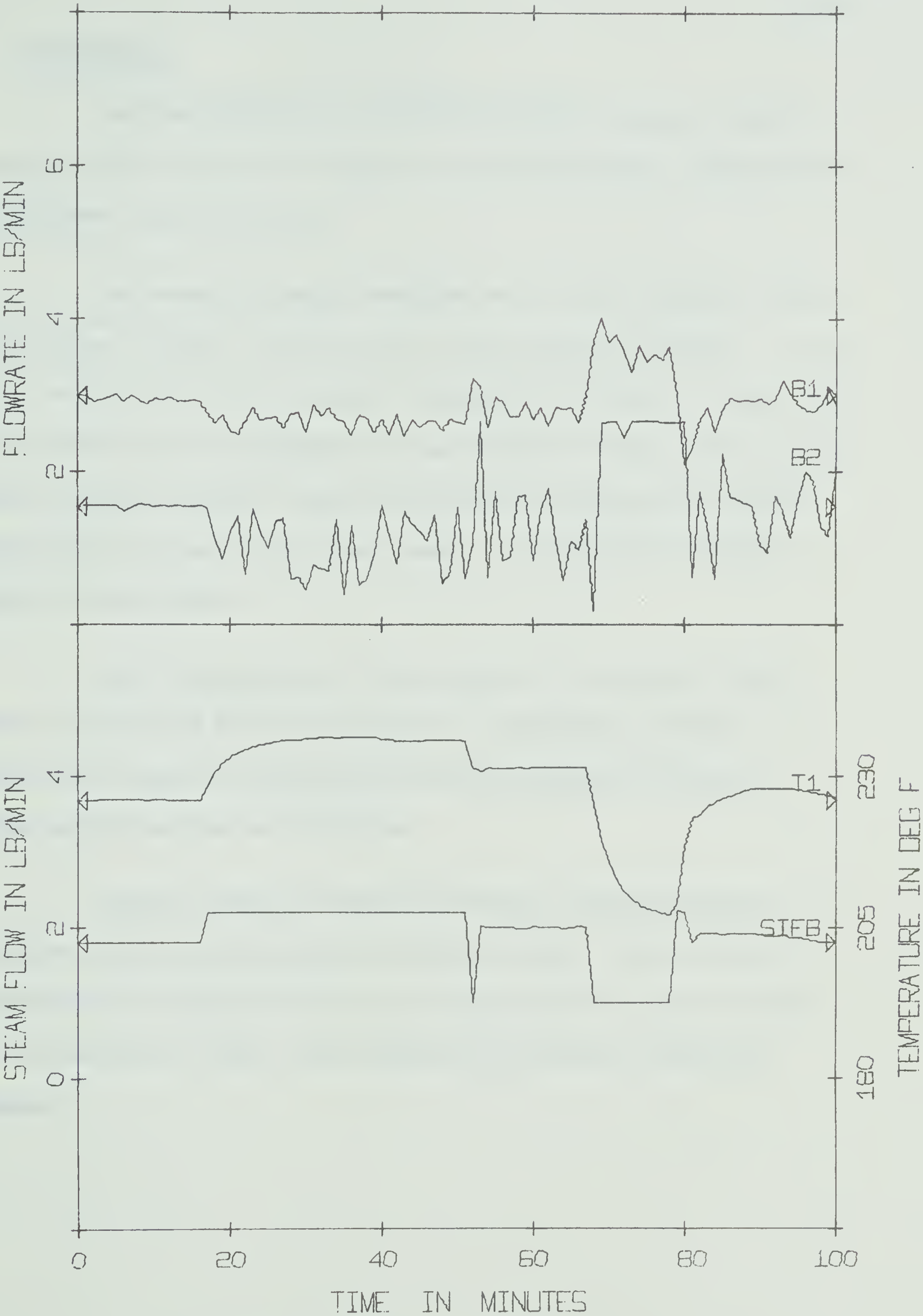


FIGURE 14b. STATE DRIVING CONTROL
(EXP/20%C2/SD/T2/JC32)

7. CONCLUSIONS

The two formulations presented produced optimal setpoint changes while requiring no changes to the multivariable feedback and feedforward control matrices.

The model following technique was the most flexible giving the designer control over the form of the setpoint transients. "Set-point models" had to be realistic, dynamically as slow or slower than the process, or the responses were equivalent to those produced under direct setpoint control. Model following also resulted in smaller interactions between the process outputs if a decoupled "setpoint model" was specified.

Load changes which "worked against" the setpoint change were found to slow down the transient. This effect could be practically removed by feedforward control provided sufficient extra control action was available.

The two optimal formulations showed improved setpoint response over conventional DDC multiloop control. They were also comparable to a minimum time state driving control system although it was based on a lower order model and had tighter constraints imposed.

CHAPTER NINE

IMPLEMENTING MULTIVARIABLE CONTROL

ABSTRACT

Implementation on a pilot plant evaporator has shown that multivariable regulatory control is a practical alternative to conventional DDC control in an industrial environment.

This paper describes the control system used in the implementation of multivariable computer control at the University of Alberta. The control system is made up of the process, a DDC monitor system, and a multivariable control program.

The process and DDC system are briefly described and the general design considerations involved in operating and programming such a system are discussed. The importance of operator-system communication is also mentioned.

The multivariable control system is now a standard package which can be used by students for laboratory exercises and as a basis for other research studies.

1. INTRODUCTION

This paper presents part of a research study whose overall purpose is to develop multivariable process control techniques that would be of advantage to industry and to demonstrate them by implementation on pilot plant processes.

A multivariable regulatory control scheme has been developed in this work and results from a pilot plant evaporator showed that it could be implemented for industrial use with improved control resulting. Figure 1 exemplifies these results, showing a comparison between conventional multiloop Direct Digital Control (DDC) and multivariable regulatory control.

The following sections discuss a general design approach for a multivariable control program. The program [1] written and operating successfully at the University of Alberta is discussed as an example. Comments are also included on the desirable features of the computer system hardware and software.

2. THE SYSTEM CONFIGURATION

The University of Alberta multivariable control system is presented as a specific example of the design approach. The system presented in block diagram form in Figure 2 consists of three subsystems:

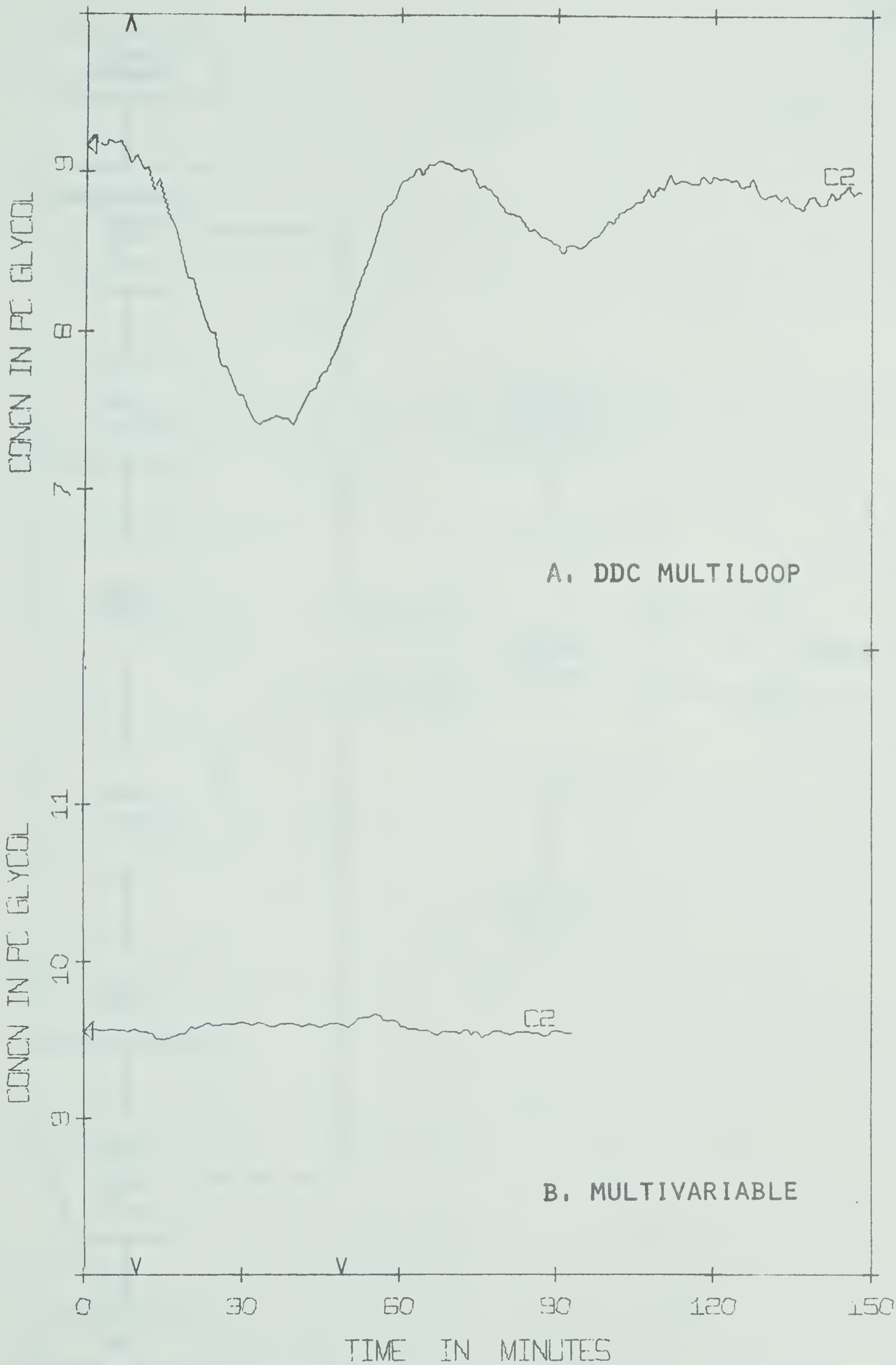


FIGURE 1. EXPERIMENTAL COMPARISON OF DDC AND MULTIVARIABLE

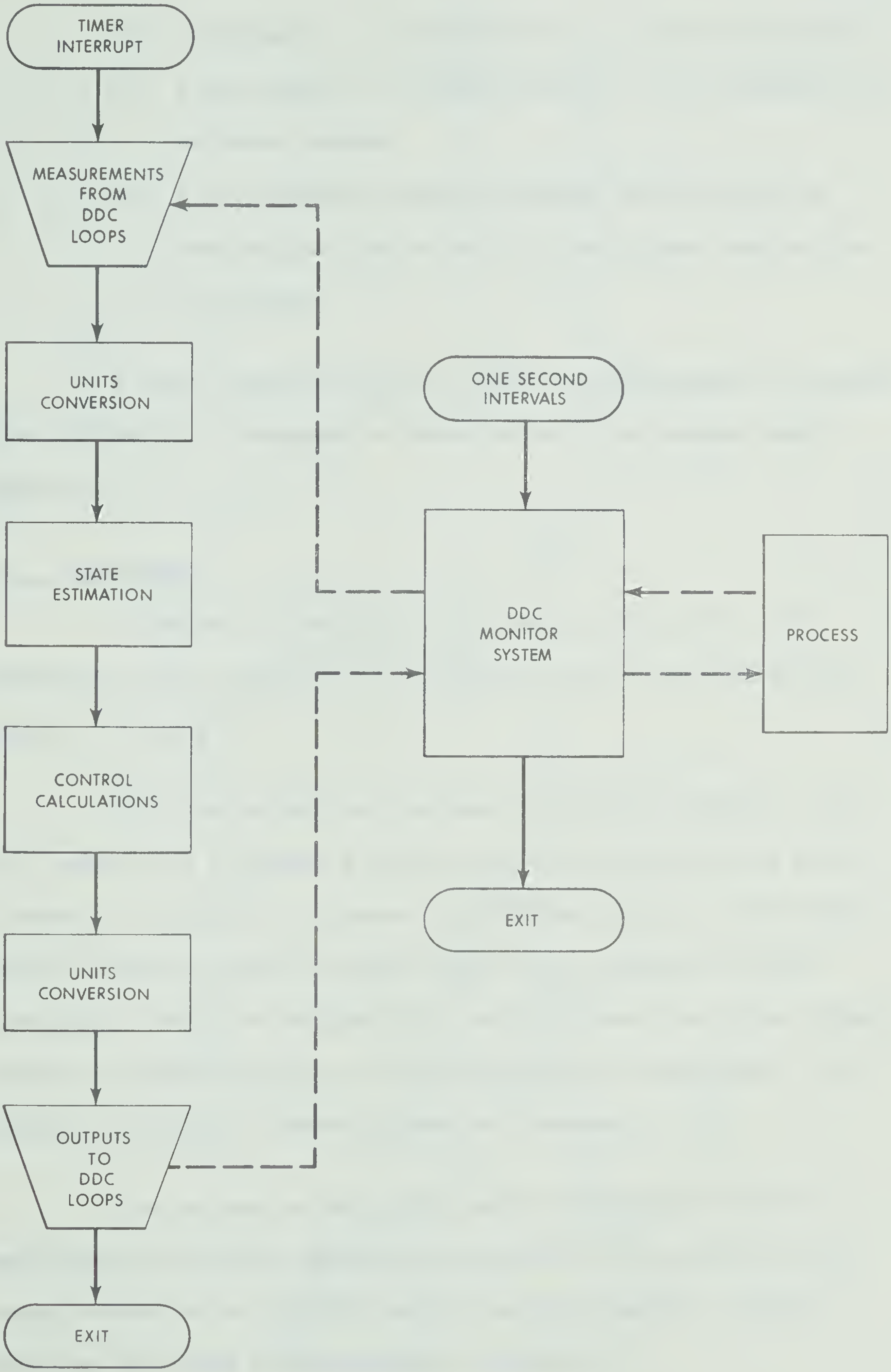


FIGURE 2. SCHEMATIC OF MULTIVARIABLE CONTROL SYSTEM

(a) The process: a double effect pilot plant evaporator.

(b) A DDC system: a modified version of the standard IBM software package.

(c) A multivariable control program: user written to make maximum use of existing facilities, such as the DDC system.

A brief description of the first two subsystems is presented here and the third subsystem is described with the general design approach.

2.1. The Process

A schematic flowsheet of the double effect pilot plant evaporator in the Department of Chemical and Petroleum Engineering appears in Figure 3.

The first effect is a natural circulation calandria type unit heated with a nominal 2 lb./min. of fresh steam and fed with a nominal 5 lb./min. of 3 percent triethylene glycol. First effect vapour is used to heat the second effect, an externally forced circulation long tube vertical unit, which concentrates first effect product to about 10 percent. The second effect is kept under tight pressure control by a vacuum system and condenser.

The process is fully instrumented for research with 54 measurements accessible through multiplexers in the process input/output section of an IBM 1800 digital control computer. Table 1 indicates the types of measurements available.

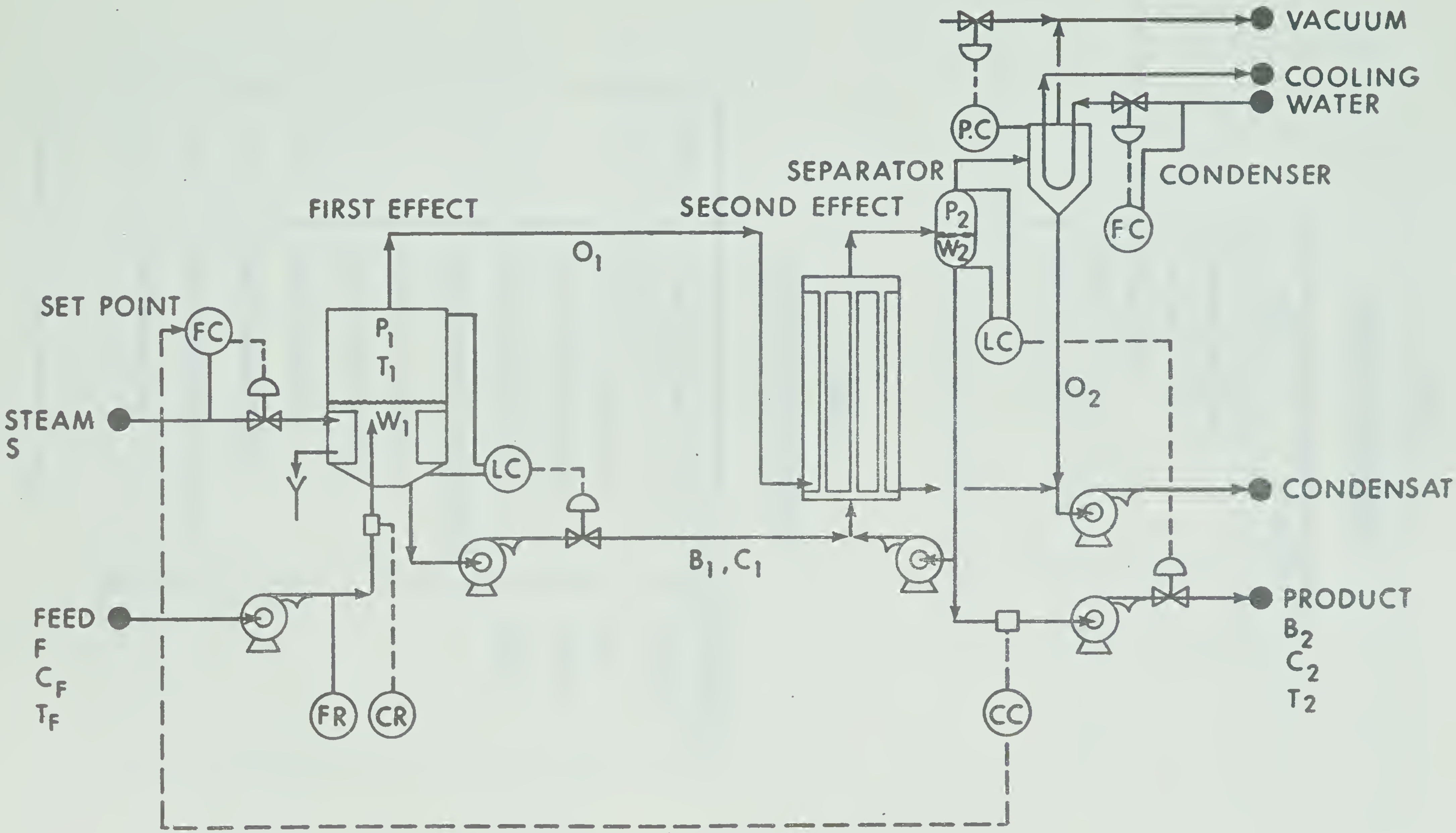


FIGURE 3. PILOT PLANT DOUBLE EFFECT EVAPORATOR

TABLE 1

PROCESS MEASUREMENTS

Type	Number
Concentrations	3
Levels	3
Pressures	2
Flows	9
Temperatures	37
	54

TABLE 2

CONTROL CONFIGURATION

Number	Description of Loop	Process Variable
1*	Product Concentration	C2/S
1*	First Effect Level	W1/B1
1*	Second Effect Level	W2/B2
1*	Second Effect Condensate	-
1	Condenser Pressure	P2
1	Cooling Water Flow	-
2	Feed Flowrates	-
2	Feed Temperatures	-
8	Data Acquisition Loops	-
32	Thermocouples	-

* Cascaded to flow control loops.

An interface unit allows computer or manual switching between local analogue control under 12 Foxboro electronic controllers, Direct Digital Control time shared on the IBM 1800, or analogue supervisory control. The evaporator normally operates under DDC with six single control loops and four cascaded loops. Other process variables are available to the user through DDC data acquisition loops or directly from multiplexers. The configuration is detailed in Table 2.

2.2. The DDC System

The IBM 1800 process control computer in the Data Acquisition, Control, and Simulation (DACS) Centre [2] with 48K core and 2 μsec. cycle time operates under a multiprogramming executive. The DDC monitor and multivariable control program execute in express partitions while lower priority partitions allow the simultaneous operation of gas chromatograph [3] and infrared analyser [4] systems as well as background processing.

The DDC program package is capable of servicing several hundred regulatory control loops, standard consoles for process operators and engineers, and basic data logging functions. Control loops and functions can be configurated safely, simply, and on-line by suitable user entries into the process variable table (PVT), an in-core table of configuration data, parameters, and measurements. The DDC program periodically reads all measurement signals, performs control computations communicating with the PVT for parameters, and sends out new signals directly to the process valves.

User communication with the DDC system through the PVT is a major advantage and this can be done through the Process Operator's Console (POC) or under program control using a set of system subroutines. The latter feature allows easy communication between DDC and user written control programs.

3. CONTROL PROGRAM DESIGN

The design of a multivariable control system is discussed under four headings: the program structure, the basic design approach, programming considerations, and operator communication.

3.1. Program Structure

The basic structure of a multivariable control program was illustrated in Figure 1. The following sections discuss the basic functions.

3.1.1. Measurements

Process variable measurements can be obtained from a DDC table as in the present case or read directly from multiplexers. In either case the designer must consider signal conditioning and units conversion.

Multivariable control systems generally derive their superiority from being able to operate with high gains while remaining stable. The high gains magnify process noise in the measurements sometimes causing rapid fluctuations in the control variables. The noise can be reduced by filtering [5] although care must be taken not to introduce undue lags. Process noise can also be reduced at the transducer site by good placement, etc.

3.1.2. State Estimation

It is quite common in many applications that certain variables are not measured for practical or economic reasons. Since multivariable control requires an estimate of the state, the control program often requires an estimation section.

The present program includes model calculations, with input from the process measurements, which parallel the real-time process operation. The predicted state from the model is combined with process measurements to give an estimated state for use by the control algorithm.

$$\underline{x}_{est} = \underline{x}_{model} - \underline{\underline{K}}_F (\underline{x}_{model} - \underline{x}_{meas}) \ . \qquad (1)$$

The "filter matrix" $\underline{\underline{K}}_F$ can have a single entry for each state $(1 \geq k_F^{ii} \geq 0)$ or be designed by optimal filter theory.

3.1.3. Control Algorithm

The control algorithm is the main function of the system and can vary with the application. In the present case a general control algorithm was used of the following form.

$$\underline{u} = \underline{\underline{K}}_{FB}\underline{x}_{est} + \underline{\underline{K}}_{FF}\underline{d} + \underline{\underline{K}}_I \int \underline{y}_{est} dt + \underline{\underline{K}}_{SP}\underline{y}_{sp} + \underline{\underline{K}}_M\underline{y}_m \ . \qquad (2)$$

This form includes constant control matrices for feedback, feed-forward, integral, setpoint, and model following control modes.

The algorithm can be condensed to incorporate the integration and units conversion. This may reduce execution time but also removes flexibility.

3.1.4. Output of Control

Control signals can be sent directly to control valves or, as in the present system, sent to the setpoint of a servo-control loop. To reduce implementation lags these loops can be tightly tuned with small control intervals. This also allows the designer to execute the multivariable program less frequently since "fast" localized disturbances are controlled by these DDC loops.

3.1.5. Data Collection

Data collection may be desirable for research purposes or for maintaining historic operating data. This function may be carried out by the DDC system if the function is available.

3.2. Basic Design Approach

The approach to writing such a control program depends to a large extent on the type of computing system available. The approach for a small dedicated computer will differ from that for a large general purpose machine such as the IBM 1800. Factors which must be decided on are system structure, program location, and program initiation.

3.2.1. System Structure

There are three alternative approaches, the latter ones dependent upon the existance of a DDC system.

(a) Completely User Written. This is the approach necessary on a dedicated machine or one without a standard DDC package. Great care is necessary if this approach is used on a time shared computer to avoid scheduling problems, particularly with process input/output hardware.

(b) User Program Interfaced to DDC. All process input/output can be handled by the existing DDC system. The control program communicates with the process through the DDC system and its data table. This arrangement gives the user the greatest flexibility without the worry of hardware scheduling.

(c) Special Algorithms Within DDC. It would be possible to build into the DDC system algorithms for conversions, state estimation, and the control calculations. This would have the advantage of an "automatic" system which is best for final implementation but inflexible and hard to "debug" in the development stages.

3.2.1. Program Location

The program can be core resident or periodically loaded into core from bulk storage, for example disk or drum. A core resident program requires either a dedicated machine or core partition. With a non-core resident program priorities should be carefully arranged so that delays in the execution of the control program do not occur. A dedicated core partition has the advantages of no

delay in, or system time needed for, loading but it takes up expensive core space.

3.2.2. Program Initiation

The control program executes periodically and can either cycle within itself for a fixed interval or it can be initiated by an interrupt [6]. A program which operates without using an interrupt requires both a core resident program and a dedicated machine. Time shared operation requires the use of an interrupt which can come from several sources.

(a) External timer.

(b) System hardware or software timer.

(c) The DDC system.

An interrupt from the DDC system can be used to synchronize the operation of the control program with DDC control loops and reduce delays in implementing calculated control action.

The control program in the present case was interfaced to the DDC system and was user written. This gave the greatest flexibility needed for the continual development work which occurs on a research application. The program was initiated by an interrupt from a system timer and executed at a high priority level in an express core partition.

3.3. Programming Considerations

There are a large number of considerations necessary in the final program layout and coding ([7] Chapter 12). The most important of these are listed below.

(a) Language. In choosing the basic language there is the usual trade off between a high level language such as FORTRAN and an assembly language [8,9]. Assembly languages are faster in execution and more efficient in core requirements. However, unless these factors are limiting, the ease of initial programming and innovations and the familiarity of high level languages to more people make them preferable. A number of vendors are releasing higher level languages specifically for process control and if they become standardized [10] and are suitable for multivariable applications they could replace the more general languages.

(b) Flexibility. A modular structure enables better organization and makes program alterations easier. Symbolic references to input/output addresses and the like enable a change in application with relatively minor program changes.

(c) Storage. In a multivariable control program the greatest savings in core requirements result from minimizing the dimension of vectors. While vectors can be over-dimensioned so that different applications can be easily handled, it is often more efficient to design the program for easy modification. With the modular design large programs can be used in less core by overlaying although this technique is undesirable because of loading time.

(d) Speed. The time consuming tasks of the control program are process input/output, disk/drum storage and retrieval, and the calculations. The calculations can be speeded up by efficient programming and the use of integer arithmetic. However, the use of integer arithmetic is a trade off between increased speed and increased complexity and loss of accuracy. Floating point hardware, not a common feature on control computers, can be resorted to in limiting cases. The time lag between process input to and output from the program can be reduced by doing calculations before or after them rather than between them.

(e) Input/Output. Communications with the process and bulk storage can be reduced by continuous data channel transfers and faster hardware, such as analogue-to-digital and digital-to-analogue convertors. There should be no typewriter or printer output unless there are system errors, which can be made self checking, or alarm conditions associated with the process.

3.4. Operator Communication

Operator communication with computer control schemes cannot be too heavily stressed. [11,12]. Direct Digital Control systems presently communicate through a process operator's console [13,14] to supply both current measurements and historic data.

Two additional features were found to be necessary in the present multivariable control system. In many instances it was desirable to ascertain whether or not the control program was executing every control interval. This was particularly so in the debug stages and on an experimental installation such as the present machine where continual debugging of other programs can interfere with system timers. Since typewriter output was undesirable, the process output hardware was used to turn on a light on the operators console while the control program was executing. This proved to be a simple but effective way to assure those running the process that the control program was "on-the-job".

The other feature incorporated was an executive program which gave the operator reasonable control over the control program. The executive could update controller matrices, automatically switch back and forth between multivariable control and conventional multi-loop DDC, and perform such tasks as initiate and terminate data collection.

It is of particular note that unlike conventional DDC control it becomes impossible, or at least extremely unwise, for control constants in multivariable control to be "adjusted" by the operator. Instead control matrices must be "updated" by an off-line program when operating conditions or control policies change significantly.

4. CONCLUSIONS

Multivariable computer control has been shown to be a practical alternative to conventional DDC multiloop systems.

The design of the multivariable algorithm is straightforward given the design equations, guidelines for parameter choice, and a process model . Even simple or approximate models can serve as the basis for significant improvements in control quality.

The computers in most DDC installations are capable of implementing advanced control algorithms with little or no expansion. The multivariable control system described used less than eight percent of system time operating at the normal 64 second control interval. This was despite the fact that the program was written in a high level language and designed for maximum flexibility rather than speed of execution.

The control system developed at the University of Alberta has proved versatile and has been successfully used to obtain test results by persons without a detailed knowledge of multivariable control or the control system. It constitutes an operating package which enables students to implement a multivariable control scheme with ease as a laboratory exercise or for use in other research studies.

CHAPTER TEN

COMPARISON OF MULTIVARIABLE CONTROL TECHNIQUES FOR A DISTILLATION COLUMN

ABSTRACT

A distillation column model is used to compare design techniques for multivariable control. A conventional multiloop control scheme, which does not account for interactions, is compared to a noninteracting design, which removes interactions by state feedback and then uses the same multiloop control, and to an optimal multivariable scheme, which makes use of some interactions to compensate for others. The noninteracting and optimal control systems resulted in faster responses to load changes and smaller deviations.

The optimal multivariable control also gave improved control compared to a system designed by the Liapunov approach. Design parameters for optimal multivariable control and multivariable feed-forward design approaches were also studied and some of the conclusions appeared generally applicable.

1. INTRODUCTION

This work is based on a nonlinear theoretical distillation column model taken from the literature [1,2]. The model was linearized into the standard state space form. Calculated open and closed loop responses from this state space model were compared with simulated response data from the original paper.

The design parameters in the optimal multivariable infinite time quadratic index formulation are examined. This optimal control scheme is compared to one based on the stepwise approach and different design approaches to multivariable feedforward control are also examined.

Many of the troubles encountered with conventional multiloop control systems result from interactions between the different control loops. An optimal multivariable system compensates for some of these interactions and makes use of others. However, a more classical approach to the problem is to firstly design controls to eliminate the interactions, and then design single variable controllers for each non-interacting pair. This non-interacting design approach is applied to the distillation column and the resulting control system is compared to a conventional multiloop scheme and optimal multivariable control.

2. DISTILLATION MODEL

The binary distillation column is presented schematically in Figure 1. It has three trays, a reboiler, and a total condenser, all with constant holdups. The feed is subcooled liquid of almost

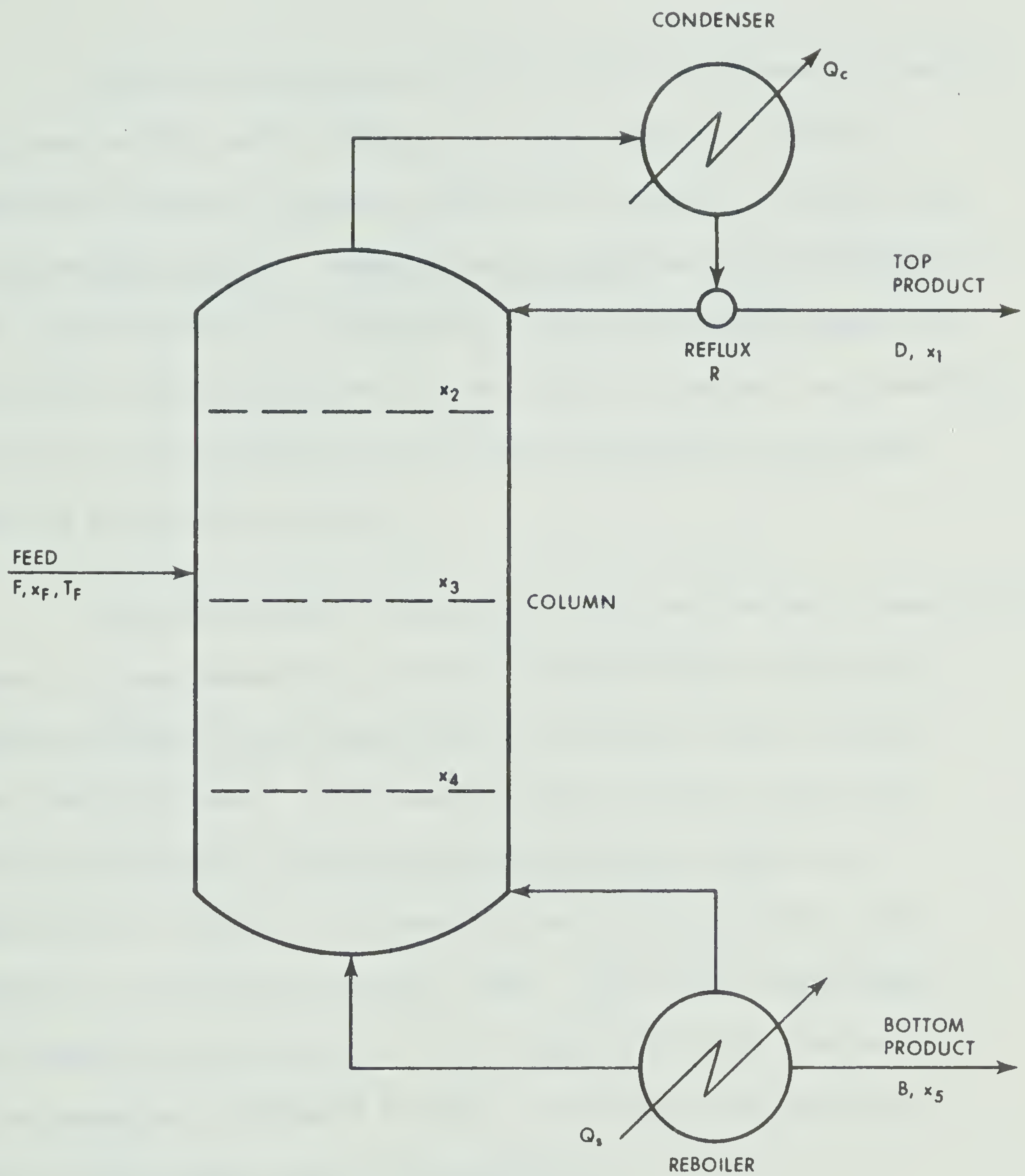
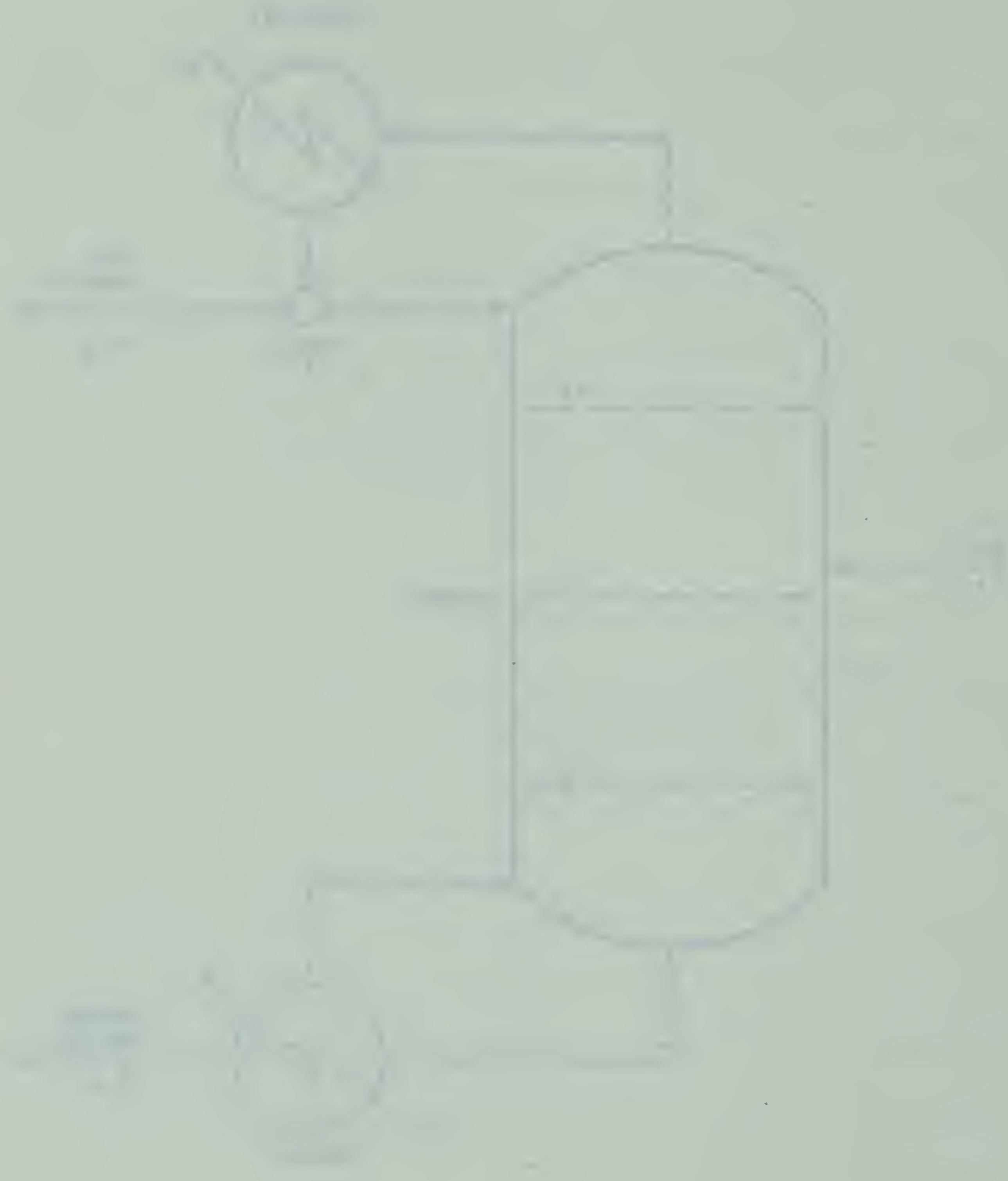


FIGURE 1. SCHEMATIC OF DISTILLATION COLUMN



equal density components and liquid is refluxed at its boiling point.

The model has been derived by Rafal and Stevens [1] and is based upon overall mass, energy, and light component balances. Assumptions include a Murphree efficiency of unity, no vapour hold-up, and no heat losses. Assuming that liquid and vapour enthalpies are a linear function of composition, Rafal and Stevens simplified the fifteen balance equations for the trays, condenser, and reboiler to a total of five nonlinear first order differential equations. These are presented in Table 1.

The multivariable design techniques used in this investigation require a linear state space model so the nonlinear differential equations (Table 1) were analytically linearized about a steady state operating point. The resulting state, control, and load variables with their reference steady state are listed in the Nomenclature. Equation (1) presents the state space model with variables in normalized deviation form. This state space model was checked by generating open loop response data and comparing it to the responses calculated by Rafal and Stevens [1] for disturbances in feed composition and temperature.

TABLE 1

DISTILLATION MODEL EQUATIONS

$$57.8 \frac{dx_1}{dt} = \frac{E(y_2 - x_1)}{y_2 + .986}$$

$$y_2 = .8197\, x_2 + .2951$$

$$5.0 \frac{dx_2}{dt} = E\left[\frac{y_3 - x_2}{y_3 + .986} + \frac{Rx_1 + x_2 + (R + 1)y_2}{(R + 1)(y_2 + .986)}\right]$$

$$y_3 = 1.017\, x_3 + .2021$$

$$5.0 \frac{dx_3}{dt} = E\left[\frac{x_2 - y_3}{y_3 + .986} + \frac{x_3 - x_2}{(R + 1)(y_2 + .986)}\right] + \frac{Q_S}{238.2}\left[\frac{y_4 - x_3}{y_4 + .986}\right] + F(x_F - x_3)$$

$$y_4 = 1.4393\, x_4 + .0355$$

$$5.0 \frac{dx_4}{dt} = (x_4 - x_3)\left[\frac{E}{(R + 1)(y_2 + .986)} - F\right] + \frac{Q_S}{238.2}\left[\frac{y_5 - x_4}{y_5 + .986} + \frac{x_3 - y_4}{y_4 + .986}\right]$$

$$y_5 = 1.615\, x_4$$

$$94.2 \frac{dx_5}{dt} = (x_5 - x_4)\left[\frac{E}{(R + 1)(y_2 + .986)} - F\right] + \frac{Q_S}{238.2}\left[\frac{x_4 - y_5}{y_5 + .986}\right]$$

$$E = \frac{Q_S - F(131. - T_F)}{238.2} .$$

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -.1094 & .0687 & 0 & 0 & 0 \\ 1.306 & -2.132 & .9807 & 0 & 0 \\ 0 & 1.595 & -3.146 & 1.547 & 0 \\ 0 & .0355 & 2.623 & -4.257 & 1.855 \\ 0 & .00227 & 0 & .1636 & -.1625 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ .0638 & 0 \\ .0838 & -.1396 \\ .1004 & -.2060 \\ .0063 & -.0128 \end{bmatrix} \begin{bmatrix} R \\ Q_S \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ .1233 & .5495 & -.0142 \\ .1960 & 0 & -.0082 \\ .0123 & 0 & -.0005 \end{bmatrix} \begin{bmatrix} F \\ x_F \\ T_F \end{bmatrix}$$

Equation (1)

3. MULTIVARIABLE STEPWISE CONTROL

Rafal and Stevens [1] worked with multivariable stepwise feedback control with a hypersurface search when constraints were encountered. The basic design technique is similar to designs based on Liapunov's second method where a Liapunov function is minimized for greatest stability. Stepwise control minimizes the quadratic criterion,

$$J = \underline{x}_i^T \underline{\underline{Q}} \, \underline{x}_i + \underline{u}_{i-1}^T \underline{\underline{R}} \, \underline{u}_{i-1} \tag{2}$$

over each control interval separately. The discrete process model can be represented as follows, neglecting the load term.

$$\underline{x}_{i+1} = \underline{\underline{A}} \, \underline{x}_i + \underline{\underline{B}} \, \underline{u}_i \, . \tag{3}$$

Differentiating Equation (2) with respect to the control vector results in a proportional multivariable feedback control law.

$$\begin{aligned} \underline{u}_i &= -(\underline{\underline{B}}^T \underline{\underline{Q}} \, \underline{\underline{B}} + \underline{\underline{R}})^{-1} \underline{\underline{B}}^T \underline{\underline{Q}} \, \underline{\underline{A}} \, \underline{x}_i \\ &= \underline{\underline{K}}_{FB} \, \underline{x}_i \, . \end{aligned} \tag{4}$$

This is the first step in the discrete dynamic programming solution of the infinite time formulation . The control matrix in Equation (4) can be evaluated by a single iteration of these full recursive relations.

Rafal and Stevens' calculated open loop and stepwise control responses to simultaneous disturbances in x_F and T_F have

been calculated and are shown in Figure 2. The control matrix for stepwise control was evaluated by one iteration of the recursive relations for the solution of the infinite time control problem since this was already available.

4. MULTIVARIABLE OPTIMAL CONTROL

The optimal multivariable control problem which minimizes the criterion,

$$J = \sum_{i=1}^{N} (\underline{x}_i^T \underline{\underline{Q}} \, \underline{x}_i + \underline{u}_{i-1}^T \underline{\underline{R}} \, \underline{u}_{i-1}) \tag{5}$$

for the discrete, linear, time-invariant state space model,

$$\underline{x}_{i+1} = \underline{\underline{A}} \, \underline{x}_i + \underline{\underline{B}} \, \underline{u}_i + \underline{\underline{D}} \, \underline{d}_i \tag{6}$$

can be solved using discrete dynamic programming [2]. The control law is of the form,

$$\underline{u}_i = \underline{\underline{K}}_{FB} \, \underline{x}_i + \underline{\underline{K}}_{FF} \, \underline{d}_i \tag{7}$$

with constant control matrices evaluated from recursive relations.

4.1. Design Parameters

The following work on the distillation column model examines the effects of the three design parameters: the control interval, Δt , the state weighting matrix, $\underline{\underline{Q}}$, and the control weighting matrix, $\underline{\underline{R}}$.

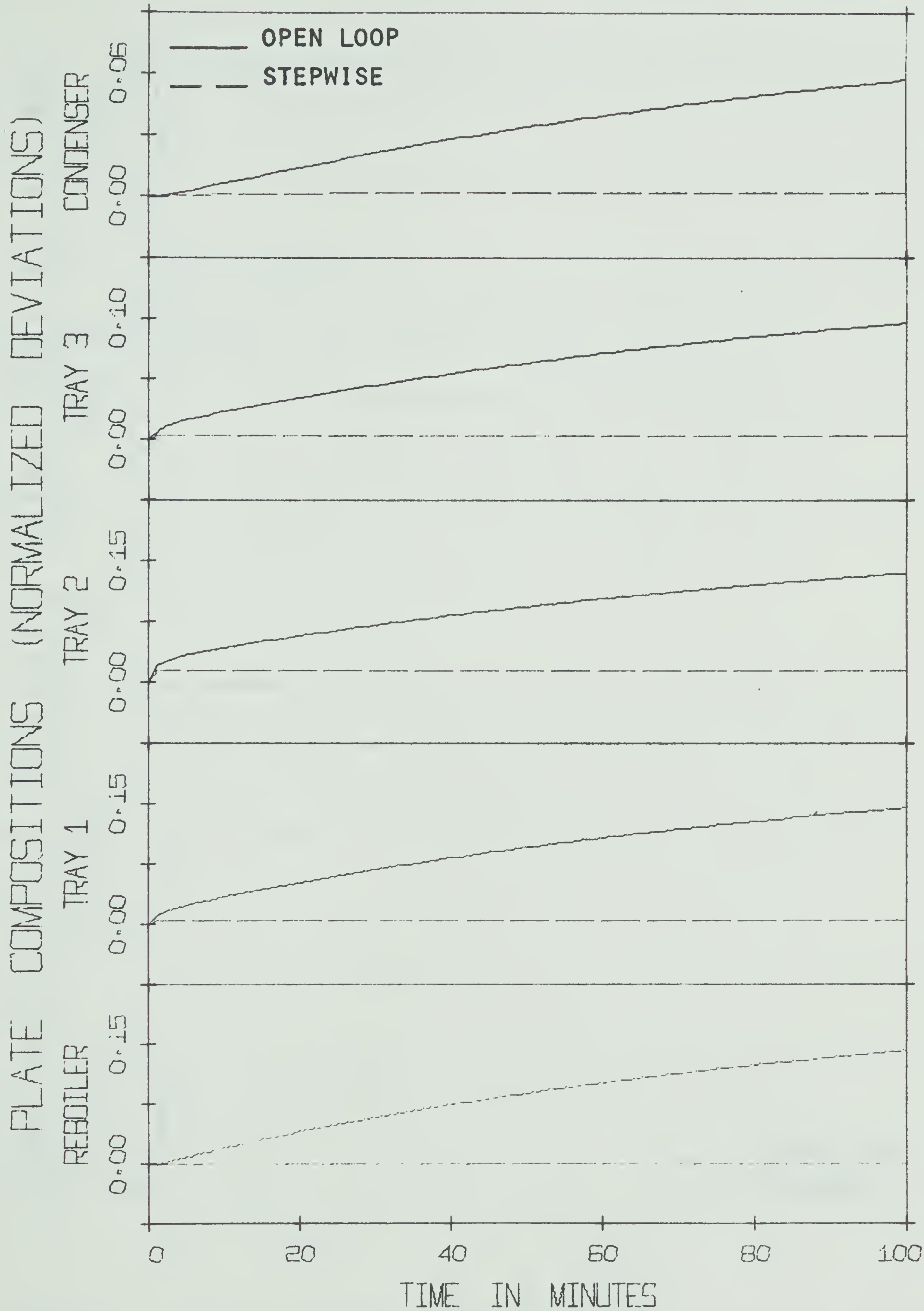


FIGURE 2a. OPEN LOOP RESPONSE VS STEPWISE CONTROL
(5L/+8%XF, +12%TF/OL, FBS/QD1/RD1)

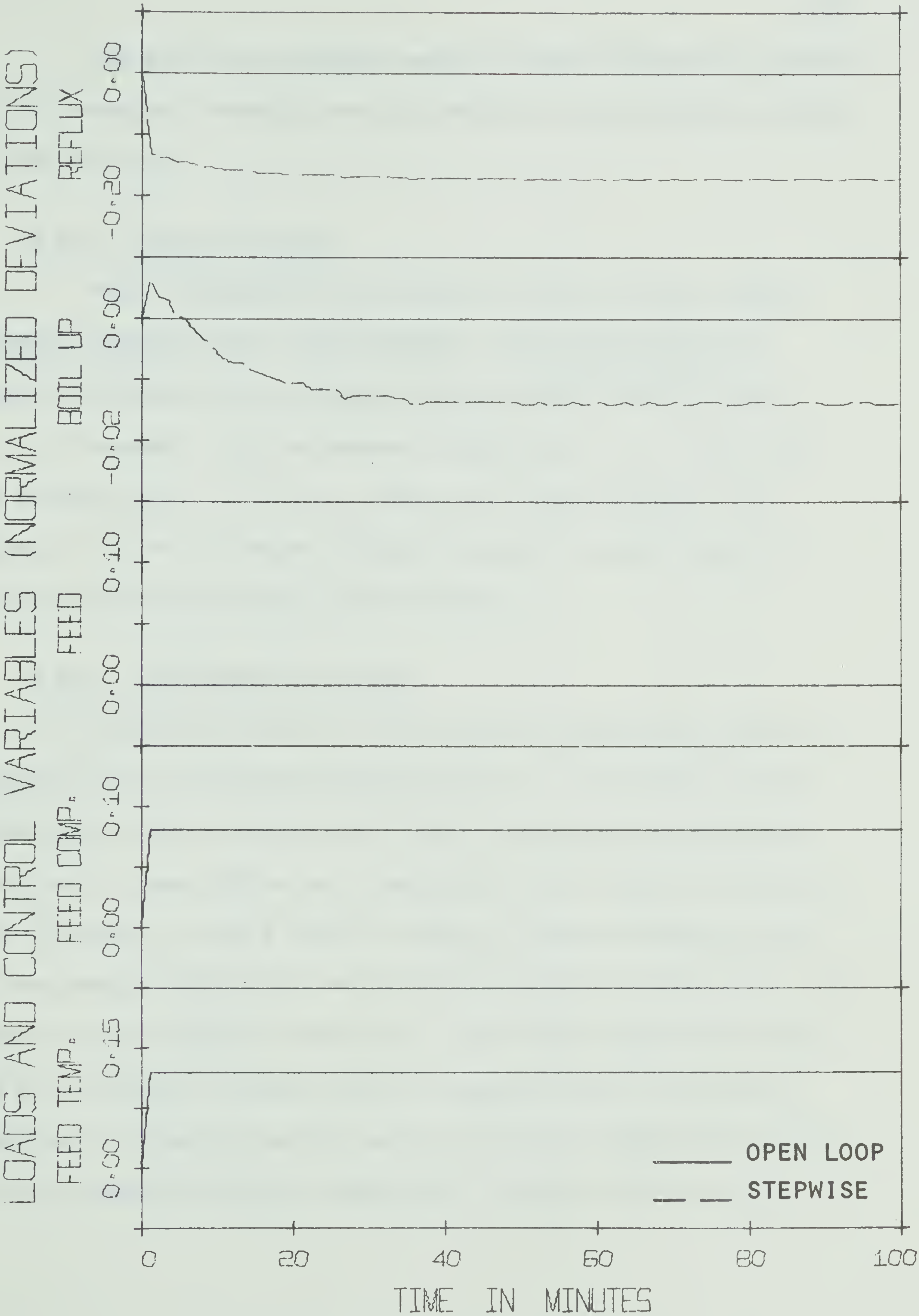


FIGURE 2b. OPEN LOOP RESPONSE VS STEPWISE CONTROL (5L/+8%XF, +12%TF/OL, FBS/QD1/RD1)

The model was simulated using the state difference equations, and all transients examined were the result of a ten percent increase in feed flowrate.

4.1.1. Control Interval

Table 2 summarizes the results of five runs with control intervals ranging from 5 to 120 seconds. The gains showed the expected decrease with increasing control interval and the offset in x_5 increased. The top product composition, x_1, will interact strongly with x_5 and its offsets show some variation. The criteria also vary in value although they show a general trend of increasing with increasing control interval.

4.1.2. State Weighting Matrix

Increased weighting on the product compositions tended to increase gains and thereby decrease offsets. The increase in the normalized criteria indicates the faster rise times and increased overshoot. These effects are illustrated by the first two series of runs in Table 3. Runs 9 and 10 in Table 3 which had weighting only on the product compositions showed little variations between the runs despite vastly different weighting. This effect suggests that there may be an "optimal" optimum when all possible direct and indirect weighting is removed from the control variables (compare the continuous case where zero control weighting is infinite control action).

TABLE 2

CONTROL INTERVALS

Run Number	Δt Seconds	x_1 Gain	x_1 Offset	x_1 ISE^*_n	x_5 Gain	x_5 Offset	x_5 ISE^*_n
17	5	-15.2	-.046	.807	24.1	.289	.981
18	15	-15.0	-.039	.765	22.1	.312	1.008
19	30	-14.6	-.028	.814	19.5	.349	.988
20	60	-13.8	-.004	7.79	15.4	.433	1.008
21	120	-12.7	.052	1.253	10.3	.629	1.017

* ISE_n is the integral of the squared errors divided by the offset squared times the run period. This reduces the effect of offsets on the criterion, but for small values of offset (eg. x_1 in Run 20) the criterion is unsatisfactory.

TABLE 3

WEIGHTING PRODUCT COMPOSITIONS

Run Number	State Weighting / Control Weighting	x_1 Gain	x_1 Offset	x_1 ISE_n	x_5 Gain	x_5 Offset	x_5 ISE_n
7	1 1 1 1 1 0 0	-29.9	0.10	.885	17.8	0.48	.977
12	10 1 1 1 10 0 0	-92.1	0.08	1.004	54.6	0.23	1.018
5	100 1 1 1 100 0 0	-319.	0.03	1.158	118.	0.11	1.129
11	10^4 1 1 1 10^4 0 0	-1616.	0.003	9.93	132.	0.08	1.136
8	1 1 1 1 1 1 1	-.54	1.28	.551	.16	3.94	.725
6	100 1 1 1 100 1 1	-2.98	.09	.977	6.42	.75	.969
10	1 0 0 0 100 0 0	-2301.	.001	27.9	119.0	.088	1.111
9	100 0 0 0 1 0 0	-2340.	.001	33.5	119.8	.087	1.122

Table 4 illustrates the effects on the product streams of increasing the weighting on the intermediate plate compositions. As might be expected the gains decrease, offsets increase, and there is less overshoot.

4.1.3. Control Weighting Matrix

Increased control weighting decreases control action (gains) and results in generally increased offsets. Responses are more damped with generally decreasing criteria. The results are summarized in Table 5.

The effects of the design parameters on system control can be summarized as follows.

(a) Larger control intervals result in lower gains but larger offsets with some overshoot.

(b) Increased weighting on one variable improves its control (by increasing its gains) and results in poorer control for the remaining variables.

(c) Removal of all direct and indirect weighting from the control variables appears to give "optimal" optimal control with relative weighting having little effect.

(d) Control variable weighting results in slower responses and larger offsets.

TABLE 4

WEIGHTING PLATE COMPOSITIONS

Run Number	State Weighting / Control Weighting	x_1 Gain	x_1 Offset	x_1 ISE_n	x_5 Gain	x_5 Offset	x_5 ISE_n
15	100 0 0 0 100 / 0 0	-2340.	.001	41.4	119.8	.076	1.137
5	100 1 1 1 100 / 0 0	-319.	.03	1.158	118.	.11	1.129
12*	100 10 10 10 100 / 0 0	-92.1	.08	1.004	54.6	.23	1.018

*multiplication of the $\underline{\underline{Q}}$ matrix by ten (compare Run 12 in Table 3) does not affect the design

TABLE 5

WEIGHTING CONTROL VARIABLES

Run Number	State Weighting / Control Weighting	x_1 Gain	x_1 Offset	x_1 ISE_n	x_5 Gain	x_5 Offset	x_5 ISE_n
5	100 1 1 1 100 0 0	-319.	.03	1.158	118.	.11	1.129
13	100 1 1 1 100 .1 .1	- 14.6	-.025	.783	19.5	.31	1.000
14	100 1 1 1 100 .5 .5	- 4.69	.028	1.603	9.15	.57	.978
6	100 1 1 1 100 1 1	- 2.98	.09	.977	6.42	.75	.969

These general conclusions correspond to those observed from a similar study on a double effect evaporator and they would appear to be reasonable for any control system design based on a quadratic index.

4.2. Simulated Control Schemes

Simulated comparisons of a number of multivariable feedback and feedforward control systems were made.

4.2.1. Multivariable Feedback

A comparison was made between the multistep formulation discussed in Section 3 and the infinite time multivariable formulation.

Results are shown in Figure 3 of both control schemes reacting to a 10 percent step increase in feed flowrate. The infinite time formulated control system reacts faster (note the control variables in Figure 3b) and the states have smaller offsets because of higher gains in the control matrix.

The solution of the stepwise formulation involves less computation and has been used where the criterion or control law are modified on-line to meet control and state constraints [1]. In this work the infinite time formulation was solved off-line and hard constraints put on the control variables at the implementation stage. This is not optimal implementation but worked well for the cases investigated.

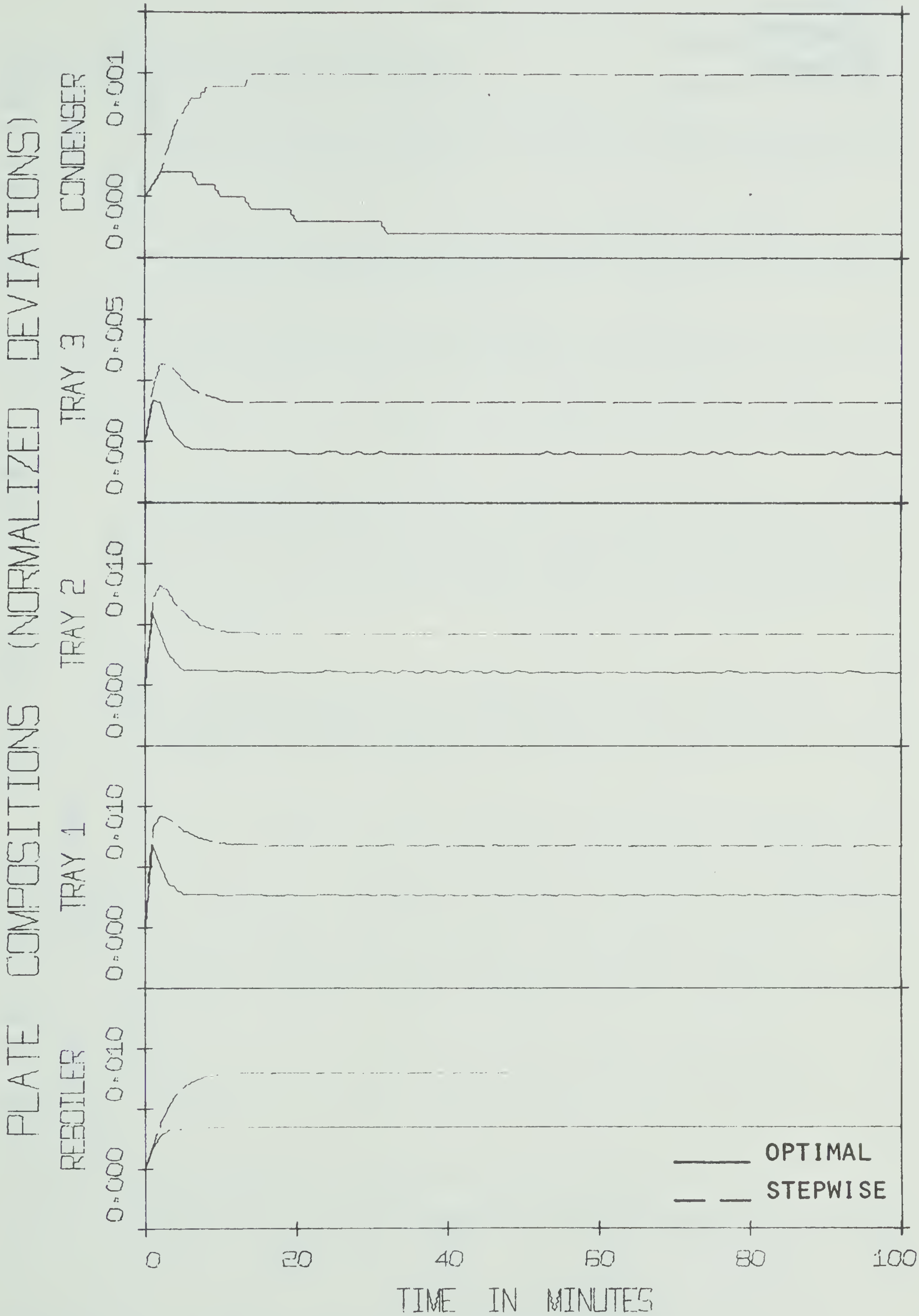


FIGURE 3a. STEPWISE VS OPTIMAL MULTIVARIABLE CONTROL (5L/+10%F/FBS, FB/QD2/RD2)

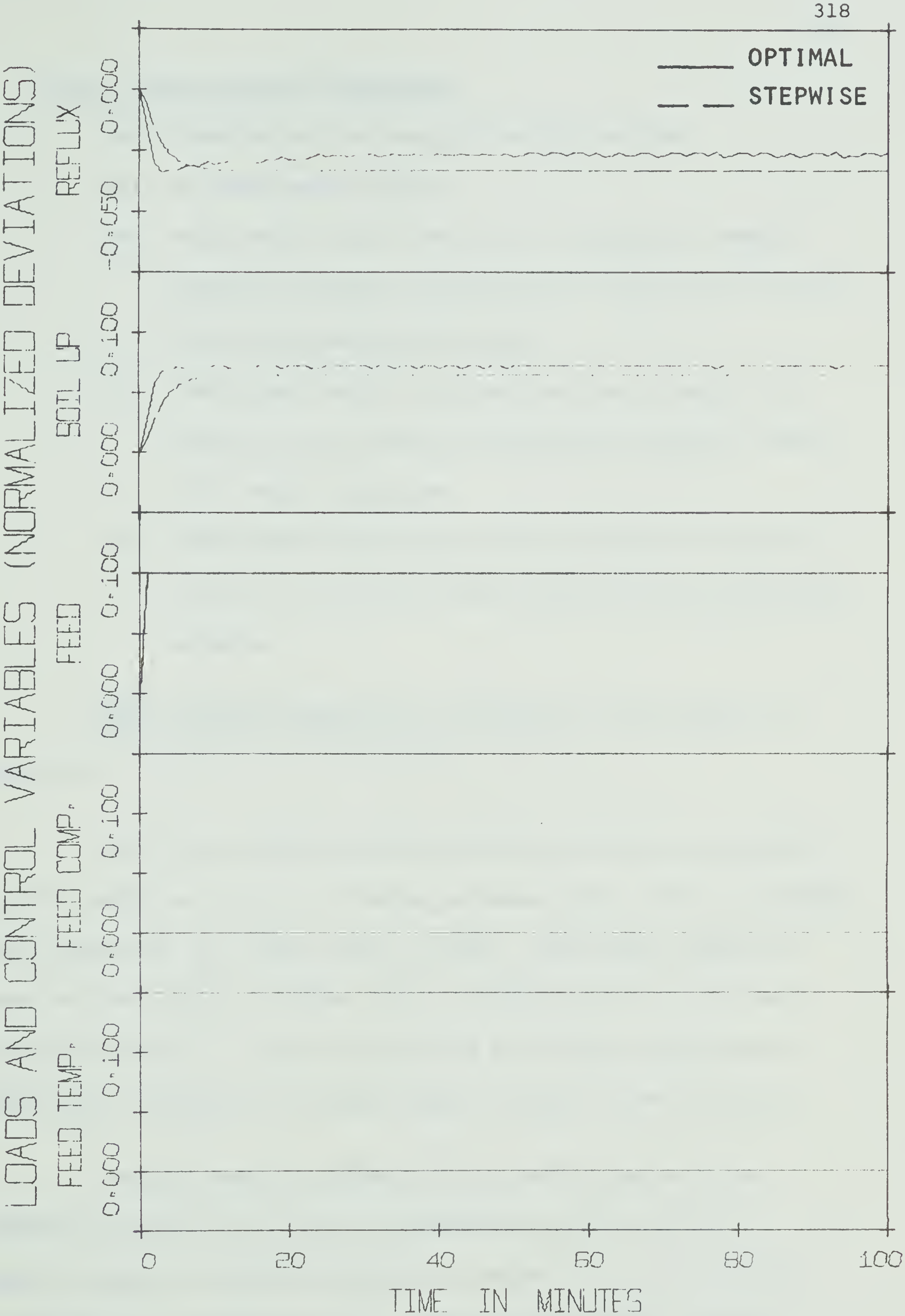


FIGURE 3b. STEPWISE VS OPTIMAL MULTIVARIABLE CONTROL
(5L/+10%F/FBS, FB/QD2/RD2)

4.2.2. Multivariable Feedforward

Four formulations are compared in this section.

(a) No feedforward control.

(b) Feedforward control designed to minimize a summed quadratic criterion (Equation (5)) using the discrete dynamic programming algorithm.

(c) Feedforward control designed for zero steady state offsets in the product compositions using the steady state design technique.

(d) Feedforward control designed to minimize a quadratic function of the final state offsets using differential calculus.

These design techniques are discussed in more detail in Chapter 6.

All three feedforward formulations have the same feedback control matrix as case (a). Design parameters were Δt = 30 seconds, state weighting $\underline{\underline{Q}}$ = diag (100,1,1,1,100), and control weighting zero for the results in Table 6 and $\underline{\underline{R}}$ = diag (0.1,0.1) for the results in Table 7. All results listed are steady state offsets caused by a persistent 10 percent step increase in feed flowrate.

In all cases the effects of the feed change were considerably reduced by the use of feedforward action (Run 30 in Table 7 showed a threefold increase in offset in x_1 but a tenfold decrease in offset in x_5). The smallest offsets resulted

TABLE 6

COMPARISON OF FEEDFORWARD SCHEMES I

(No control weighting, $\underline{\underline{R}} = 0$)

Run	Case	x_1 Offset	x_5 Offset
5	(a)	.03	.11
33	(b)	$.294 \times 10^{-3}$	$.251 \times 10^{-3}$
34	(c)	$-.594 \times 10^{-7}$	$-.91 \times 10^{-7}$
35	(d)	$.294 \times 10^{-3}$	$.250 \times 10^{-3}$

TABLE 7

COMPARISON OF FEEDFORWARD SCHEMES II

(Control weighting, $\underline{\underline{R}} = \text{diag}\ (.1\ .1)$)

Run	Case	x_1 Offset	x_5 Offset
13	(a)	-.025	.310
30	(b)	-.085	.037
31	(c)	$-.448 \times 10^{-6}$	$.596 \times 10^{-6}$
32	(d)	$.294 \times 10^{-3}$	$.250 \times 10^{-3}$

from the use of the feedforward matrix designed to give zero offsets, as might have been expected. The control feedforward matrices from the minimization of the summed quadratic criterion and of the quadratic function of offsets gave equivalent results when there was no control weighting (Table 6). However, the design based on the summed criterion is affected considerably by control weighting (Run 30 in Table 7). The other feedforward design methods (Run 31 and 32) do not involve the control weighting matrix, $\underline{\underline{R}}$, and the feedforward control matrices are therefore independent of it. (The variations between Runs 31 and 34 are a result of round-off error.)

Similar comparisons were made on a double effect evaporator which also indicated that the zero offset technique appeared to give the best results independent of control weighting.

5. NONINTERACTING CONTROL SYSTEM DESIGN

There are basically three steps in completing the design of a noninteracting control system. Firstly, it is necessary to determine whether the process can, in fact, be made noninteracting. Secondly, a system of compensators must be designed to achieve noninteraction. Finally, a set of single-variable controllers must be designed for the noninteracting pairs of outputs and control variables.

5.1. Condition for Noninteraction

Since a state space model was available for the distillation column, synthesis of noninteraction by state variable feedback was chosen. Falb and Wolovich [3] have presented necessary and sufficient

conditions for noninteraction by state feedback. The distillation model was found to satisfy these conditions with the top and bottom product compositions as outputs and the reflux and boilup as the corresponding control variables. This configuration could be chosen by a multiloop sensitivity analyses such as presented in Chapter 4.

5.2. Noninteracting Design

Gilbert [4] has developed a synthesis procedure for noninteraction by state feedback and this was developed into a computer algorithm by Gilbert and Pivnichny [5]. The program written in FORTRAN was obtained from the University of Michigan and was adapted for execution on the IBM 360 Model 67 at the University of Alberta.

The computer algorithm accepts the state space process model in the form

$$\dot{\underline{x}} = \underline{\underline{A}}_c \underline{x} + \underline{\underline{B}}_c \underline{u} \tag{8}$$

$$\underline{y} = \underline{\underline{C}} \, \underline{x} \tag{9}$$

and synthesizes a control law of the following form where $\underline{v}$ is the new control vector.

$$\underline{u} = \underline{\underline{F}} \, \underline{x} + \underline{\underline{G}} \, \underline{v} \, . \tag{10}$$

The control matrices $\underline{\underline{F}}$ and $\underline{\underline{G}}$ are synthesized as functions of parameters λ_i and σ_{ij} which are defined by the transfer function model of the synthesized noninteracting system, Equation (13). Equations (11) and (12) express the $\underline{\underline{F}}$ and $\underline{\underline{G}}$ matrices for the distillation column model, Equation (1). The transfer function model for the distillation column is of the following form.

$$\underline{\underline{F}} = -\begin{bmatrix} 23.20 & -35.14 & 15.37 & 0 & 0 \\ 11.39 & -17.43 & 7.55 & -12.77 & 12.69 \end{bmatrix} + \sigma_{11}\begin{bmatrix} -24.96 & 15.68 & 0 & 0 & 0 \\ -12.25 & 7.70 & 0 & 0 & 0 \end{bmatrix}$$

$$+ \sigma_{12}\begin{bmatrix} 228.2 & 0 & 0 & 0 & 0 \\ 112.0 & 0 & 0 & 0 & 0 \end{bmatrix} + \sigma_{21}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -78.06 \end{bmatrix}$$

Equation (11)

$$\underline{\underline{G}} = \lambda_1\begin{bmatrix} 228.2 & 0 \\ 112.0 & 0 \end{bmatrix} + \lambda_2\begin{bmatrix} 0 & 0 \\ 0 & -78.06 \end{bmatrix}$$

Equation (12)

$$\begin{bmatrix} x_1(s) \\ x_5(s) \end{bmatrix} = \begin{bmatrix} \dfrac{\lambda_1}{s^2 - \sigma_{11}s - \sigma_{12}} & 0 \\ 0 & \dfrac{\lambda_2}{s - \sigma_{21}} \end{bmatrix} \begin{bmatrix} R(s) \\ Q_s(s) \end{bmatrix} \qquad (13)$$

where $\underline{y}^T = (x_1\ x_2)$ and $\underline{v}^T = (R\ Q_s)$.

In the case of the distillation column Gilbert's synthesis leaves five parameters to be chosen by the designer. For this design the two transfer functions were fitted to the step responses of x_1 and x_5 to R and Q_s respectively from the original model (Equation (1)). Parameter values chosen from the responses were as follows:

$$\lambda_1 = .00193\ , \qquad \sigma_{11} = -.3084$$

$$\sigma_{12} = -.00320$$

$$\lambda_2 = -.0239\ , \qquad \sigma_{21} = -.0115$$

A comparison of the original responses used for the fitting and the responses from the synthesized noninteracting system is made in Figures 4 and 5 for 10 percent step increases in reflux and boil-up respectively. It showed that almost complete noninteraction was achieved. Other bases for the choice of the parameters λ_i and σ_{ij} exist, although for the present situation where the effects of the interactions are to be examined the present basis is the best. It can be noted that if the parameters are chosen for faster responses in x_1 and x_5 the gain elements in $\underline{\underline{F}}$ and $\underline{\underline{G}}$ increase, and the system may even become unstable if it is attempted to obtain "impossible" responses.

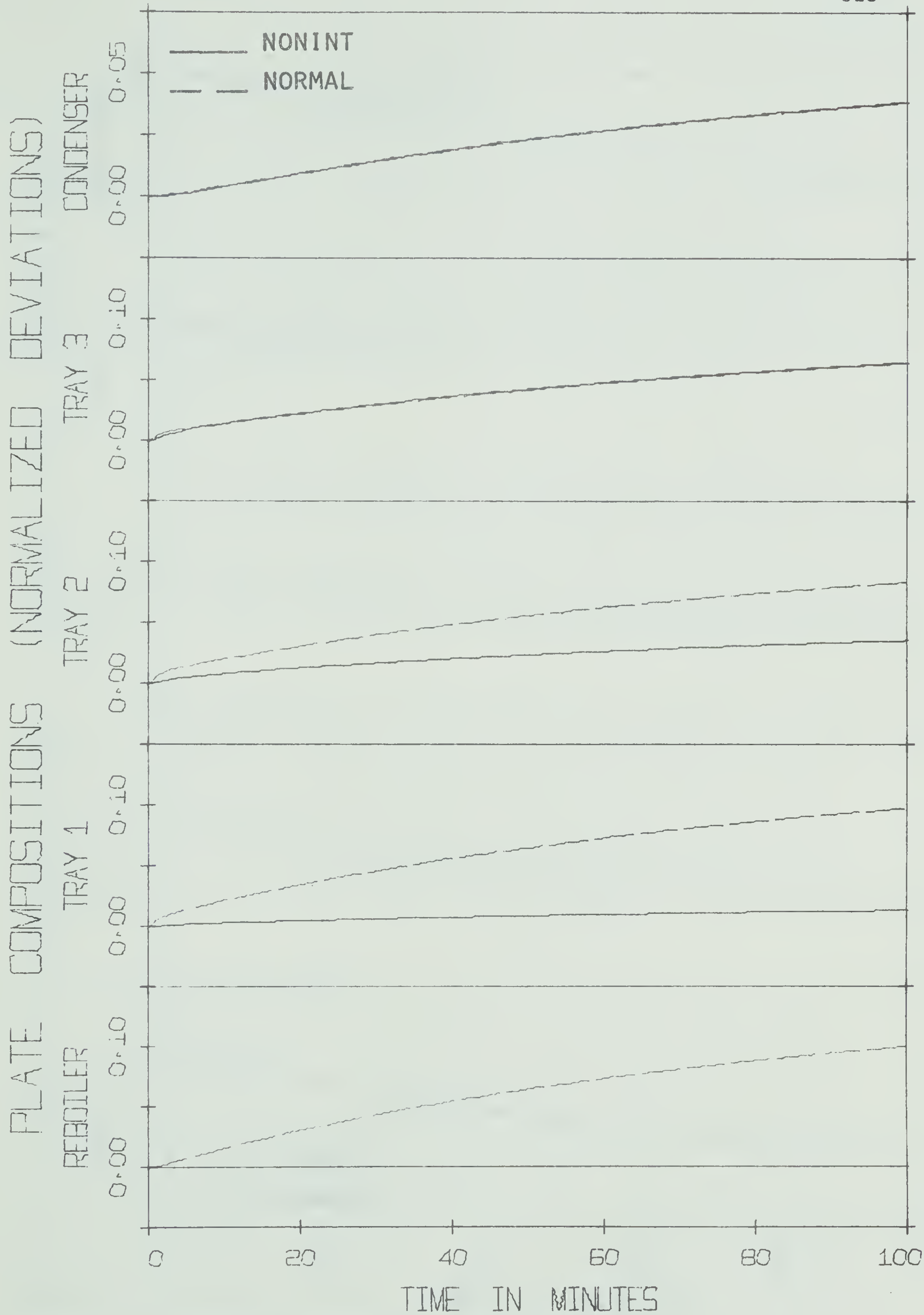


FIGURE 4. COLUMN RESPONSES TO REFLUX STEP
(5L, 5LDC/+10%R/OL)

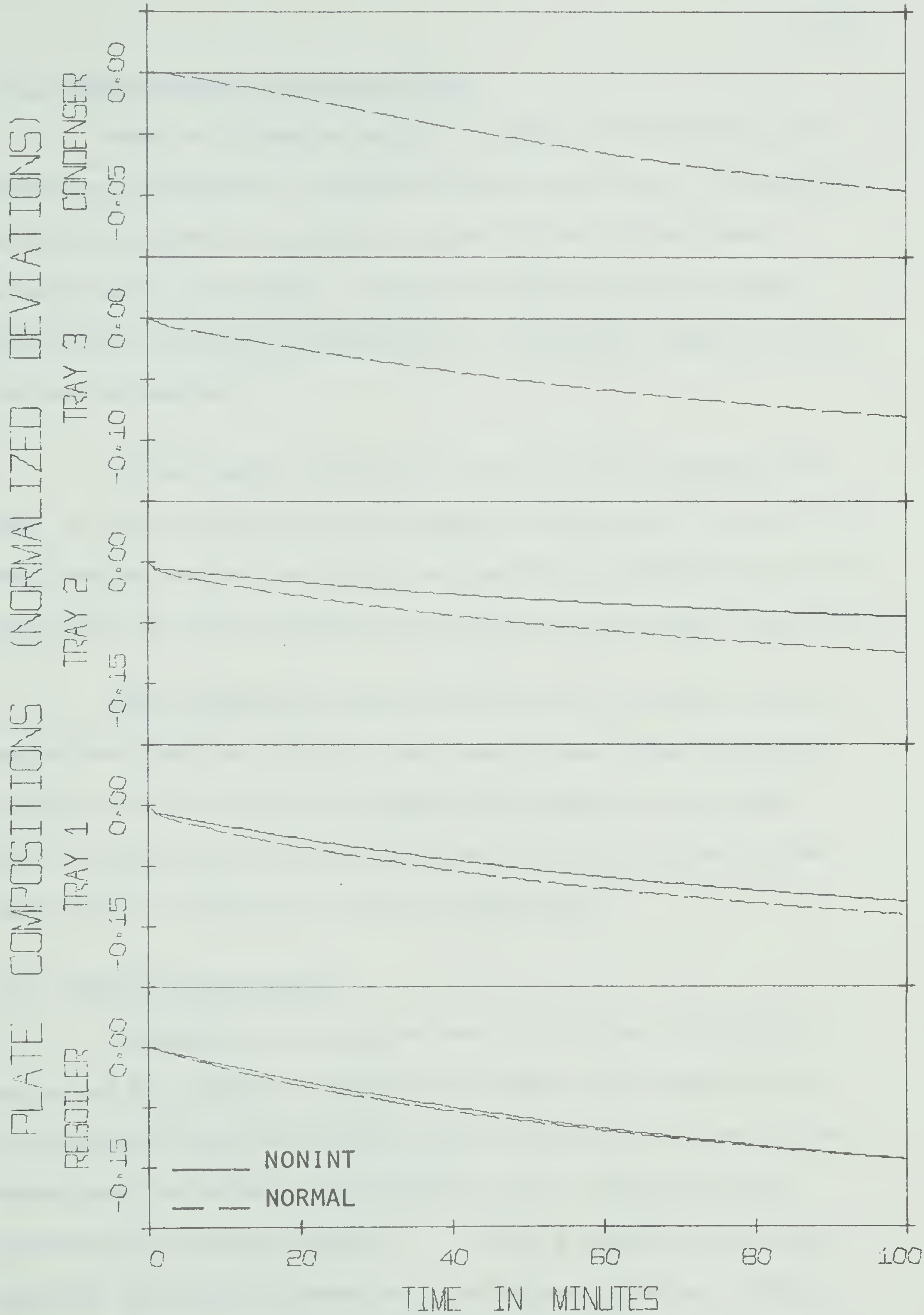


FIGURE 5. COLUMN RESPONSES TO BOIL UP STEP
(5L, 5LDC/+10%QS/OL)

5.3. Single-Variable Controller Design

There is a large selection of methods for evaluating the controller constants for a single-variable control loop. Analytical methods such as Bode and root loci plots [6] can be used when a reliable model is available. Empirical formula based on a first order plus time delay approximation to the process response [7] are also used extensively.

In the present circumstance where the time constants are large (87 and 93 minutes) and the apparent delays small (0 and 3 minutes) both approaches, analytical and empirical, predict impractically large gains for single-variable proportional-plus-integral controllers.

Both controllers were given a practical maximum 2 percent proportional band and integral times were chosen within the normal range of 1 to 50 minutes to minimize the integral of the squared error. Integral time values chosen for the top and bottom product loops were 2.5 minutes and 1 minute respectively.

5.4. Simulated Comparisons

A comparison was made between the original interacting system and the designed noninteracting system, both controlled by a conventional proportional-plus-integral algorithm and the original system under an infinite time formulation for multivariable proportional-plus-integral control . Table 8 presents the details regarding the control systems and a performance index and Figure 6 shows the responses of the three controlled systems to a 10 percent step increase in feedrate.

TABLE 8

CONTROL SYSTEM COMPARISON

Run	Control	Parameters	Quadratic Criteria x_1	x_2	x_3	x_4	x_5	J^*
1207	Conventional P + I	$K = 50$ $\tau_I = 2.5$ and 1.0	$\frac{.14}{10^3}$	$\frac{.34}{10^3}$	$\frac{.24}{10^2}$	$\frac{.21}{10^2}$	$\frac{.40}{10^5}$	.0189
1209	Noninteracting State Feedback and Conventional P + I	Matched Responses $K = 50$ $\tau_I = 2.5$ and 1.0	$\frac{.33}{10^4}$	0	$\frac{.79}{10^3}$	$\frac{.49}{10^3}$	0	.0046
1208	Multivariable P + I	$\underline{\underline{Q}} = \text{diag}(100,1,1,1,100,50,50)$ $\underline{\underline{R}} = \text{diag}(.01,.01)$ $\Delta t = 1$ second	$\frac{.28}{10^4}$	$\frac{.92}{10^4}$	$\frac{.77}{10^3}$	$\frac{.36}{10^3}$	$\frac{.35}{10^5}$	.0043

* $J = \underline{x}^T \underline{\underline{Q}}\, \underline{x}$; $\underline{\underline{Q}} = \text{diag}(100,1,1,1,100)$

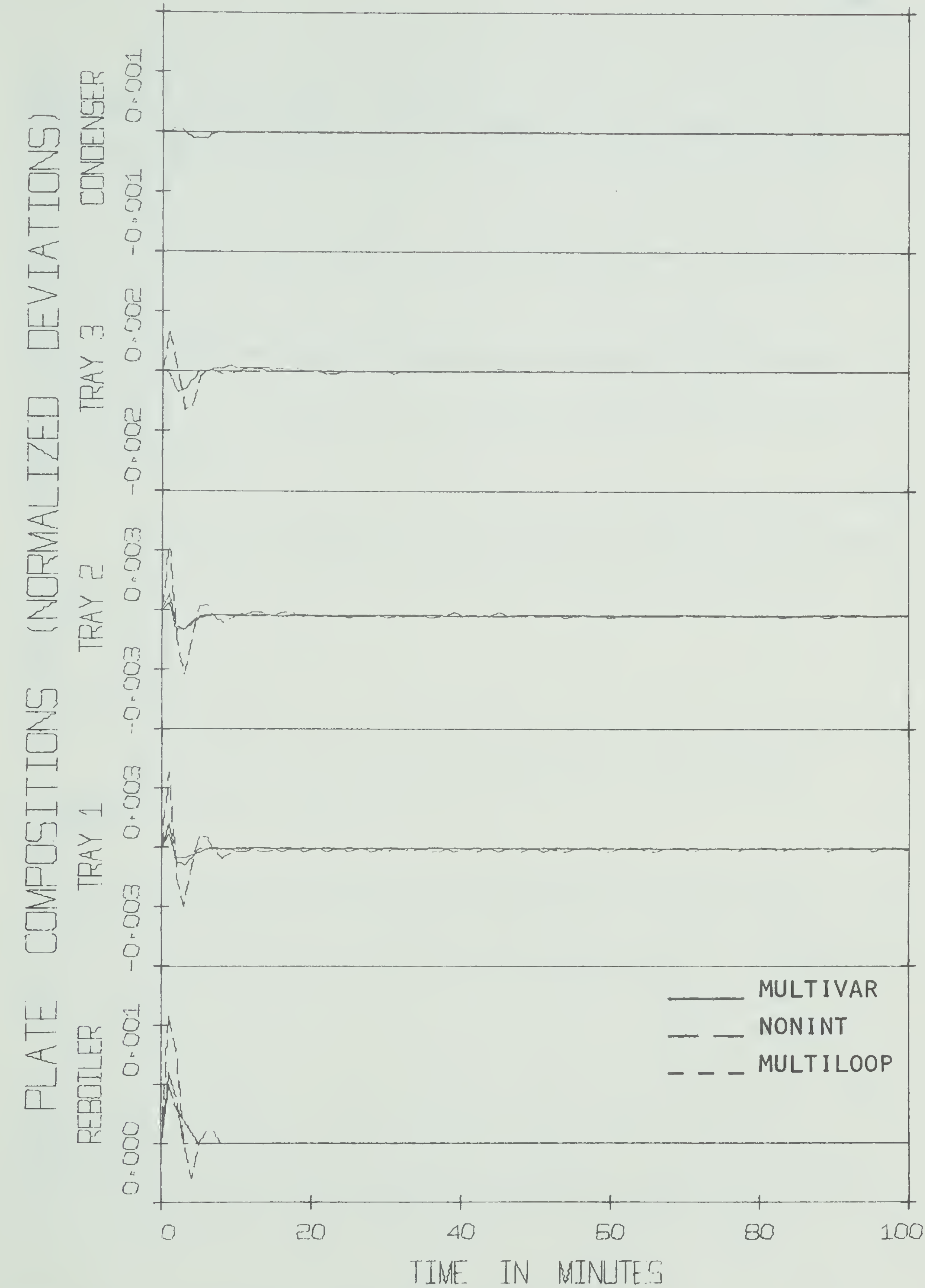


FIGURE 6a. COMPARISON OF CONTROL SYSTEMS
(5L/+10%F/FB, MLDC, ML/QD3/RD3)

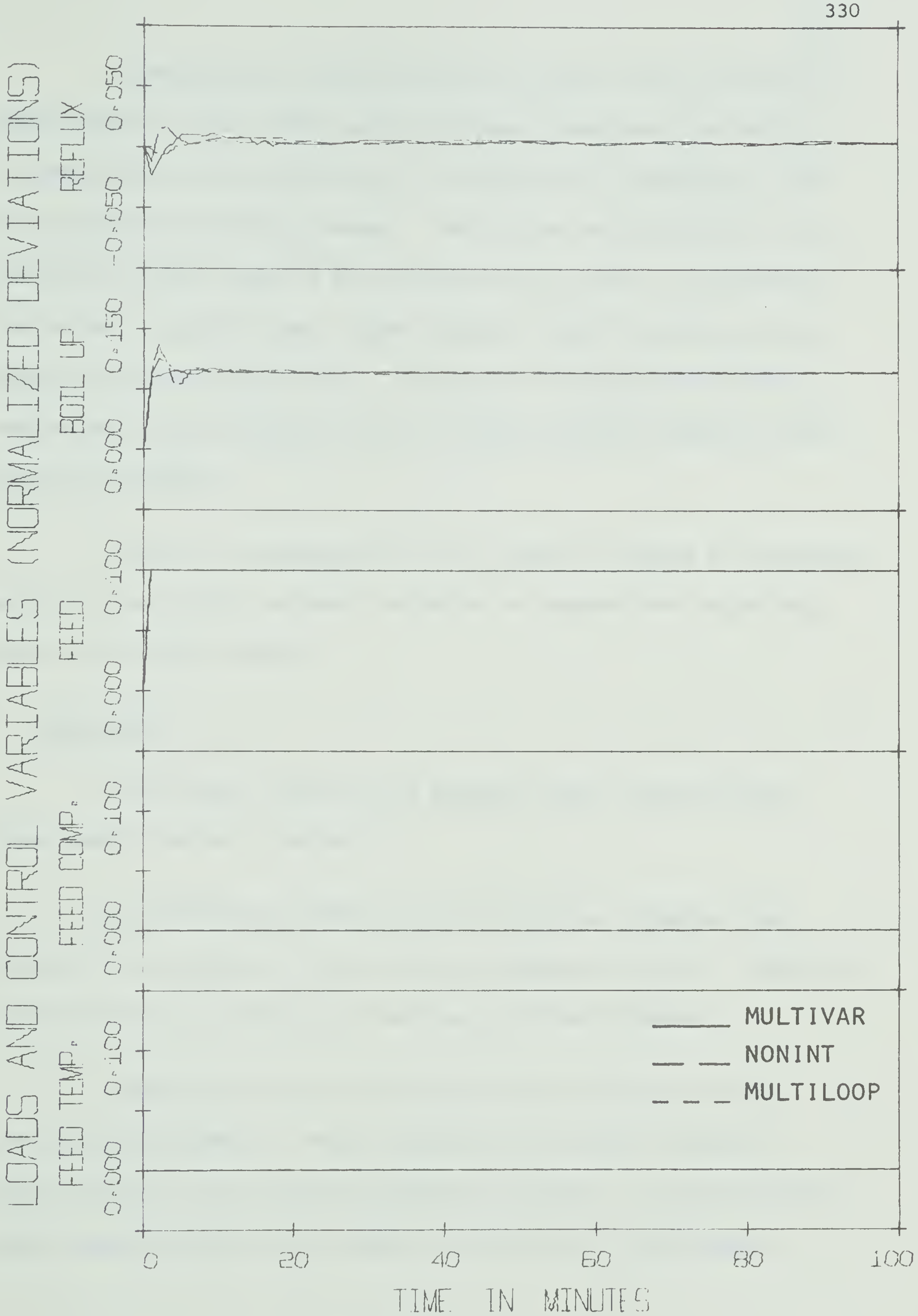


FIGURE 6b. COMPARISON OF CONTROL SYSTEMS
(5L/+10%F/FB, MLDC, ML/QD3/RD3)

A comparison of the multiloop (Run 1207) and the noninteracting multiloop (Run 1209) control schemes illustrate graphically the advantages in control quality of removing the interactions from the conventional multiloop scheme. The optimal multivariable control (Run 1208) is very close to the noninteracting scheme in performance with perhaps a slight edge. (Note that the "noise" in the simulated results in Figure 6 is due to a truncation of significant figures (simulating a DDC algorithm) and the limited figures punched by the simulation program).

However, an examination of the scales in Figure 6a indicates that all three control schemes including the conventional multiloop scheme give "good" control.

6. CONCLUSIONS

Four control schemes were compared using a distillation column model from the literature.

Multivariable feedback control using the infinite time formulation was superior to that using a stepwise approach. Responses to load changes were faster and had smaller final offsets.

Compensators for noninteraction were synthesized using state variable feedback. When controlled by the same multiloop control scheme as the original interacting column, the noninteracting system responded faster with smaller deviations to load changes.

An optimal multivariable proportional-plus-integral control scheme based on the infinite time formulation gave improved responses to load changes when compared to the noninteracting scheme. Furthermore, the optimal scheme involved less design effort and would be no more complex to implement.

A study of optimal multivariable design parameters and different multivariable feedforward design techniques led to conclusions which agreed with those of an evaporator study. The conclusions drawn appear to be generally applicable.

CHAPTER ELEVEN

DISCUSSION AND CONCLUSIONS

The purpose was to develop, implement, and evaluate multivariable control systems that would be suitable for industrial application. The control systems examined were multiloop, noninteracting, and optimal multivariable control with emphasis on the last system. Evaluation and comparison of the different methods was completed using a total of 73 experimental runs on a pilot plant evaporator and over 250 simulation runs on models of the evaporator and a distillation column.

1. DISCUSSION

Multiloop control system design was examined in Chapter Four. It was found that a state space model could be used in conjunction with a sensitivity (to the control variables) analysis to design a configuration for the single-variable controllers. Although the fifth-order linear model adequately represented the process and proved a satisfactory basis for multivariable control, it was not a suitable basis for selecting constants for single-variable controllers. The fifth-order model with three outputs implied approximately first or second-order dynamics between output and control variable pairs. Hence "theoretical" constants were too high. It would be necessary to consider time delays and more of the dynamics for a satisfactory basis. Empirical techniques based on actual process responses appear to be the most practical way to obtain constants.

Control quality under multiloop control was fair in the case of the evaporator where large interactions existed but was good for the simulated distillation column. The model also had interactions but its almost first-order behavior again resulted in the choice of high controller constants. These could be used in simulations whereas in practice the delays inherent in columns [[1] Chapter 11] would likely remove this advantage.

Noninteracting control is restricted to processes which can be decoupled. The evaporator could not be, so the study was restricted to the distillation model (see Chapter Ten). A computational algorithm exists for the synthesis of noninteraction by state feedback. The designer is presented with the form of a model of the decoupled system and relations for the control matrices but must choose a number of parameters in the model of the decoupled system. In addition, single-variable controller constants must be chosen to control the decoupled pairs. The removal of the interactions gave a significant improvement over multiloop control and was comparable to an optimal multivariable control system. The performance of the noninteracting system may not be comparable if the high controller constants could not be used. This was the experience with the evaporator.

Optimal multivariable control does not suffer from the decoupling restriction. Its application is dependent only on controllability and observability. The design parameters, the control interval and the weighting matrices, must be chosen by the designer and a simulation study proved valuable here. Some general effects of changing the parameters were presented in Chapter Five. These were

applicable to both the evaporator and distillation column models. The design algorithm developed together with different formulations could be used to evaluate multivariable proportional control constants for feedback, feedforward, integral, and setpoint control modes. The control quality observed under these modes (and any combination) was excellent.

The proportional feedback control mode was found to optimally drive the process to some offset if a persistent load was placed on the process. This offset was a function of the process, the feedback control, and the load. However these offsets were very small because of the high gains associated with the optimal control. The feedforward control mode could be designed in a variety of ways and in all cases almost completely removed the offset resulting from measurable loads. Integral feedback control was formulated and it "trimmed" offsets to zero where loads could not be measured or were unknown. The integral control raised both the order of the control problem and the resulting feedback gains which could possibly be disadvantageous in some cases. It was shown that a multivariable system has the same number of degrees of freedom as control variables in so far as driving states or outputs to specific values. This was of great importance in both the integral and setpoint formulations. Finally two formulations were presented for optimal setpoint control which did not interfere with the regulatory modes. A "model following" formulation gave the designer control over the processes response to a step change in setpoint.

A process model was required for the noninteracting synthesis as well as for the optimal multivariable control design. The model

required for the multivariable design need include only the significant dynamics and reasonably accurate gains. Reduction of model order by neglecting small time constants did not appear to unduly upset the design. State feedback contributes phase leads to the system which are approximately in proportion to the respective state "time constants". These phase leads account for the process's acceptance of high gains [2]. Hence there is relatively little reduction in the total phase lead, and hence little degradation of control quality, when states with small "time constants" are neglected in the analysis. However, the ultimate test of the model remains the successful implementation of the task required of it.

Implementation of both noninteracting and optimal multivariable control requires a measurement or estimate of the state. The evaporator application had one state which was not measured and was estimated from the model. An "exponential" filter was used to combine measurements and model predictions and proved adequate although it did not result in optimal implementation. Optimal implementation is possible deterministically with Luenberger predictors or stochastically with a Kalman filter. Process noise in the measurements can be troublesome with the high gains and should be reduced to a minimum. It is obviously preferable to work on obtaining a noise-free measurement rather than filtering a noisy one. The latter alternative introduces unwanted lags into the system. Implementation of noninteracting and optimal multivariable control almost certainly requires a digital control computer particularly if such overhead as state estimation is necessary. It was found that interfacing a control program with a Direct Digital

Control (DDC) system simplifies the implementation. Computer usage for a five dimensional application, including state estimation, was not greater than eight percent of system time including some slow disk read/writes.

2. FURTHER WORK

The results of this work indicated a number of areas where further development might be of advantage in the design and application of multivariable control.

2.1. Design Parameters

Further work is always possible relating the parameters in the summed quadratic criterion to more familiar measures of control quality. Some work has been done on relating weighting parameters to system poles [3,4]. It may be possible to extend the relations to familiar time domain criteria such as overshoots, settling times, decay ratios, etc.

2.2. Control Problem Formulations

It may be possible to formulate the optimal control problem so that other modes of control can be designed and other system properties allowed for. Two possibilities are mentioned.

(a) Process Time Delays. The present problem formulation will handle the presence of delayed state variables although the derivation of the recursive relations becomes exceedingly complex for time delays greater than about four control intervals. It may also be possible to use the process model to predict over the time delay and

combine this prediction with the state estimation [5].

(b) Dynamic Lead/Lag Feedforward Compensation. Although the examples used in this study controlled so well with static feedforward, other systems, especially those involving time delays, could benefit from a lead/lag type of dynamic compensation.

2.3. Partial State Feedback Control

Partial state feedback is a favoured topic in the recent literature and some of the papers are mentioned in Chapter Two, Section 8.

An examination of the phase leads contributed by feedback from different states [2] may give an understanding of the most desirable states to feedback for control. It may be that partial state feedback is not the entire answer and that it shall remain desirable to estimate some states. Model reduction is an approach to the partial state feedback problem and should also be considered in this light.

2.4. State Estimation

A more satisfactory state estimation procedure than that used should probably be considered. The two outstanding approaches are Luenberger's deterministic predictor and Kalman's stochastic filter. The predictor approach has the advantages of a lower order problem although it does not result in optimal implementation in the presense of process noise.

2.5. Empirical Multivariable Control

The current approach to multivariable design requires a process model which is generally derived theoretically and/or "identified". An empirical approach could be developed. Diagonal control matrices are equivalent to multiloop control and could be used as a basis for an adaptive optimization of the process's performance with respect to the elements of the control matrices. This work has shown that many of these off-diagonal elements are relatively significant. Advanced optimization techniques [6] would allow the use of constraints and arbitrary (informed or otherwise) fixing of elements.

3. CONCLUSIONS

An optimal multivariable regulatory control system was designed and developed for industrial processes. It proved superior to noninteracting and multiloop control on the basis of both simulated and experimental tests.

The optimal control problem formulations developed are straight forward and their solutions readily obtainable from a calculational algorithm derived using discrete dynamic programming. Design parameters can be chosen using guidelines developed for the parameter effects and can be checked by simulation. Practical factors such as on-line computer usage and process noise must be considered in choosing the design parameters. The feedback and setpoint control modes should be sufficient in most cases. Feedforward and/or integral control modes should be added when persistent loads affect the process

and the resulting offsets are not desirable.

Noninteracting design is restricted to decouplable systems and involves design parameters with no established basis for their selection. The resulting control system requires the same implementation as an optimal system but does not perform as well for regulatory control.

Multiloop controller configurations can be designed by "experience" or using a sensitivity technique which was developed for state space models. Controller constants are still best obtained empirically from plant data. Control quality suffers significantly from process or control interactions.

The state space model required for the optimal and non-interacting designs need only have reasonable gains and contain the dynamics of the states with the most significant "time constants". State estimation requires more accurate gains particularly if used with setpoint control.

Implementation is straightforward on a control computer particularly if the control program can be interfaced to a DDC system. With an appropriate choice of control interval the implementation of state estimation and optimal multivariable control should be within the capabilities of the computer in most computer controlled industrial processes.

NOMENCLATURE FOR CHAPTER TWO

(a) Alphabetic

$\underline{\underline{A}}$	State equation coefficient matrix
$\underline{\underline{B}}$	State equation coefficient matrix
$\underline{\underline{C}}$	Output equation coefficient matrix
$\underline{\underline{D}}$	State equation coefficient matrix
$\underline{d}$	Load vector
d	Derivative
$\underline{\underline{E}}$	Output equation coefficient matrix
e	Exponential
$\underline{\underline{F}}$	Output equation coefficient matrix
$\underline{\underline{G}}$	Output equation coefficient matrix
J	Control criterion
$\underline{\underline{K}}$	Control matrix
$\underline{\underline{Q}}$	State weighting matrix
$\underline{\underline{R}}$	Control weighting matrix
$\underline{\underline{S}}$	Final state weighting matrix
T	Final time
t	Intermediate time
$\underline{u}$	Control vector
$\underline{v}$	Delayed state vector
$\underline{x}$	State vector
$\underline{y}$	Output vector

(b) Greek Alphabetic

β	Time weighting in exponential term

Nomenclature for Chapter Two (continued)

(c) Subscripts

FB Feedback

FF Feedforward

= Matrix

_ Vector

(d) Superscripts

T Matrix or vector transpose

NOMENCLATURE FOR CHAPTER THREE

(a) Alphabetic

A	Heat transfer area
B	Bottoms flowrate
C	Solution concentration
C_p	Heat capacity
F	Feed flowrate
$\underline{F}$	Vector function
f	Scalar function
H	Vapour enthalpy
h	Liquid enthalpy
K	Transfer function gain
k	Evaporation per heating vapour rate
L	Heat loss
O	Vapour flowrate
P	Pressure
Q	Heat flowrate
S	Steam flowrate
Sc	Steam condensate flowrate
s	Laplace transform variable
T	Temperature
t	Time
U	Heat transfer coefficient
u	Forcing function
V	Volume; variable
W	Solution holdup

Nomenclature for Chapter Three (continued)

W_w Wall mass

x State vector

(b) Greek

α Fractional recycle

β Fractional feed

δ Superheat

λ Heat of vapourization

ρ Density

ϕ Heat of solution effect

τ Time constant

(c) Subscripts

c Condenser; condensate; condensing

cw Cooling water

d Time delay

F Feed

i Element subscript in a vector, inlet value

j Element subscript in a vector

m Mean value

n Normalized

o Outlet

PR Product

s Steam; solution

v Vapour

w Wall of tubes

Nomenclature for Chapter Three (continued)

ss Steady state

(d) Superscripts

$\_$ Mean value

i Effect or unit number

j Effect or unit number

(e) Abbreviations

CC Concentration controller

CR Concentration recorder

FC Flow controller

FR Flow recorder

LC Level controller

PC Pressure controller

NOMENCLATURE FOR CHAPTER FOUR

(a) Process Variables

		Steady State
B1	Product flow from first effect	1.69 lb./min.
B2	Product flow from second effect	0.98 lb./min.
CF1	Concentration of solution to feed section	
CF	Concentration of first effect feed	3.02 percent
C1	Product concentration from first effect	4.34 percent
C2	Product concentration from second effect	7.51 percent
F	Feed flow to first effect	2.43 lb./min.
F1	Feed flow of solution to feed section	
F2	Feed flow of solute to feed section	
HF1	Enthalpy of solution to feed section	
HF2	Enthalpy of solute to feed section	
HF	Enthalpy of feed to first effect	55.3 Btu/lb.
HOF1	Enthalpy of solution from feed section	
HOF2	Enthalpy of solute from feed section	
H1	Enthaply of product from first effect	152. Btu/lb.
O1	Overheads from first effect	
O2	Overheads from second effect	
P1	Pressure in first effect	
P2	Pressure in separator	
S	Steam flow to first effect	1.00 lb./min.
S1	Steam flow to feed solution heater	
S2	Steam flow to feed solute heater	
TF	Temperature of first effect feed	
T1	Temperature in first effect	

Nomenclature for Chapter Four (continued)

		Steady State
T2	Temperature in second effect	
W1	Holdup in first effect	37.5 lb.
W2	Holdup in second effect	26.9 lb.

(b) Alphabetic

$\underline{\underline{A}}$	Coefficient matrix in state space model
$\underline{\underline{B}}$	Coefficient matrix in state space model
$\underline{\underline{C}}$	Coefficient matrix in state space model
$\underline{\underline{D}}$	Coefficient matrix in state space model
$\underline{d}$	Load and disturbance vector
e	Exponential
$\underline{\underline{F}}$	Coefficient matrix in model solution
$\underline{f}$	Vector of functions in model solution
f_{ij}	Elements of matrix $\underline{\underline{F}}$
I	Dynamic interaction index
J	Integral of squared output variable
m	Dimension of manipulated vector
n	Dimension of state vector
p	Dimension of load vector
q	Dimension of output vector
$\underline{\underline{R}}$	Matrix of sensitivity ratios
R_{ij}	Sensitivity ratio (element of $\underline{\underline{R}}$)
s	Laplace variable
t	Time
$\underline{u}$	Manipulated vector

Nomenclature for Chapter Four (continued)

u_i	Element of vectur $\underline{u}$
$\underline{x}$	State vector
$\underline{y}$	Output vector
y_i	Element of vector $\underline{y}$

(c) Greek Letters

$\underline{\underline{\mu}}$	Matrix of interaction measures
μ_{ij}	Interaction measure (element of $\underline{\underline{\mu}}$)
τ	Dummy time variable

(d) Superscript

*	Indicates all control loops closed

(e) Subscripts

i	Position index for matrix elements
j	Position index for matrix elements
k	Position index for matrix elements
_	Indicates a vector
=	Indicates a matrix

(f) Abbreviations

CC	Concentration controller
CR	Concentration recorder
DDC	Direct Digital Control
FC	Flow controller
FR	Flow recorder
LC	Level Controller

Nomenclature for Chapter Four (continued)

PC Pressure controller

NOMENCLATURE FOR CHAPTER FIVE

(a) Process Variables — Steady State

			Steady State
$\underline{x}$		Five element state vector	
	W1	Holdup in the first effect	30 lb.
	C1	Concentration in the first effect	4.85% glycol
	H1	First effect solution enthalpy	194 Btu/lb.
	W2	Holdup in the second effect	35 lb.
	C2	Concentration in the second effect	9.64% glycol
$\underline{u}$		Three element control vector	
	S	Steam flowrate to the first effect	2.0 lb./min.
	B1	First effect bottoms flowrate	3.3 lb./min.
	B2	Second effect bottoms flowrate	1.66 lb/min.
$\underline{d}$		Three element load vector	
	F	Feed flowrate	5.0 lb./min.
	CF	Feed concentration	3.2% glycol
	HF	Feed enthalpy	162 Btu/lb.

Other Process Variables

O1	Overheads from first effect
O2	Overheads from second effect
P1	Pressure in first effect
P2	Pressure in second effect
TF	Temperature of feed
T1	Temperature in first effect
T2	Temperature in second effect

Nomenclature for Chapter Five (continued)

(b) Alphabetic

$\underline{\underline{A}}$	State equation coefficient matrix
$\underline{\underline{B}}$	State equation coefficient matrix
$\underline{\underline{C}}$	Output equation coefficient matrix
$\underline{d}$	Load vector
$\underline{\underline{D}}$	State equation coefficient matrix
i	Recursive relation counter
J	Criterion
k	Time interval counter
$\underline{\underline{K}}$	Control matrices
$\underline{\underline{M}}$	Recursive matrix
n	Time interval counter
N	Final time interval
$\underline{\underline{N}}$	Recursive matrix
$\underline{\underline{O}}$	Recursive matrix
$\underline{\underline{P}}$	Recursive matrix
$\underline{\underline{Q}}$	State weighting matrix
$\underline{\underline{R}}$	Control weighting matrix
$\underline{\underline{S}}$	Final state weighting matrix
t	Time
$\underline{\underline{T}}$	Closed loop state equation coefficient matrix
$\underline{u}$	Control vector
$\underline{x}$	State vector
$\underline{y}$	Output vector

Nomenclature for Chapter Five (continued)

(c) Greek Alphabetic

β	Time weighting parameter
Δt	Control interval

(d) Subscripts

C	Continuous time
calc	Calculated by model
d	Desired value
est	Estimated by filter
FB	Feedback
FF	Feedforward
meas	Measured from process
N	Normalized
SP	Setpoint

(e) Superscripts

T	Matrix transpose
-1	Matrix inverse

(f) Abbreviations

CC	Concentration controller
CR	Concentration recorder
DDC	Direct Digital Control
FC	Flow controller
FR	Flow recorder
ISE	Integral of Squared Error criterion
ISTSE	Integral of Squared Time and Squared Error

Nomenclature for Chapter Five (continued)

ITSE	Integral of Time and Squared Error
LC	Level controller
PC	Pressure controller
SS	Steady State

NOMENCLATURE FOR CHAPTER SIX

(a) Process Variables

State Vector		Steady State
W1	First effect holdup	30 lb.
C1	First effect concentration	4.85 percent
H1	First effect enthalpy	194 Btu./lb.
W2	Second effect holdup	35 lb.
C2	Second effect concentration	9.64 percent
Control Vector		
S	Steam	1.9 lb./min.
B1	First effect bottoms	3.3 lb/min.
B2	Second effect bottoms	1.66 lb./min.
Load Vector		
F	Feed flowrate	5.0 lb./min.
CF	Feed concentration	3.2 percent
HF	Feed enthalpy	162 Btu./lb.
Other Process Variables		
O1	Overheads from first effect	
O2	Overheads from second effect	
P1	Pressure in first effect	
P2	Pressure in second effect	
TF	Temperature of feed	
T1	Temperature in first effect	
T2	Temperature in second effect	

Nomenclature for Chapter Six (continued)

(b) Alphabetic

$\underline{\underline{A}}$	State equation coefficient matrix
$\underline{\underline{A}}_i$	Partition of $\underline{\underline{A}}$
$\underline{\underline{A}}^*$	Intermediate matrix
a_i	Element of $\underline{\underline{A}}$
$\underline{\underline{B}}$	State equation coefficient matrix
$\underline{\underline{B}}_i$	Partition of $\underline{\underline{B}}$
b_i	Element of $\underline{\underline{B}}$
$\underline{\underline{C}}$	Output equation coefficient matrix
$\underline{\underline{D}}$	State equation coefficient matrix
$\underline{d}$	Load vector
d_i	Element of $\underline{\underline{D}}$
$\underline{\underline{I}}$	Unit matrix
J	Criterion
$\underline{\underline{K}}$	Control matrix
k_i	Element of $\underline{\underline{K}}$
m	Dimension of control vector
N	Indicator of final time period
n	Dimension of state vector
$\underline{\underline{O}}$	Recursive matrix
$\underline{\underline{P}}$	Recursive matrix
p	Dimension of load vector
$\underline{\underline{Q}}$	State weighting matrix
q	Dimension of output vector
$\underline{\underline{R}}$	Control weighting matrix

Nomenclature for Chapter Six (continued)

r	Number of feedforward actions
$\underline{\underline{S}}$	Final state weighting matrix
s	Laplace transform variable
$\underline{\underline{T}}$	Closed loop system matrix
t	Time
$\underline{u}$	Control vector
$\underline{u}_i$	Partition of $\underline{u}$
u	Control variable
$\underline{x}$	State vector
$\underline{x}_i$	Partition of $\underline{x}$
x_i	Element of $\underline{x}$
$\underline{y}$	Output vector

(c) Subscripts

d	Desired value
e	Equilibrium value
FF	Feedforward
FB	Feedback
i	Iteration counter; vector/matrix element; or partition
n	Time interval counter
s	Steady state

(d) Greek

Δ	Interval

Nomenclature for Chapter Six (continued)

(e) Superscripts

N-i	Iteration counter
T	Matrix or vector transpose
-1	Matrix inverse

(f) Abbreviations

CC	Concentration controller
CPU	Central processing unit
CR	Concentration recorder
DDC	Direct digital control
FC	Flow controller
FR	Flow recorder
LC	Level controller
PC	Pressure controller

NOMENCLATURE FOR CHAPTER SEVEN

(a) Process Variables

			Steady State
$\underline{x}$	five element state vector		
	W1	Holdup in the first effect	30 lb.
	C1	Concentration in the first effect	4.85% glycol
	H1	First effect solution enthalpy	194 Btu./lb.
	W2	Holdup in the second effect	35 lb.
	C2	Concentration in the second effect	9.64% glycol
$\underline{u}$	three element control vector		
	S	Steam flowrate to the first effect	1.9 lb./min.
	B1	First effect bottoms flowrate	3.3 lb./min.
	B2	Second effect bottoms flowrate	1.66 lb./min.
$\underline{d}$	three element load vector		
	F	Feed flowrate	5.0 lb./min.
	CF	Feed Concentration	3.2% glycol
	HF	Feed enthalpy	162 Btu./lb.

Other Process Variables

O1 Overheads from first effect

O2 Overheads from second effect

P1 Pressure in first effect

P2 Pressure in separator

TF Temperature of feed

T1 Temperature in first effect

T2 Temperature in second effect

Nomenclature for Chapter Seven (continued)

(b) Alphabetic

$\underline{\underline{A}}$	Coefficient matrix in state space model
$\underline{a}_i$	Row of matrix $\underline{\underline{A}}$
$\underline{\underline{B}}$	Coefficient matrix in state space model
$\underline{b}_i$	Row of matrix $\underline{\underline{B}}$
$\underline{\underline{C}}$	Coefficient matrix in output equation
c_i	A constant
$\underline{\underline{D}}$	Coefficient matrix in state space model
$\underline{d}$	Load vector
$\underline{d}_i$	Row of matrix $\underline{\underline{D}}$
$\underline{\underline{I}}$	Unit matrix
J	Performance index
$\underline{\underline{K}}$	Control matrix
k	Integer time counter
m	Dimension of control vector
N	Final value of time counter k
n	Dimension of state vector
$\underline{\underline{P}}_i$	Intermediate matrix in recursive relations
p	Dimension of load vector
$\underline{\underline{Q}}$	State weighting matrix
q	Dimension of "integral" state vector
$\underline{\underline{R}}$	Control weighting matrix
$\underline{\underline{S}}$	Final state weighting matrix
$\underline{\underline{S}}_i'$	Intermediate matrix in recursive relations
t	Time

Nomenclature for Chapter Seven (continued)

$\underline{u}$ Control vector

$\underline{x}$ State vector

$\underline{x}'$ Augmented state vector

x_i Element of vector $\underline{x}$

$\underline{y}$ Output vector

$\underline{z}$ "Integral" state vector

(c) Greek Letters

$\underline{\underline{\alpha}}$ Coefficient matrix in state estimation equation

β Time weighting factor

Δ Prefix indicating an increment (eg Δt control interval)

(d) Subscripts

c Continuous time

FB Feedback

I Integral

i Indicates incremental time counter or a matrix row

= Double underline specifies a matrix

_ Single underline specifies a vector

(e) Abbreviations

CC Concentration controller

CR Concentration recorder

DDC Direct Digital Control

FC Flow controller

FR Flow recorder

Nomenclature for Chapter Seven (continued)

LC	Level controller
PC	Pressure controller
P+I	Proportional-plus-Integral

NOMENCLATURE FOR CHAPTER EIGHT

(a) Process Variables

State Vector		Steady State
W1	First effect holdup	30 lb.
C1	First effect concentration	4.85 percent
H1	First effect enthalpy	194 Btu./lb.
W2	Second effect holdup	35 lb.
C2	Second effect concentration	9.64 percent
Control Vector		
S	Steam flowrate	1.9 lb./min.
B1	First effect bottoms	3.3 lb./min.
B2	Second effect bottoms	1.66 lb./min.
Load Vector		
F	Feed flowrate	5.0 lb./min.
CF	Feed concentration	3.2 percent
HF	Feed enthalpy	162 Btu./lb.
Other Variables		
O1	Overheads from first effect	
O2	Overheads from second effect	
P1	Pressure in first effect	
P2	Pressure in second effect	
TF	Temperature of feed	
T1	Temperature in first effect	
T2	Temperature in second effect	

Nomenclature for Chapter Eight (continued)

(b) Alphabetic

$\underline{\underline{A}}$	State coefficient matrix
$\underline{\underline{B}}$	Control coefficient matrix
$\underline{\underline{C}}$	Output coefficient matrix
$\underline{\underline{D}}$	Load coefficient matrix
$\underline{d}$	Load vector
$\underline{\underline{E}}$	Setpoint steady state matrix
$\underline{\underline{G}}$	Model gain matrix
$\underline{\underline{H}}$	Model dynamic matrix
i	Control interval counter
J	Control criterion
$\underline{\underline{K}}$	Control matrix
$\underline{\underline{M}}$	Recursive matrix
m	Dimension of control vector
$\underline{\underline{N}}$	Recursive matrix
N	Number of control intervals in recursive calculations
n	Dimension of state vector
$\underline{\underline{O}}$	Recursive matrix
$\underline{\underline{P}}$	Recursive matrix
p	Dimension of load vector
$\underline{\underline{Q}}$	State weighting matrix
q	Dimension of output vector
$\underline{\underline{R}}$	Control weighting matrix
$\underline{\underline{T}}$	Closed loop state coefficient matrix
$\underline{u}$	Control vector

Nomenclature for Chapter Eight (continued)

$\underline{x}$ State vector

$\underline{y}$ Output vector

(c) Greek

τ Time constant

(d) Subscripts

c Continuous

FB Feedback

FF Feedforward

i Control interval counter

M Model

m Model

SP Setpoint

sp Setpoint

ss Steady state

_ Vector

= Matrix

(e) Superscripts

i Recursive iteration counter

T Matrix/vector transpose

· Derivative with time

* Unpartitioned control matrices

NOMENCLATURE FOR CHAPTER NINE

(a) Alphabetic

$\underline{d}$	Load vector
$\underline{\underline{K}}$	Control matrix
t	Time
$\underline{u}$	Control vector
$\underline{x}$	State vector
$\underline{y}$	Output vector

(b) Subscripts

est	Estimated
F	Filter
FB	Feedback
FF	Feedforward
I	Integral
M	Model
m	Model
meas	Measurement
model	Model
SP	Setpoint
sp	Setpoint

(c) Superscript

i	Vector element

Nomenclature for Chapter Nine (continued)

(d) Abbreviations

DACS	Data Acquisition, Control, and Simulation
DDC	Direct Digital Control
POC	Process Operator's Console
PVT	Process Variable Table

NOMENCLATURE FOR CHAPTER TEN

(a) Process Variables

State Vector		Steady State
x_1	Top product composition	0.793
x_2	Top plate composition	0.608
x_3	Feed plate composition	0.434
x_4	Bottom plate composition	0.293
x_5	Bottom product composition	0.187
Control Vector		
R	Reflux ratio	3.79
Q_s	Reboiler heat flow	166800 Btu./hr.
Load Vector		
F	Feed flowrate	125.4 lb./hr.
x_F	Feed composition	0.570
T_F	Feed temperature	83 deg. F.
Other Process Variables		
B	Bottom product flowrate	
D	Top product flowrate	
E	Intermediate variable	
Q_c	Condenser heat flow	
y_i	Vapour composition at position i	

Nomenclature for Chapter Ten (continued)

(b) Alphabetic

$\underline{\underline{A}}$	State equation coefficient matrix
$\underline{\underline{B}}$	State equation coefficient matrix
$\underline{\underline{C}}$	Output equation coefficient matrix
$\underline{\underline{D}}$	State equation coefficient matrix
$\underline{d}$	Load vector
$\underline{\underline{F}}$	Noninteracting control law matrix
$\underline{\underline{G}}$	Noninteracting control law matrix
J	Criterion
$\underline{\underline{K}}$	Control matrix
K	Proportional controller constant
N	Number of time intervals for summation
$\underline{\underline{Q}}$	State weighting matrix
$\underline{\underline{R}}$	Control weighting matrix
s	Laplace transform variable
t	Time
$\underline{u}$	Control vector
$\underline{v}$	Noninteracting control vector
$\underline{x}$	State vector
$\underline{y}$	Output vector

(c) Greek Alphabetic

Δ	Interval
λ	Gain constant
σ	Characteristic equation constant

Nomenclature for Chapter Ten (continued)

τ Reset time

(d) Subscripts

c Continuous time

FB Feedback

FF Feedforward

I Integral

i Element counter, time interval counter

j Element counter

n Normalized

- Vector

= Matrix

(e) Superscripts

T Matrix transpose

-1 Matrix inverse

(f) Abbreviations

ISE Integral of Squared Error

P+I Proportional-plus-Integral

NOMENCLATURE FOR COMPUTER GRAPHS

(a) Symbols

Δ ∇	Y - axes: steady state values
$\wedge$ $\vee$	X - axes: times of disturbances or changes

(b) Run Codes

Data Source:

EXP	Experimental evaporator data
10NL	Tenth-order nonlinear model
5NL	Fifth-order nonlinear model
5L	Fifth-order linear model
5LDC	Fifth-order linear decoupled model
3LR	Third-order linear reduced model
3LD	Third-order linear derived model
2LD	Second-order linear derived model

Load/disturbance:

+,-	Positive or negative step (no sign means both)
x%	Step size as percentage of steady state
XX	Process variable disturbed

Control mode:

OL	Open loop
DDC	Closed loop multiloop DDC
INF	Inferential control of C2
FBS	Stepwise multivariable feedback
FB	Multivariable feedback

Nomenclature for Computer Graphs (continued)

FF-QI	Feedforward and feedback - quadratic index
FF-ZO	Feedforward and feedback - zero offsets
FF-MO	Feedforward and feedback - minimized offsets
SP	Setpoint control
P+I-O	Proportional - plus - integral, loose, weights = 1
P+I-1	Proportional - plus - integral, tight, weights = 10
MF1	Model following (1 minute model)
MF5	Model following (5 minute model)
ML	Multiloop control
MLDC	Decoupled multiloop control

State weighting matrix:

Code	Diagonal elements
Q1	10, 1, 1, 10, 100
Q2	1, 1, 1, 1, 1
Q3	1, 1, 1, 1, 10000
Q4	1, 1, 1, 1, 100
Q5	100, 1, 1, 100, 100
QD1	1, 1, 1, 1, 1
QD2	100, 1, 1, 1, 100

Multiloop tuning

T1	Averaging level control
T2	Tight level control

Nomenclature for Computer Graphs (continued)

Control weighting matrix:

Code	Diagonal elements
R1	0, 0, 0
R2	.5, .5, .5
R3	.02, 0, 0
R4	.2, 0, 0
RD1	0, 0
RD2	.1, .1

Control interval:

Code	Seconds
D1	64
D2	16
D3	448
D4	256

State estimation matrix:

Code	Diagonal elements
A1	.9, 0, .9, .9, .5
A2	.9, 0, .9, .9, 1.

Run number:

Last alphanumeric code of the string for experimental data

"Standard conditions" are Q1, R1, and D1. These conditions were used for all simulation runs except where explicitly noted in the tables in the text.

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CHAPTER THREE

APPENDIX A

SIX EFFECT MODEL AND CONFIGURATION

A six effect evaporation process modelled by Nisenfeld and Hoyle [2] as two stirred tanks is used as an illustration of model building from general unit model relations and configuration relations.

Figure 2 shows a process flowsheet with unit numbers and stream letters.

A. Units 1 to 6 - Evaporator Effects

The general model relations for a single effect are as follows with $i = 1,2,3,4,5,6$ for each of the process effects.

$$V_s^i \frac{d\rho_s^i}{dt} = S^i - S_c^i \qquad \text{(A-1)}$$

$$V_s^i \frac{d}{dt} (\rho_s^i H_s^i) = S^i H_{si} - S_c^i h_c^i - Q_s^i - L_s^i \qquad \text{(A-2)}$$

$$Q_s^i = U_v^i A^i (T_c^i - T_w^i) = S_c^i (\lambda_s^i + \delta_s^i) \qquad \text{(A-3)}$$

$$W_w^i C_{pw}^i \frac{dT_w^i}{dt} = U_v^i A^i (T_c^i - T_w^i) - U_s^i A^i (T_w^i - T^i) \qquad \text{(A-4)}$$

$$\frac{dW^i}{dt} = F^i - B^i - O^i \qquad \text{(A-5)}$$

$$\frac{d}{dt} (W^i C^i) = F^i C_F^i - B^i C^i \qquad \text{(A-6)}$$

Chapter Three, Appendix A (continued)

$$\frac{d}{dt}(W^i h^i) = F^i h_F^i - B^i h^i - O^i H_v^i + Q^i - L^i + \phi^i \quad \text{(A-7)}$$

$$Q^i = U_s^i A^i (T_w^i - T^i)$$

Other data which are required are listed below.

(a) ρ_s^i, H_s^i, h_c^i, λ_s^i, δ_s^i as functions of T_c^i and P_c^i.

(b) h^i, ϕ^i, H_v^i as functions of T^i and C^i or P^i.

(c) relations for U_v^i and U_s^i and L_s^i and L^i.

(d) values for V_s^i, A^i, W_w^i, C_{pw}^i.

B. <u>Unit 7 - Condenser</u>

The general model for the condenser can be listed as follows:

$$V_c^7 \frac{d\rho_c^7}{dt} = O_v^7 - O_c^7 \quad \text{(A-9)}$$

$$V_c^7 \frac{d}{dt} \rho_c^7 H_c^7 = O_v^7 H_v^7 - O_c^7 h_c^7 - Q_c^7 - L_c^7 \quad \text{(A-10)}$$

$$Q_c^7 = U_{vc}^7 A_c^7 (T_{cc}^7 - T_{wc}^7) \quad \text{(A-11)}$$

$$W_{wc}^7 C_{pc}^7 \frac{dT_{wc}^7}{dt} = U_{vc}^7 A_c^7 (T_{cc}^7 - T_{wc}^7) - U_{sc}^7 A_c^7 (T_{wc}^7 - T_m^7) \quad \text{(A-12)}$$

$$T_m^7 = 0.5 (T_{co}^7 + T_{ci}^7) \quad \text{(A-13)}$$

$$Q_c^7 = F_{cw}^7 \; C_{pcw}^7 \; (T_{co}^7 - T_{ci}^7) \tag{A-14}$$

Other data required are listed below.

(a) ρ_c^7, H_c^7, h_c^7 as functions of T_c^7 and P_c^7 .

(b) relations for U_{vc}^7, U_{sc}^7, and L_c^7 .

(c) values for V_c^7, A_c^7, C_{pc}^7, C_{pcw}^7 .

C. Unit 8 - Soap Tank

General model relations for a soap tank as illustrated in Figure A-1 may be written as follows.

$$\frac{dW^8}{dt} = F^8 - B^8 - B_s^8 \tag{A-15}$$

$$\frac{d}{dt}(W^8 \; h^8) = F^8 \; h_F^8 - B^8 \; h^8 - B_s^8 \; h_s^8 - L^8 \tag{A-16}$$

D. Configuration Relations

Configuration statements are given in Table A-1 relating enthalpy, mass flow, and concentration for the solution process streams and enthalpy and mass flow for the vapour process streams.

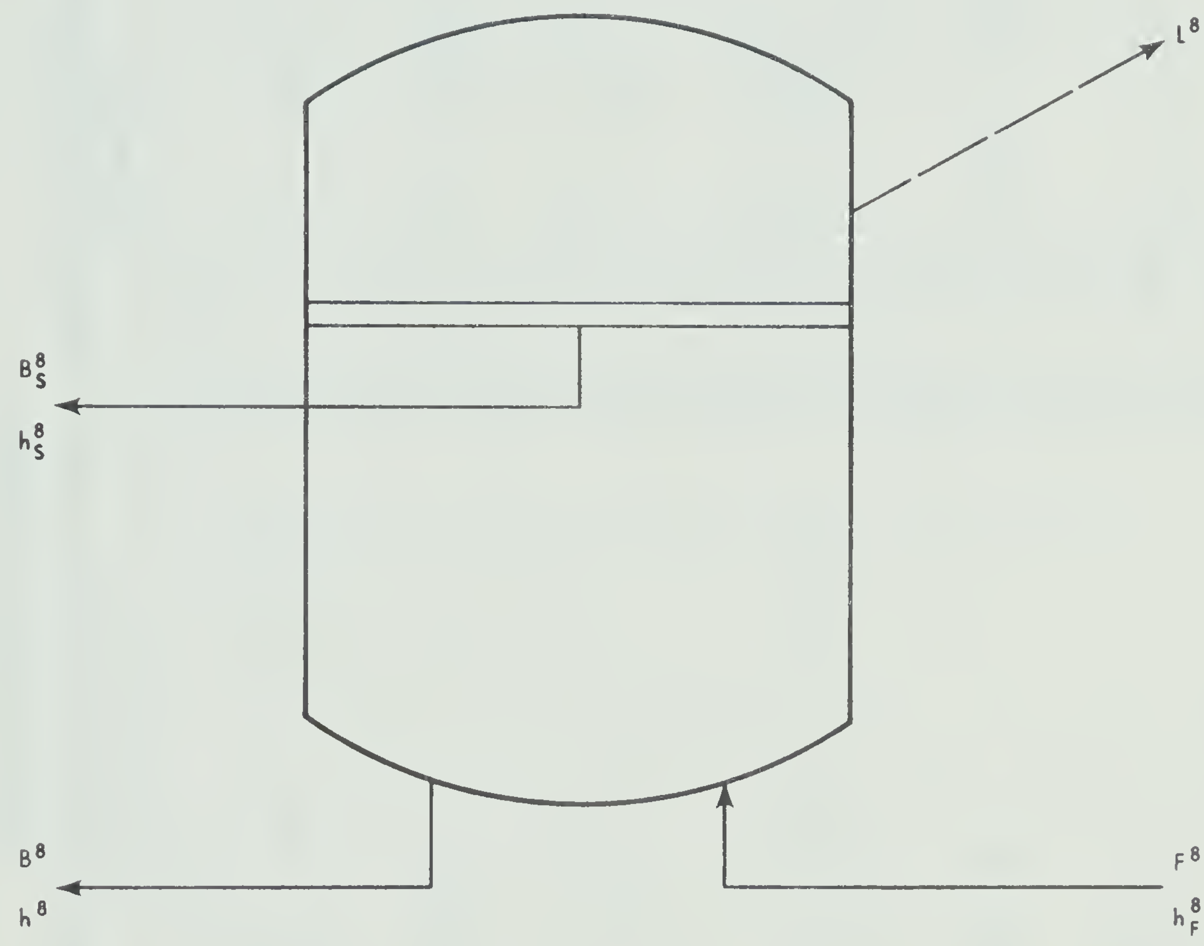

FIGURE A-1. SCHEMATIC OF SOAP TANK

TABLE A-1

CONFIGURATION RELATIONS

Stream	Mass Flow	Concentration	Enthalpy	
A	$B_{PR} = (1 - \alpha)B^1$	$C_{PR} = C^1$	$h_{PR} = h^1$	
B	$F^1 = B^2 + \alpha B^1$	$C_F^1 = (B^2 C^2 + \alpha B^1 C^1)/F^1$	$h_F^1 = (B^2 h^2 + \alpha B^1 h^1 - L_B)/F^1$	
C	$S^2 = O^1$	-	$H_{si}^2 = H_v^1$	*
D	$F^2 = B^3$	$C_F^2 = C^3$	$h_F^2 = (B^3 h^3 - L_D)/F^2$	
E	$S^3 = O^2$	-	$H_{si}^3 = H_v^2$	*
F	$F^3 = B^8$	$C_F^3 = C^4$	$h_F^3 = (B^8 h^8 - L_F)F^3$	
G	$S^4 = O^3$	-	$H_{si}^4 = H_v^3$	*
H	$F^8 = B^4$	-	$h_F^8 = (B^4 h^4 - L_H)/F^8$	
I	$F^4 = B^5 + B^6$	$C_F^4 = (B^5 C^5 + B^6 C^6)/F^4$	$h_F^4 = (B^5 h^5 + B^6 h^6 - L_I)/F^4$	
J	$S^5 = O^4$	-	$H_{si}^5 = H_v^4$	*
K	$F^5 = (1 - \beta)F$	$C_F^5 = C_F$	$h_F^5 = ((1-\beta)\ Fh_F - L_K)/F^5$	
L	$S^6 = O^5$	-	$H_{si}^6 = H_v^5$	*
M	$F^6 = \beta F$	$C_F^6 = C_F$	$h_F^6 = (\beta F h_F - L_M)/F^6$	
N	$O_v^7 = O^6$	-	$H_v^7 = H_v^6$	*

* The volume of vapour spaces and heat losses are added to the appropriate terms in the relations for steam chests and condenser vapour space.

CHAPTER THREE

APPENDIX B

COMPLETE PILOT PLANT MODEL

A model of the pilot plant double effect evaporator illustrated in Figure 3 is derived using the general model building approach.

The model equations will include those for the two effects and the condenser as well as the necessary configuration statements.

Major assumptions involved in the derivation are as follows:

(a) All vapours are saturated so that the following property relations are applicable.

$$\rho_{vap} = \alpha_1 T_{vap} - \alpha_2 \tag{B-1}$$

$$H_{vap} = 0.4 T_{vap} + 1066.0 \tag{B-2}$$

$$h_{cond} = T_{vap} - 32.0 \tag{B-3}$$

$$\lambda_{vap} = 1098.0 - 0.6 T_{vap} \tag{B-4}$$

(b) There is no boiling point rise for the Triethylene Glycol solution.

$$\delta^i = 0, \quad T^i_{BP} = 0, \quad \text{i.e.} \quad T^i_c = T^i_s \tag{B-5}$$

(c) There are no heat of solution effects

$$\phi^i = 0$$

(d) Heat transfer coefficients are constants. Solution property relations are as follows:

$$h_{soln} = T_{soln} (1 - .16\, C_{soln}) - 32.1 \tag{B-6}$$

(e) Heat losses are assumed constant.

The general equations for an evaporator effect are as follows:

$$V_s^i \frac{d\rho_s^i}{dt} = S^i - S_c^i \tag{B-7}$$

$$V_s^i \frac{d}{dt} (\rho_s^i H_s^i) = S^i H_{si}^i - S_c^i h_c^i - Q_s^i - L_s^i \tag{B-8}$$

$$Q_s^i = U_v^i A^i (T_c^i - T_w^i) = S_c^i (\lambda_s^i + \delta_s^i) \tag{B-9}$$

$$W_w^i C_{pw}^i \frac{dT_w^i}{dt} = U_v^i A^i (T_c^i - T_w^i) - U_s^i A^i (T_w^i - T^i) \tag{B-10}$$

$$\frac{dW^i}{dt} = F^i - B^i - O^i \tag{B-11}$$

$$\frac{d}{dt} (W^i C^i) = F^i C_F^i - B^i C^i \tag{B-12}$$

$$\frac{d}{dt} (W^i h^i) = F^i h_F^i - B^i h^i - O^i H_v^i + Q^i - L^i + \phi^i \tag{B-13}$$

$$Q^i = U_s^i A^i (T_w^i - T^i) \tag{B-14}$$

A. First Effect Equations

The condensate can be expressed from equation (B-9) as:

$$S_c^1 = U_v^1 A^1 (T_s^1 - T_w^1)/\lambda_s^1 \tag{B-15}$$

where U_v^1 and A^1 are constant parameters and λ_s^1 is defined by equation (B-4).

Eliminating the derivative of ρ_s^1 from equations (B-7) and (B-8) gives:

$$V_s^i \rho_s^i \frac{dH_s^1}{dt} = V_s^1 H_s^1 (S_c^1 - S^1) + S^1 H_{si}^1 - S_c^1 h_c^1$$

$$- U_v^1 A^1 (T_s^1 - T_w^1) - L_s^1 \tag{B-16}$$

where H_s^1 and ρ_s^1 and h_c^1 are related to T_s^1 by equations (B-2), (B-1), and (B-3), respectively and V_s^1, H_{si}^1 and L_s^1 are constants.

The tube wall temperature is defined by the differential equation (B-10),

$$W_w^1 C_{pw}^1 \frac{dT_w^1}{dt} = U_v^1 A^1 (T_s^1 - T_w^1) - U_s^1 A^1 (T_w^1 - T^1) \tag{B-17}$$

and the solution balances follow from equations (B-11) to (B-14).

$$\frac{dW^1}{dt} = F^1 - B^1 - O^1 \tag{B-18}$$

$$\frac{d}{dt} (W^1 C^1) = F^1 C_F^1 - B^1 C^1 \tag{B-19}$$

Chapter Three, Appendix B (continued)

$$\frac{d}{dt}(W^1 h^1) = F^1 h_F^1 - B^1 h^1 - O^1 H_v^1$$

$$+ U_s^1 A^1 (T_w^1 - T^1) - L^1 \qquad \text{(B-20)}$$

where h^1 and H_v^1 are related to T^1 by equations (B-6) and (B-2) respectively and U_s^1 and L^1 are constants.

B. Second Effect Equations

The vapour space of the first effect is combined with the steam chest of the second effect to give the equations:

$$(V^1 + V_s^2)\frac{d\rho_s^2}{dt} = S^2 - S_c^2 \qquad \text{(B-21)}$$

$$(V^1 + V_s^2)\frac{d}{dt}(\rho_s^2 H_s^2) = S^2 H_{si}^2 - S_c^2 h_c^2 - U_v^2 A^2 (T_s^2 - T_w^2)$$

$$- L_s^2 - L_v^1 \qquad \text{(B-22)}$$

where V^1, V_s^2, U_v^2, A^2, L_s^2, and L^1 are constants and ρ_s^2, H_s^2, and h_c^2 are defined by T_s^2 and property relations.

The other four relations for the second effect are analagous to equations (B-17) to (B-20) for the first effect.

C. Condenser Equations

The vapour spaces of the second effect and condenser are combined to give the equations:

$$(V^2 + V_c^3) \frac{d\rho_c^3}{dt} = O_v^3 - O_c^3 \tag{B-23}$$

$$(V^2 + V_c^3) \frac{d}{dt} (\rho_c^3 H_c^3) = O_v^3 H_v^3 - O_c^3 h_c^3 - \left(U_{vc}^3 A_c^3 (T_c^3 - T_{wc}^3)\right) - L_c^3 - L_v^2 \tag{B-24}$$

where V^2, V_c^3, U_{vc}^3, A_c^3, L_c^3, and L_v^2 are constant and ρ_c^3, H_c^3, and h_c^3 are related to T_c^3 by the vapour property relations.

The remaining condenser relations are as follows:

$$W_{wc}^3 C_{pc}^3 \frac{dT_{wc}^3}{dt} = U_{vc}^3 A_c^3 (T_c^3 - T_{wc}^3) - U_{sc}^3 A_c^3 (T_{wc}^3 - T_m^3) \tag{B-25}$$

$$T_m^3 = 0.5 (T_{co}^3 + T_{ci}^3) \tag{B-26}$$

$$F_{cw}^3 C_{pcw}^3 (T_{co}^3 - T_{ci}^3) = U_{sc}^3 A_c^3 (T_{wc}^3 - T_m^3) \tag{B-27}$$

where W_{wc}^3, C_{pc}^3, U_{vc}^3, A_c^3, U_{sc}^3, and C_{pcw}^3 are constant.

D. Configuration Equations

(a) First effect vapour.

$S^2 = O^1$, $\rho_s^2 = \rho_v^1$, $H_{si}^2 = H_v^1 = H_s^2$

(b) First effect bottoms.

$F^2 = B^1$, $C_F^2 = C^1$, $h_F^2 = h^1$

(c) Second effect vapour

$O_v^3 = O^2$, $\rho_c^3 = \rho_v^2$, $H_v^3 = H_c^3 = H_v^2$

Chapter Three, Appendix B, (continued)

E. Model Equations

Due to the assumption of saturated vapours there are two major simplifications that can be made.

Considering the first effect vapour space and second effect steam chest, the two variables ρ_s^2 and H_s^2 can be related to h^1 through their dependence on the common temperature $T^1 = T_s^2$. Making use of this fact and substituting equation (B-21) into (B-22), these equations will be algebraic except for the derivative of h^1 which can be removed using equation (B-20). These equations can then be used to evaluate O^1 and S_c^2. The relation for O^1 is

$$O^1 \left[1 - \left(\frac{.4\,\rho_s^2}{H_v^1 - h_c^2} + \alpha_1\right) \frac{(V^1+V_s^2)(.16C^1T^1 - H_v^1 + h^1)}{W^1(1-.16C^1)}\right] = \frac{L_s^2 + L_v^1 + U_v^2A^2(T^1 - T_w^2)}{H_v^1 - h_c^2}$$

$$+ \left[\left(\frac{.4\,\rho_s^2}{H_v^1 - h_c^2} + \alpha_1\right)\left(\frac{V^1+V_s^2}{W^1(1-.16C^1)}\right)(.16T^1F^1(C_F^1 - C^1) + F^1(h_F^1 - h^1) + U_s^1A^1(T_w^1 - T^1) - L^1)\right] \tag{B-28}$$

A similar expression can be derived for O^2 using equations (B-23), (B-24), and the second effect equivalent of (B-20).

This leaves ten differential equations for the variables H_s^1, T_w^1, W^1, C^1, h^1, T_w^2, W^2, C^2, h^2, and T_{wc}^3 (the state variables) with the independent variables F^1, C_F^1, h_F^1, S^1, H_{si}^1, B^1, B^2, F_{cw}^3, and T_{ci}^3.

CHAPTER THREE

APPENDIX C

LINEARIZATION

Nonlinear models can be represented in state space form as follows.

$$\dot{\underline{x}} = \underline{F}\,(\underline{x},\ \underline{u}) \tag{C-1}$$

where $\underline{x}$ is an n dimensional state vector and $\underline{u}$ an m dimensional forcing function vector.

It is required to linearize this model into the form,

$$\dot{\underline{x}} = \underline{\underline{A}}\ \underline{x} + \underline{\underline{B}}\ \underline{u} \tag{C-2}$$

where $\underline{\underline{A}}$ and $\underline{\underline{B}}$ are constant coefficient matrices. The linearization is about an operating point $\underline{x}_s$ and $\underline{u}_s$.

There are two procedures for determining the coefficient matrices.

(a) Analytical linearization involves $n\ (n + m)$ partial derivatives which must be fully evaluated algebraically.

$$a_{ij} = \left.\frac{\delta f_i}{\delta x_j}\right|_{\underline{x}_s,\ \underline{u}_s} \tag{C-3}$$

$$b_{ij} = \left.\frac{\delta f_i}{\delta u_j}\right|_{\underline{x}_s,\ \underline{u}_s} \tag{C-4}$$

This approach has been used in the fifth-order nonlinear model [19].

(b) Numerical linearization uses some relationship such as central differences to evaluate the coefficients by numerically differentiating the nonlinear model.

$$a_{ij} = \frac{f_i\big|_{\underline{x}_s + \delta \underline{x}_{sj}, \underline{u}_s} - f_i\big|_{\underline{x}_s - \delta \underline{x}_{sj}, \underline{u}_s}}{2\,\delta \underline{x}_{sj}} \qquad \text{(C-5)}$$

where $\delta \underline{x}_{sj}$ is such that the elements

$$\begin{aligned} \delta x_{sji} &= 0, && i \neq j \\ &= \delta x_{sj}, && i = j \end{aligned}$$

A similar expression exists for b_{ij} .

The accuracy of the coefficients depends on the size of the perturbation $\delta \underline{x}_{sj}$ which can be reduced until the coefficients are approximately constant.

f

The method requires at least $2n\ (n + m)$ evaluations of $\underline{x}$ in the nonlinear model (equation (C-1)).

After linearization all variables were normalized to the normalized perturbation form.

$$x_{ni} = \frac{x_i - x_{si}}{x_{si}} \tag{C-6}$$

and

$$u_{ni} = \frac{u_i - u_{si}}{u_{si}} \tag{C-7}$$

CHAPTER THREE

APPENDIX D

SIMPLIFIED MODELS

A. Third-Order Reduced Model

The reduction begins with the linearized fifth-order model presented in the text as equations (21) to (30).

Some basic assumptions are made.

(a) No heat losses

$$L^1 = L^2 = 0 \tag{D-1}$$

(b) Solution enthalpy has negligible dependence on concentration.

$$\frac{\delta h^2}{\delta C^2} = \frac{\delta h^1}{\delta C^1} = 0 \tag{D-2}$$

Substituting equations (D-1), (D-2), and (26) into equation (30) gives:

$$O^2 \ (H_v^2 - h^2) = O^1 \ (H_v^1 - h_c^2) + B^1 \ (h^1 - h^2) \tag{D-3}$$

Assuming the difference in sensible heat between the effects is small compared to the heat load $(O^1 \ (H_v^1 - h_c^2))$.

$$O^2 = \frac{H_v^1 - h_c^2}{H_v^2 - h^2} O^1 = k^2 \ O^1 \tag{D-4}$$

Assuming first effect temperature is approximately constant implies that

$$\frac{dh^1}{dt} = 0 \qquad \text{(D-5)}$$

It follows from equations (D-5), (21), (25), and (24) that

$$O = F^1 (h_F^1 - h^1) - O^1 (H_v^1 - h^1) + S^1 \lambda_s^1 \qquad \text{(D-6)}$$

and assuming the sensible heat difference between the feed and bottoms is small compared to the heat load results in the following relation.

$$O^1 = \frac{\lambda_s^1}{H_v^1 - h^1} S^1 = k^1 S^1 \qquad \text{(D-7)}$$

Assuming constant holdup in the second effect the mass balance, equation (28), reduces to

$$B^2 = B^1 - O^2 = B^1 - k^2 k^1 S^1 \qquad \text{(D-8)}$$

Making the necessary substitutions into the remaining differential equations, (22), (23), and (29), results in the following equations for the model which can be linearized to the form of equations (34) to (36) in the text.

$$\frac{dW^1}{dt} = F^1 - B^1 - k^1 S^1 \qquad \text{(D-9)}$$

$$W^1 \frac{dC^1}{dt} = F^1 (C_F^1 - C^1) + k^1 S^1 C^1 \qquad \text{(D-10)}$$

Chapter Three, Appendix D (continued)

$$W^2 \frac{dC^2}{dt} = B^1 (C^1 - C^2) + k^2 k^1 S^1 C^2 \qquad \text{(D-11)}$$

B. Third-Order Derived Model

An alternative procedure for deriving a third-order model is to assume directly that second effect holdup is constant and evaporation is proportional to the heating vapours.

$$\text{i.e.} \qquad O^1 = k^1 S^1 \text{ and } O^2 = k^2 O^1 \qquad \text{(D-12)}$$

Hence a first effect mass balance and two solute balances give the following relations.

$$\frac{dW^1}{dt} = F^1 - B^1 - k^1 S^1 \qquad \text{(D-13)}$$

$$\frac{d}{dt} (W^1 C^1) = F^1 C_F^1 - B^1 C^1 \qquad \text{(D-14)}$$

$$W^2 \frac{dC^2}{dt} = B^1 C^1 - (B^1 - k^1 k^2 S^1) C^2 \qquad \text{(D-15)}$$

Expanding the left hand side of equation (D-14) and substituting (D-13) gives the same set of equations as (D-9) to (D-11).

In this case constants k^1 and k^2 are derived from equation (D-13) at steady state and the second effect mass balance,

$$\bar{B}^1 = \bar{B}^2 + k^1 k^2 \bar{S}^1 \qquad \text{(D-16)}$$

$$k^1 = \frac{\bar{F}^1 - \bar{B}^1}{\bar{S}^1} \quad \text{and} \quad k^2 = \frac{\bar{B}^1 - \bar{B}^2}{\bar{F}^1 - \bar{B}^1} \tag{D-17}$$

C. Second-Order Derived Model

Assuming constant holdups in both effects reduces the model to that of two stirred tanks for which the solute balances are as follows.

$$W^1 \frac{dC^1}{dt} = F^1 C^1_F - B^1 C^1 \tag{D-18}$$

$$W^2 \frac{dC^2}{dt} = B^1 C^1 - B^2 C^2 \tag{D-19}$$

It follows from the algebraic mass balances and the assumption of evaporation proportional to heating vapour rate that:-

$$B^1 = F^1 - k^1 S^1 \tag{D-20}$$

$$B^2 = F^1 - k^1 S^1 - k^1 k^2 S^1 \tag{D-21}$$

whence the model equations can be expressed and subsequently linearized.

$$W^1 \frac{dC^1}{dt} = F^1 (C^1_F - C^1) + k^1 S^1 C^1 \tag{D-22}$$

$$W^2 \frac{dC^2}{dt} = (F - k^1 S^1)(C^1 - C^2) + k^1 k^2 S^1 C^2 \tag{D-23}$$

D. Transfer Function Model

Consider the linearized form of the second-order model, equations (D-22) and (D-23), with the terms in F^1 and C_F^1 neglected.

$$\bar{W}^1 \frac{dC^1}{dt} = (k^1 \bar{S}^1 - \bar{F}^1)C^1 + k^1 \bar{C}^1 S^1 \tag{D-24}$$

$$\bar{W}^2 \frac{dC^2}{dt} = \bar{B}^1 C^1 + (k^1 k^2 \bar{S}^1 - \bar{B}^1)C^2$$

$$+ (k^1 k^2 \bar{C}^2 - k^1 (\bar{C}^1 - \bar{C}^2))S^1 \tag{D-25}$$

Taking Laplace transforms of equation (D-24) gives:

$$C^1(s) = \frac{k^1 \bar{C}^1}{\bar{W}^1 s - k^1 \bar{S}^1 + \bar{F}^1} S^1(s) \tag{D-26}$$

Taking Laplace transforms of equation (D-25) gives:

$$C^2(s) = \frac{\bar{B}^1}{\bar{W}^2 s - k^1 k^2 \bar{S}^1 + \bar{B}^1} C^1(s)$$

$$+ \frac{k^1 k^2 \bar{C}^2 - k^1(\bar{C}^1 - \bar{C}^2)}{\bar{W}^2 s - k^1 k^2 \bar{S}^1 + \bar{B}^1} S^1(s) \tag{D-27}$$

It follows from steady state mass balances that

$$\bar{F}^1 = \bar{B}^1 + k^1 \bar{S}^1 \quad \text{and} \quad \bar{B}^1 = \bar{B}^2 + k^1 k^2 \bar{S}^1$$

and eliminating $C^1(s)$ from equations (D-26) and (D-27) gives

the following expression.

$$\frac{C^2(s)}{S^1(s)} = \frac{k^1 \bar{B}^1 \bar{C}^1}{(\bar{W}^1 s + \bar{B}^1)(\bar{W}^2 s + \bar{B}^2)} + \frac{k^1 k^2 \bar{C}^2 - k^1(\bar{C}^1 - \bar{C}^2)}{\bar{W}^2 s + \bar{B}^2}$$

Substituting the steady state values listed in Table 1 of the text gives the expression

$$\frac{C^2(s)}{S^1(s)} = 10.2 \frac{6.7s + 1}{(9.1s + 1)(21.1s + 1)}$$

$$\simeq \frac{10.2}{\tau s + 1}$$

cancelling the pole and zero.

CHAPTER FIVE

APPENDIX A

CONTROL LAW DERIVATION

Consider the process as described by the discrete state equation,

$$\underline{x}_{n+1} = \underline{\underline{A}}\,\underline{x}_n + \underline{\underline{B}}\,\underline{u}_n + \underline{\underline{D}}\,\underline{d}_n \tag{A-1}$$

and the output equation,

$$\underline{y}_n = \underline{\underline{C}}\,\underline{x}_n \tag{A-2}$$

where $\underline{y}$ is a subset of $\underline{x}$ of dimension less than or equal to the dimension of $\underline{u}$.

The control criterion is expressed as follows

$$\begin{aligned} J = {} & \beta^N (\underline{x}_N - \underline{\underline{C}}^T \underline{y}_d)^T \underline{\underline{S}} (\underline{x}_N - \underline{\underline{C}}^T \underline{y}_d) \\ & + \sum_{k=1}^{N} \beta^k \left[(\underline{x}_k - \underline{\underline{C}}^T \underline{y}_d)^T \underline{\underline{Q}} (\underline{x}_k - \underline{\underline{C}}^T \underline{y}_d) + \underline{u}_{k-1}^T \underline{\underline{R}}\, \underline{u}_{k-1} \right] \end{aligned} \tag{A-3}$$

The system is divided into subsystems on a time basis with each control interval being separately optimized in a "backwards" sense.

Consider the Nth control interval and optimize with respect to $\underline{u}_{N-1}$

$$\frac{\partial J}{\partial \underline{u}_{N-1}} = \frac{\partial}{\partial \underline{u}_{N-1}} \left(\beta^N (\underline{x}_N - \underline{\underline{C}}^T \underline{y}_d)^T (\underline{\underline{Q}} + \underline{\underline{S}}) (\underline{x}_N - \underline{\underline{C}}^T \underline{y}_d) + \beta^N \underline{u}_{N-1}^T \underline{\underline{R}}\, \underline{u}_{N-1} \right) \tag{A-4}$$

neglecting the terms of the criterion independent of $\underline{u}_{N-1}$. Setting the partial differential to zero,

$$\frac{\partial \underline{x}_N^T}{\partial \underline{u}_{N-1}} (\underline{\underline{Q}} + \underline{\underline{S}})(\underline{x}_N - \underline{\underline{C}}^T \underline{y}_d) + \underline{\underline{R}}\, \underline{u}_{N-1} = 0 \qquad \text{(A-5)}$$

Substituting for the partial differential and $\underline{x}_N$ from the state equation (A-1) an equation of the following form results,

$$\underline{\underline{C}}_x^{N-1} \underline{x}_{N-1} + \underline{\underline{C}}_u^{N-1} \underline{u}_{N-1} + \underline{\underline{C}}_d^{N-1} \underline{d}_{N-1} + \underline{\underline{C}}_y^{N-1} \underline{y}_d = 0 \qquad \text{(A-6)}$$

where the coefficients are

$$\underline{\underline{C}}_x^{N-1} = \underline{\underline{B}}^T (\underline{\underline{Q}} + \underline{\underline{S}}) \underline{\underline{A}} \qquad \text{(A-7)}$$

$$\underline{\underline{C}}_u^{N-1} = \underline{\underline{B}}^T (\underline{\underline{Q}} + \underline{\underline{S}}) \underline{\underline{B}} + \underline{\underline{R}} \qquad \text{(A-8)}$$

$$\underline{\underline{C}}_d^{N-1} = \underline{\underline{B}}^T (\underline{\underline{Q}} + \underline{\underline{S}}) \underline{\underline{D}} \qquad \text{(A-9)}$$

$$\underline{\underline{C}}_y^{N-1} = -\underline{\underline{B}}^T (\underline{\underline{Q}} + \underline{\underline{S}}) \underline{\underline{C}}^T \qquad \text{(A-10)}$$

and the control law follows

$$\underline{u}_{N-1} = \underline{\underline{K}}_{FB}^{N-1} \underline{x}_{N-1} + \underline{\underline{K}}_{FF}^{N-1} \underline{d}_{N-1} + \underline{\underline{K}}_{SP}^{N-1} \underline{y}_d \qquad \text{(A-11)}$$

where

Chapter Five, Appendix A (continued)

$$\underline{\underline{K}}_{FB}^{N-1} = -[\underline{\underline{C}}_u^{N-1}]^{-1} \; \underline{\underline{C}}_x^{N-1} \qquad \text{(A-12)}$$

$$\underline{\underline{K}}_{FF}^{N-1} = -[\underline{\underline{C}}_u^{N-1}]^{-1} \underline{\underline{C}}_d^{N-1} \qquad \text{(A-13)}$$

$$\underline{\underline{K}}_{SP}^{N-1} = -[\underline{\underline{C}}_u^{N-1}]^{-1} \; \underline{\underline{C}}_y^{N-1} \qquad \text{(A-14)}$$

Now consider the N-1th control interval similarily,

$$\frac{\partial J}{\partial \underline{u}_{N-2}} = \frac{\partial}{\partial \underline{u}_{N-2}} \Big(\beta^N (\underline{x}_N - \underline{\underline{C}}^T \underline{y}_d)^T (\underline{\underline{Q}} + \underline{\underline{S}})(\underline{x}_N - \underline{\underline{C}}^T \underline{y}_d) + \beta^N \underline{u}_{N-1}^T \underline{\underline{R}} \underline{u}_{N-1}$$

$$+ \beta^{N-1} (\underline{x}_{N-1} - \underline{\underline{C}}^T \underline{y}_d)^T \underline{\underline{Q}} (\underline{x}_{N-1} - \underline{\underline{C}}^T \underline{y}_d) + \beta^{N-1} \underline{u}_{N-2}^T \underline{\underline{R}} \underline{u}_{N-2}\Big)$$

$$= 0 \qquad \text{(A-15)}$$

The following expressions for the coefficients result after substituting the state equation (A-1) and control law (A-11). It is also assumed that the load vector is constant i.e.

$\underline{d}_N = \underline{d}_{N-1} = \ldots = \underline{d}_1 = \underline{d}$,

$$\underline{\underline{C}}_x^{N-2} = \beta \underline{\underline{B}}^T \underline{\underline{T}}_{N-1}^T (\underline{\underline{Q}} + \underline{\underline{S}}) \underline{\underline{T}}_{N-1} \underline{\underline{A}} + \beta \underline{\underline{B}}^T \underline{\underline{K}}_{FB}^{N-1T} \underline{\underline{R}} \; \underline{\underline{K}}_{FB}^{N-1} \underline{\underline{A}} + \underline{\underline{B}}^T \underline{\underline{Q}} \; \underline{\underline{A}} \qquad \text{(A-16)}$$

$$\underline{\underline{C}}_u^{N-2} = \beta \underline{\underline{B}}^T \underline{\underline{T}}_{N-1}^T (\underline{\underline{Q}} + \underline{\underline{S}}) \underline{\underline{T}}_{N-1} \underline{\underline{B}} + \beta \underline{\underline{B}}^T \underline{\underline{K}}_{FB}^{N-1T} \underline{\underline{R}} \; \underline{\underline{K}}_{FB}^{N-1} \underline{\underline{B}} + \underline{\underline{B}}^T \underline{\underline{Q}} \; \underline{\underline{B}} + \underline{\underline{R}} \qquad \text{(A-17)}$$

Chapter Five, Appendix A (continued)

$$\underline{\underline{C}}_d^{N-2} = \beta \underline{\underline{B}}^T \underline{\underline{T}}_{N-1}^T (\underline{\underline{Q}} + \underline{\underline{S}}) \underline{\underline{T}}_{N-1} \underline{\underline{D}} + \beta \underline{\underline{B}}^T \underline{\underline{T}}_{N-1}^T (\underline{\underline{Q}} + \underline{\underline{S}})(\underline{\underline{B}}\, \underline{\underline{K}}_{FF}^{N-1} + \underline{\underline{D}})$$

$$+ \beta \underline{\underline{B}}^T \underline{\underline{K}}_{FB}^{N-1} \underline{\underline{R}}\, (\underline{\underline{K}}_{FB}^{N-1} \underline{\underline{D}} + \underline{\underline{K}}_{FF}^{N-1}) + \underline{\underline{B}}^T \underline{\underline{Q}}\, \underline{\underline{D}} \tag{A-18}$$

$$\underline{\underline{C}}_y^{N-2} = -\beta \underline{\underline{B}}^T \underline{\underline{T}}_{N-1}^T (\underline{\underline{Q}} + \underline{\underline{S}})(\underline{\underline{C}}^T - \underline{\underline{B}}\, \underline{\underline{K}}_{SP}^{N-1}) + \beta \underline{\underline{B}}^T \underline{\underline{K}}_{FB}^{N-1} \underline{\underline{R}}\, \underline{\underline{K}}_{SP}^{N-1} - \underline{\underline{B}}^T \underline{\underline{Q}}\, \underline{\underline{C}}^T \tag{A-19}$$

where the closed loop matrix

$$\underline{\underline{T}}_{N-1} = \underline{\underline{A}} + \underline{\underline{B}}\, \underline{\underline{K}}_{FB}^{N-1} \tag{A-20}$$

Now considering the N-2th control interval,

$$\frac{\partial J}{\partial \underline{u}_{N-3}} = \frac{\partial}{\partial \underline{u}_{N-3}} \Big(\beta^N (\underline{x}_N - \underline{\underline{C}}^T \underline{y}_d)^T (\underline{\underline{Q}} + \underline{\underline{S}})(\underline{x}_N - \underline{\underline{C}}^T \underline{y}_d)$$

$$+ \beta^N \underline{u}_{N-1}^T \underline{\underline{R}}\, \underline{u}_{N-1} + \beta^{N-1} (\underline{x}_{N-1} - \underline{\underline{C}}^T \underline{y}_d)^T \underline{\underline{Q}} (\underline{x}_{N-1} - \underline{\underline{C}}^T \underline{y}_d)$$

$$+ \beta^{N-1} \underline{u}_{N-2}^T \underline{\underline{R}}\, \underline{u}_{N-2}$$

$$+ \beta^{N-2} (\underline{x}_{N-2} - \underline{\underline{C}}^T \underline{y}_d)\, \underline{\underline{Q}}\, (\underline{x}_{N-2} - \underline{\underline{C}}^T \underline{y}_d)$$

$$+ \beta^{N-2} \underline{u}_{N-3}^T \underline{\underline{R}}\, \underline{u}_{N-3}\Big)$$

$$= 0 \tag{A-21}$$

Substituting state relations and control laws the following expressions

define the coefficients.

$$C_x^{N-3} = \beta^2 B^T T_{N-2}^T T_{N-1}^T (Q + S)\, T_{N-1}\, T_{N-2}\, A + \beta^2 B^T T_{N-2}^T\, K_{FB}^{N-1T}\, R\, K_{FB}^{N-1}\, T_{N-2} A$$

$$+ \beta B^T T_{N-2}^T Q\, T_{N-2} A + \beta B^T K_{FB}^{N-2T}\, R\, K_{FB}^{N-2} A \qquad \text{(A-22)}$$

$$C_u^{N-3} = \beta^2 B^T T_{N-2}^T T_{N-1}^T (Q + S)\, T_{N-1} T_{N-2} B + \beta^2 B^T T_{N-2}^T K_{FB}^{N-1T}\, R\, K_{FB}^{N-1}\, T_{N-2} B$$

$$+ \beta B^T T_{N-2}^T Q\, T_{N-2} B + \beta B^T K_{FB}^{N-2T}\, R\, K_{FB}^{N-2} B + B^T Q\, B + R \qquad \text{(A-23)}$$

$$C_d^{N-3} = \beta^2 B^T T_{N-2}^T T_{N-1}^T (Q + S) \left(T_{N-1} T_{N-2} D + T_{N-1} (B\, K_{FF}^{N-2} + D) + B\, K_{FF}^{N-1} + D \right)$$

$$+ \beta^2 B^T T_{N-2}^T\, K_{FB}^{N-1T} R \left(K_{FB}^{N-1}\, T_{N-2} D + K_{FB}^{N-1} (B\, K_{FF}^{N-2} + D) + K_{FF}^{N-1} \right)$$

$$+ \beta B^T T_{N-2}^T Q \left(T_{N-2} D + B\, K_{FF}^{N-2} + D \right) + \beta B^T K_{FB}^{N-2T}\, R\, (K_{FB}^{N-2} D + K_{FF}^{N-2})$$

$$+ B^T Q\, D \qquad \text{(A-24)}$$

$$C_y^{N-3} = \beta^2 B^T T_{N-2}^T T_{N-1}^T (Q + S) \left(T_{N-1} B\, K_{SP}^{N-2} + B\, K_{SP}^{N-1} - C^T \right)$$

$$+ \beta^2 B^T T_{N-2}^T K_{FB}^{N-1T}\, R \left(K_{FB}^{N-1} B\, K_{SP}^{N-2} + K_{SP}^{N-1} \right) + \beta B^T T_{N-2}^T Q (B\, K_{SP}^{N-2} - C^T)$$

$$+ \beta B^T K_{FB}^{N-2T}\, R\, K_{SP}^{N-2} - B^T Q\, C^T \qquad \text{(A-25)}$$

Further steps can be considered although a study of the coefficients

for the above three control intervals will reveal the following recursive relations.

$$\text{(i)}\quad \underline{\underline{C}}_x^{N-i} = \underline{\underline{B}}^T \underline{\underline{P}}_{i-1}\underline{\underline{A}} \qquad i = 1,\ldots,N \tag{A-26}$$

where

$$\underline{\underline{P}}_{i-1} = \beta \underline{\underline{T}}_{N-i+1}^T \underline{\underline{P}}_{i-2} \underline{\underline{T}}_{N-i+1} + \beta \underline{\underline{K}}_{FB}^{N-i+1T} \underline{\underline{R}}\, \underline{\underline{K}}_{FB}^{N-i+1} + \underline{\underline{Q}} \tag{A-27}$$

initialized by

$$\underline{\underline{P}}_o = \underline{\underline{Q}} + \underline{\underline{S}}$$

$$\text{(ii)}\quad \underline{\underline{C}}_u^{N-i} = \underline{\underline{B}}^T \underline{\underline{P}}_{i-1}\underline{\underline{B}} + \underline{\underline{R}} \tag{A-28}$$

$$\text{(iii)}\quad \underline{\underline{C}}_d^{N-i} = \underline{\underline{B}}^T \underline{\underline{P}}_{i-1}\underline{\underline{D}} + \underline{\underline{B}}^T \underline{\underline{O}}_{i-1} \tag{A-29}$$

where

$$\underline{\underline{O}}_{i-1} = \beta \underline{\underline{T}}_{N-i+1}^T \underline{\underline{O}}_{i-2} + \beta \underline{\underline{T}}_{N-i+1}^T \underline{\underline{P}}_{i-2} \left(\underline{\underline{B}}\, \underline{\underline{K}}_{FF}^{N-i+1} + \underline{\underline{D}}\right)$$

$$+ \beta \underline{\underline{K}}_{FB}^{N-i+1T} \underline{\underline{R}}\, \underline{\underline{K}}_{FF}^{N-i+1} \tag{A-30}$$

initialized by $\underline{\underline{O}}_o = 0$

$$\text{(iv)}\quad \underline{\underline{C}}_y^{N-i} = \underline{\underline{B}}^T \left(\underline{\underline{M}}_{i-1} - \underline{\underline{N}}_{i-1}\underline{\underline{C}}^T\right) \tag{A-31}$$

where

$$\underline{\underline{M}}_{i-1} = \beta \underline{\underline{T}}_{N-i+1}^T \left(\underline{\underline{P}}_{i-2}\underline{\underline{B}}\, \underline{\underline{K}}_{SP}^{N-i+1} - \underline{\underline{M}}_{i-2}\right) + \beta \underline{\underline{K}}_{FB}^{N-i+1T} \underline{\underline{R}}\, \underline{\underline{K}}_{SP}^{N-i+1} \tag{A-32}$$

initialized by $\underline{\underline{M}}_o = 0$ and where

$$\underline{\underline{N}}_{i-1} = \beta \underline{\underline{T}}^{T}_{N-i+1} \underline{\underline{N}}_{i-2} + \underline{\underline{Q}} \qquad \text{(A-33)}$$

initialized by $\underline{\underline{N}}_{o} = \underline{\underline{Q}} + \underline{\underline{S}}$. It follows from equations (A-12) to (A-14) that

$$\underline{\underline{K}}^{N-i}_{FB} = - (\underline{\underline{B}}^{T} \underline{\underline{P}}_{i-1} \underline{\underline{B}} + \underline{\underline{R}})^{-1} \underline{\underline{B}}^{T} \underline{\underline{P}}_{i-1} \underline{\underline{A}} \qquad \text{(A-34)}$$

$$\underline{\underline{K}}^{N-i}_{FF} = - (\underline{\underline{B}}^{T} \underline{\underline{P}}_{i-1} \underline{\underline{B}} + \underline{\underline{R}})^{-1} (\underline{\underline{B}}^{T} \underline{\underline{P}}_{i-1} \underline{\underline{D}} + \underline{\underline{B}}^{T} \underline{\underline{O}}_{i-1}) \qquad \text{(A-35)}$$

$$\underline{\underline{K}}^{N-i}_{SP} = - (\underline{\underline{B}}^{T} \underline{\underline{P}}_{i-1} \underline{\underline{B}} + \underline{\underline{R}})^{-1} (\underline{\underline{B}}^{T} \underline{\underline{M}}_{i-1} - \underline{\underline{B}}^{T} \underline{\underline{N}}_{i-1} \underline{\underline{C}}^{T}) \qquad \text{(A-36)}$$

These relations for the control matrices converge to give constant values as i increases [1]. They reduce to those presented in Lapidus and Luus [1] by substituting $\underline{d} = 0$, $\underline{\underline{S}} = 0$, and $\beta = 1$.

NOTE: In order that setting the differential to zero locates a minimum, double differentiation sets the condition that $\underline{\underline{R}}$ is positive definite. However, $\underline{\underline{R}}$ can be arbitrarily set infinitely small and in the limit to zero.

CHAPTER SIX

APPENDIX A

EQUIVALENCE OF STEADY STATE AND ERROR COORDINATE APPROACHES

The "error coordinate" approach to providing feedforward control in a multivariable system was introduced by Anderson [4]. A second order system is used below to show that the approach is equivalent to the steady state design approach which is perhaps conceptually easier to interpret. The equivalence is true for a system of any order.

1. Steady State Approach

Consider the system,

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_1 & a_3 \\ a_2 & a_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} d \tag{A-1}$$

with an optimal feedback control matrix calculated by dynamic programming.

$$u_{FB} = [k_1 \quad k_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{A-2}$$

The closed loop system can be written as follows

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_1 + b_1k_1 & a_3 + b_1k_2 \\ a_2 + b_2k_1 & a_4 + b_2k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u_{FF}$$

$$+ \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} d \tag{A-3}$$

The steady state approach is to set m states, where m is the dimension of the control vector, to their desired values, the state derivatives to zero, and evaluate the m control variables and $n - m$ states from the n equations.

In this case $m = 1$ so that putting $x_1 = x_{1d}$ and $\dot{x}_1 = \dot{x}_2 = 0$ gives the following set of equations

$$\therefore\ 0 = \begin{bmatrix} b_1 & a_3 + b_1 k_2 \\ b_2 & a_4 + b_2 k_2 \end{bmatrix} \begin{bmatrix} u_{FF} \\ x_2 \end{bmatrix} + \begin{bmatrix} a_1 + b_1 k_1 & d_1 \\ a_2 + b_2 k_1 & d_2 \end{bmatrix} \begin{bmatrix} x_{1d} \\ d \end{bmatrix} \tag{A-4}$$

which simplifies to:

$$\begin{bmatrix} u_{FF} \\ x_2 \end{bmatrix} = \frac{-\,1.0}{b_1 a_4 - b_2 a_3} \begin{bmatrix} a_4 + b_2 k_2 & -a_3 - b_1 k_2 \\ -b_2 & b_1 \end{bmatrix} \begin{bmatrix} a_1 + b_1 k_1 & d_1 \\ a_2 + b_2 k_1 & d_2 \end{bmatrix} \begin{bmatrix} x_{1d} \\ d \end{bmatrix} \tag{A-5}$$

from which

$$u_{FF} = -\frac{(a_4 + b_2 k_2)(a_1 + b_1 k_1) - (a_3 + b_1 k_2)(a_2 + b_2 k_1)}{b_1 a_4 - b_2 a_3}\, x_{1d}$$

$$-\frac{d_1 (a_4 + b_2 k_2) - d_2 (a_3 + b_1 k_2)}{b_1 a_4 - b_2 a_3}\, d \tag{A-6}$$

Chapter Six, Appendix A (continued)

2. Error Coordinate Approach

Error coordinates are introduced as

$$\bar{\underline{x}} = \underline{x} - \underline{x}_e \qquad \text{(A-7)}$$

and

$$\bar{\underline{u}} = \underline{u} - \underline{u}_e \qquad \text{(A-8)}$$

where $\underline{x}_e$ and $\underline{u}_e$ are equilibrium values such that at steady state

$$\underline{\underline{A}}\,\underline{x}_e + \underline{\underline{B}}\,\underline{u}_e + \underline{\underline{D}}\,\underline{d} = 0 \qquad \text{(A-9)}$$

and the new state equation can be written as follows

$$\dot{\bar{\underline{x}}} = \underline{\underline{A}}\,\bar{\underline{x}} + \underline{\underline{B}}\,\bar{\underline{u}} \qquad \text{(A-10)}$$

for which the optimal feedback matrix can be found by dynamic programming.

$$\bar{\underline{u}} = \underline{\underline{K}}_{FB}\,\bar{\underline{x}} \qquad \text{(A-11)}$$

The equilibrium vectors, $\underline{x}_e$ and $\underline{u}_e$, can be evaluated in terms of an m dimensioned partition of $\underline{x}_e$, $\underline{x}_{1e}$, and $\underline{d}$. This is detailed in the appendix of the paper by Anderson [4].

$$\begin{bmatrix} \underline{x}_{1e} \\ \cdots \\ \underline{x}_{2e} \\ \underline{u}_e \end{bmatrix} = \begin{bmatrix} \underline{\underline{I}} \\ \cdots\cdots\cdots\cdots \\ -[\underline{\underline{A}}_2 \quad \underline{\underline{B}}]^{-1}\ \underline{\underline{A}}_1 \end{bmatrix} \underline{x}_{1e} + \begin{bmatrix} 0 \\ \cdots\cdots\cdots\cdots \\ -[\underline{\underline{A}}_2 \quad \underline{\underline{B}}]^{-1}\ \underline{\underline{D}} \end{bmatrix} d \qquad \text{(A-12)}$$

where

$$[\underline{\underline{A}}_1 \quad \underline{\underline{A}}_2] \begin{bmatrix} \underline{x}_{1e} \\ \underline{x}_{2e} \end{bmatrix} + \underline{\underline{B}}\,\underline{u} + \underline{\underline{D}}\,\underline{d} = 0 \qquad \text{(A-13)}$$

Chapter Six, Appendix A (continued)

Using Equations (A-12), (A-7), (A-8) substituted into (A-11) gives a control law in terms of $\underline{x}$, $\underline{x}_{1e}$, and $\underline{d}$. Considering the second order system in Equation (A-1) then (A-12) can be written as follows

$$\begin{bmatrix} x_{1e} \\ \cdots \\ x_{2e} \\ u_e \end{bmatrix} = \begin{bmatrix} 1 \\ \cdots\cdots\cdots \\ -\begin{bmatrix} a_3 & b_1 \\ a_4 & b_2 \end{bmatrix}^{-1} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \end{bmatrix} x_{1e} + \begin{bmatrix} 0 \\ \cdots\cdots\cdots \\ -\begin{bmatrix} a_3 & b_1 \\ a_4 & b_2 \end{bmatrix}^{-1} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \end{bmatrix} d \qquad \text{(A-14)}$$

from which

$$\begin{bmatrix} x_{1e} \\ \cdots \\ x_{2e} \end{bmatrix} = \begin{bmatrix} 1 \\ \cdots\cdots\cdots \\ -\dfrac{b_1 a_2 - b_2 a_1}{a_4 b_1 - a_3 b_2} \end{bmatrix} x_{1e} - \begin{bmatrix} 0 \\ \cdots\cdots\cdots \\ \dfrac{b_1 d_2 - b_2 d_1}{a_4 b_1 - a_3 b_2} \end{bmatrix} d \qquad \text{(A-15)}$$

and

$$u_e = -\frac{a_1 a_4 - a_2 a_3}{a_4 b_1 - a_3 b_2} x_{1e} - \frac{a_4 d_1 - a_3 d_2}{a_4 b_1 - a_3 b_2} d \qquad \text{(A-16)}$$

The error coordinate system (A-10) results in the same control law as the original system

$$\bar{u} = (k_1 \quad k_2) \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} \qquad \text{(A-17)}$$

Substituting (A-7), (A-8), (A-15), and (A-16) into Equation (A-17) results in the control law

Chapter Six, Appendix A (continued)

$$u = (k_1 \; k_2)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + u_{FF} \qquad \text{(A-18)}$$

where

$$u_{FF} = -\left[\frac{a_1a_4 - a_2a_3 + k_1a_4b_1 - k_1a_3b_2 - k_2b_1a_2 + k_2b_2a_1}{a_4b_1 - a_3b_2}\right] x_{1e}$$

$$-\left[\frac{a_4d_1 - a_3d_2 - k_2b_1d_2 + k_2b_2d_1}{a_4b_1 - a_3b_2}\right] d \qquad \text{(A-19)}$$

which, since the values of a_i and k_i are the same in both derivations, is exactly equivalent to Equation (A-6) derived by the alternate procedure.